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Carrier Synchronization Based on Renyi's Entropy

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***Abstract*— We consider the problem of carrier frequency recovery for linearly, digitally modulated signals. Presented algorithm can be applied before modulation classification and/or demodulation of the M-ary PSK and M-ary QAM signals. It is based on the properties of the Renyi's entropy of the instantaneous phase probability density function, and uses the fact that it reaches minimum when the receiver is fine-tuned to unknown carrier frequency. This estimator is applicable to algorithms requiring high accuracy without a priori knowledge concerning modulation scheme, signal contents nor its timing parameters.**

I. Introduction

For at last two decades, automatic modulation recognition [1]–[6] is a topic of interest for the scientific community working on both military and commercial communication systems. Performance and complexity of these algorithms depend on the number of unknown parameters in the intercepted transmission. One of the most important signal features is its carrier frequency, since it allows to stabilize the constellation of a signal and then to recognize underlying modulation type.

Most of the published papers in the field of carrier recovery deal with the cases where some signal parameters are known. In [7], authors obtained their estimator in case where training bits and symbol timing are known. Similar assumptions were made in [8], where perfect symbol synchronization, no inter-symbol interferences (ISI), and known symbol rate were taken into considerations. Low estimator's variance was observed in a two-step approach proposed in [9], where the estimators of unknown channel characteristics and frequency offset are based on the information provided by the second or fourth-order cyclostationary (CS) statistics. In [10], authors presented Data-Aided (DA) Maximum Likelihood (ML) estimator which performance is close to the Modified Cramér-Rao Bound (MCRB) for low Signal to Noise Ratio (SNR) and when data sequence is known. They compared their method to standard ML algorithms proposed by Kay [11] and Fitz [12].

In this paper we propose a Non-Data-Aided (NDA), asynchronous approach, i.e. there is no preamble sequence available nor prior knowledge concerning the data-stream and timings. In the following sections, we assume the signal to be MPSK or MQAM without any additional information concerning the number of states, initial carrier phase, nor the transmission baud rate.

II. Signal Model

Let us assume that the received complex signal can be expressed as a sum of two uncorrelated components

$$r(t) = Ax(t)\mathrm{e}^{j(\omega_c t+\Theta_c)} + z(t) \tag{1}$$

where $x(t)$ is a signal complex envelope, A is a carrier amplitude, ω_c is a carrier frequency, Θ_c is a carrier phase, and $z(t)$ corresponds to a complex, zero mean, Additive White Gaussian Noise (AWGN).

Using the concept of the complex envelope, one can express any linearly modulated signal as

$$x(t) = \sum_{k=1}^{K} d_k h(t-kT) \tag{2}$$

where K denotes the number of observed symbols, T is a symbol duration, $h(t)$ is a pulse shaping function, and d_k describe constellation of a signal

$$d_k^{\text{MPSK}} = \mathrm{e}^{j\varphi_k}, \quad d_k^{\text{MQAM}} = a_k + jb_k \tag{3}$$

$$\varphi_k \in \{\tfrac{2\pi}{M}(m-1) : \ m = 1, 2, \ldots, M\} \tag{4}$$

$$a_k, b_k \in \{\pm(2m-1) : \ m = 1, 2, \ldots, \log_2(M)-2\} \tag{5}$$

Without loss of generality, we assume that all modulation states (φ_k, a_k, b_k) are equiprobable and the pulse shaping function $h(t)$ is rectangular (or $x(t)$ is the output of a matched filter $h^*(-t)$).

III. Instantaneous Phase Probability Density Function

By substituting $x(t) = 1$ in (1), one can obtain an analytic form of a Carrier Wave (CW) signal. If ω_c and Θ_c are exactly known, then after complex mixing with a local oscillator $e^{-j(\omega_{\text{LO}}t+\Theta_{\text{LO}})}$ $(\omega_{\text{LO}} = \omega_c, \ \Theta_{\text{LO}} = \Theta_c)$ one obtains a complex baseband signal, which probability density function (PDF) of its instantaneous phase ψ (IP) may be written as in [13]

