# Navigation by weighted Chance

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## Abstract

This paper deals with the problem of how to control the movement of a simple robot which has the goal to reach a specified target