$$\begin{aligned} p_{\text{cw}}(\psi;\,\gamma) = \frac{\mathrm{e}^{-\gamma}}{2\pi} + \frac{\mathrm{e}^{-\gamma}}{2}\sqrt{\frac{\gamma}{\pi}}\cos(\psi)\mathrm{e}^{\gamma\cos^2(\psi)} \\ \cdot\{1 + \text{erf}\,[\sqrt{\gamma}\cos(\psi)]\}, \quad \psi \in [-\pi,\,\pi[\end{aligned} \tag{6}$$

or in terms of a Fourier series ([14], [15]) as

$$p_{\text{cw}}(\psi;\,\gamma) = \frac{1}{2\pi}\left[1 + \sum_{l=1}^{\infty}\alpha_l(\gamma)\cos(l\psi)\right] \tag{7}$$

with the Fourier series coefficients

$$\alpha_l(\gamma) = \sqrt{\pi\gamma}\, \mathrm{e}^{-\frac{\gamma}{2}} \left[I_{\frac{l-1}{2}}\left(\frac{\gamma}{2}\right) + I_{\frac{l+1}{2}}\left(\frac{\gamma}{2}\right) \right] \qquad (8)$$

where $\mathrm{erf}(x) \triangleq \frac{2}{\sqrt{\pi}} \int_0^x \mathrm{e}^{-t^2} dt$ is an error function, σ_z^2 is a noise variance, $\gamma = \frac{A^2}{2\sigma_z^2} = 10^{\mathrm{SNR}/10}$, and $I_l(x)$ is the modified Bessel function of order l.

When carrier frequency ω_c is unknown, one can write resulting PDF as a function of time t and frequency error $\Delta_\omega = \omega_c - \omega_{LO}$ as

$$p_{\mathrm{cw}}^{\Delta_\omega}(\psi, t; \gamma) = p_{\mathrm{cw}}(\psi + \Delta_\omega t; \gamma) \qquad (9)$$

Finally, an asymptotic distribution of IP during observation period T_o can be expressed as

$$p_{\mathrm{cw}}^{T_o}(\psi; \gamma) = \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} p_{\mathrm{cw}}^{\Delta_\omega}(\psi, t; \gamma)\, dt \qquad (10)$$

and using (7) and (10), it is straightforward to write

$$p_{\mathrm{cw}}^{T_o}(\psi; \gamma) = \frac{1}{2\pi} \left[1 + \sum_{l=1}^{\infty} \frac{\sin(\frac{l\Delta_\omega T_o}{2})}{\frac{l\Delta_\omega T_o}{2}} \alpha_l(\gamma) \cos(l\psi) \right] \qquad (11)$$

In the general case of MPSK and MQAM constellations, and initial phase errors $\Delta_\Theta = \Theta_c - \Theta_{LO}$, one can write resulting PDF as

$$p(\psi) = \frac{1}{M} \sum_{k=1}^{M} p_{\mathrm{cw}}^{T_o}\left(\psi + \arg\{d_k\} + \Delta_\Theta; \gamma |d_k|^2\right) \qquad (12)$$

IV. Renyi's Entropy and Objective Function

The entropy of a random variable X is a quantitative measure of the randomness of the corresponding experiment. The most known definitions are the Shannon's entropy [16]

$$H_S = -\int_{-\infty}^{+\infty} p(x) \log[p(x)]\, dx \qquad (13)$$

and the Renyi's entropy [17]

$$H_R^\alpha = \frac{1}{1-\alpha} \log \left[\int_{-\infty}^{+\infty} p^\alpha(x)\, dx \right], \alpha > 0,\, \alpha \neq 1 \qquad (14)$$

It is known that

$$\lim_{\alpha \to 1} H_R^\alpha = H_S, \quad H_R^\beta \geqslant H_S \geqslant H_R^\gamma \qquad (15)$$

for $0 < \beta < 1$ and $1 < \gamma$, so, Shannon's entropy can be viewed as a member of Renyi's entropy family [18]. When $\alpha = 2$, Renyi's entropy H_R^2 is also called quadratic entropy

$$H_Q = -\log \left[\int_{-\infty}^{+\infty} p^2(x)\, dx \right] \qquad (16)$$

Using the relations (11), (16), and the orthogonality of a Fourier series expansion in the range $[-\pi, \pi[$

$$\int_{-\pi}^{+\pi} \left(\sum_{i=0}^{+\infty} a_i \cos(it) \right)^2 dt = \pi \left(2a_0^2 + \sum_{i=1}^{+\infty} a_i^2 \right) \qquad (17)$$

the quadratic entropy of the IP PDF can be expressed as

$$H_Q^{\mathrm{cw}} = -\log \left[\frac{1}{2\pi} \left(1 + \frac{1}{2} \sum_{l=1}^{+\infty} \mathrm{Sinc}\left(\tfrac{l\Delta_\omega T_o}{2}\right)^2 \alpha^2(\gamma) \right) \right] \qquad (18)$$

where $\mathrm{Sinc}(x) \triangleq \sin(x)/x$. For the MPSK and MQAM signals, corresponding quadratic entropies can be found via the equations (11), (12) and (16).

It is evident that H_Q^{cw} in (18) reaches its global minimum when $\Delta_\omega T_o = 0$, so the carrier synchronization can be viewed as the minimum searching

$$\widehat{\omega}_c = \min_\omega [H_Q] \qquad (19)$$

and equally as

$$\widehat{\omega}_c = \min_\omega [Q] \qquad (20)$$

where the objective function Q is defined as

$$Q \triangleq \left[\int_{-\infty}^{+\infty} p^2(x)\, dx \right]^{-1} \qquad (21)$$

To provide an insight how the objective function Q changes for different signals, numerical integration was conducted and the corresponding results (for CW, BPSK, QPSK, 16QAM, and 32QAM) are presented in the figures 1 (as a function of $\Delta_\omega T_o$) and 2 (as a function of SNR).

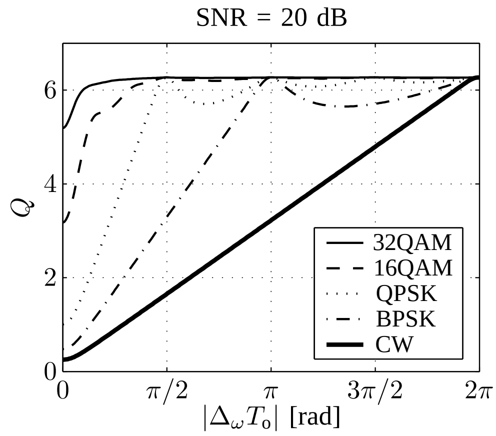

Fig. 1. Objective function Q for different frequencies

It is worth noticing, that both quadratic entropy H_Q, and the objective function Q depend only on the shape of the IP PDF. Neither the amplitude of the received signal nor its initial carrier and local oscillator phases do not affect these values.

V. Algorithm Description

The algorithm can be decomposed on two main parts: a raw estimation, and a fine-tuning part. The aim of the first part is to provide the approximate value of the carrier frequency around which, "the fine-tuning part" can search for the minimum value of the objective function Q.

First of all, the algorithm estimates the Power Spectral Density (PSD) of a signal by using Welch [19] modified periodogram method. Next, an heuristic threshold is applied

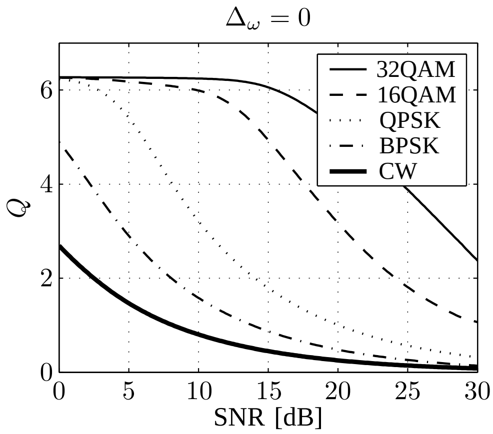

Fig. 2. Objective function Q for different SNR

to the PSD to extract only the meaningful part of the signal spectrum. Finally, the mean value of the extracted part is calculated. This value is used as the raw carrier frequency estimator around which the Frequency Raster is constructed (the surroundings of the mean frequency).

The fine-tuning part is implemented as a downconversion with the frequencies chosen from the Frequency Raster. Resulting baseband signal samples are used to extract the IP PDF and calculate Q. Finally, the minimum searching algorithm is applied to find the minimum value.

One has to pay attention on two topics: the minimum searching algorithm and the computational complexity. The fact that we did not assumed any a priori knowledge concerning the location of the true carrier frequency, causes the necessity of applying the ”raw estimation part”. The quality of this estimator is limited by the resolution of the PSD and to overcome this limit, it is necessary to make the Frequency Raster adequately large (large neighborhood and large number of frequencies). This implicates that there is a sharp peak in the Frequency Raster (corresponding to the minimal entropy), and the rest of it is almost constant. Such a minimum is hard to find using classical gradient techniques, and as a consequence, on has to apply the linear search algorithm instead of more efficient ones. To sum up, the errors due to first part of the method increase the size of the Frequency Raster what makes the application of any gradient techniques impossible and the computational complexity important. The methods of reducing the influence on performance of these factors are currently examined.

VI. Experimental Results

The performance of the proposed carrier frequency estimation algorithm was assessed by computer simulations using the following scenario: CW, BPSK, QPSK, 16QAM, and 32QAM modulation types; 5 and 10 samples per symbol, 100 and 400 symbols in the signal; 1000 trials for each signal, source signals were modeled as uniformly distributed on all constellation states; additive noise was modeled as Gaussian; SNR was varying from 0 to 30 dB with the step of 2 dB; sampling frequency F_s was equal 8 kHz.

The experimental results have proved that the proposed estimator is unbiased and independent of the baud rate (number of samples per symbol) for all signals. Variances of the normalized frequency error

$$\mathrm{Var}(\delta_\omega) = \mathrm{Var}\left(\frac{\widehat{\omega}_c - \omega_c}{2\pi F_s}\right) \quad (22)$$

for all signals and different SNR values are presented in the figures 3 (100 symbols, 5 samples per symbol) and 4 (400 symbols, 10 samples per symbol).

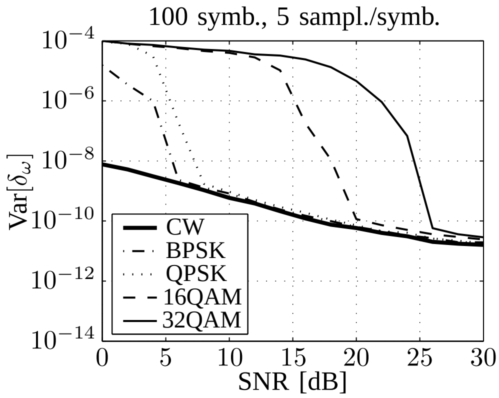

Fig. 3. Experimental results I

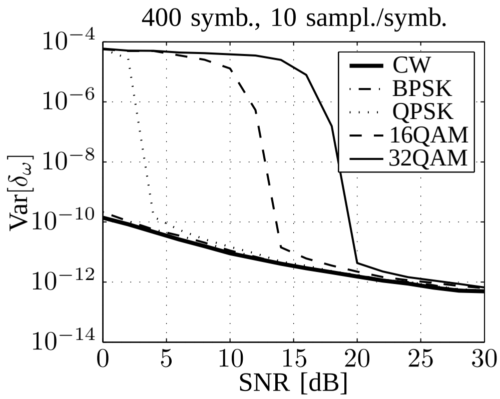

Fig. 4. Experimental results II

VII. Conclusion

Our new algorithm can be applied for a whole branch of digital transmissions – it estimates the mean frequency of the signal spectrum (raw estimation part), and in the case of the linearly, digitally modulated signals as MPSK and MQAM – it estimates the corresponding carrier frequency. It is independent of the initial carrier and local oscillator phases, as well as timings. The a priori knowledge of the constellation shape or number of characteristic states is not necessary for correct estimation of the carrier frequency (providing that SNR is big enough).

Increasing the number of available symbols and the number of samples per symbol, greatly improve performance of the method – for almost all of the signals the gain is ≈ 6 dB for the correct estimation of carrier frequency, and the final relative variance $\text{Var}[\delta_\omega]$ improves almost 1000 times.

Further work will be conducted to improve the quality of the ”raw estimator” (e.g. using a Yule-Walker AR modeling to estimate the PSD), ”fine-tuning part” (e.g. two or more Frequency Rasters), as well as the computational overhead.

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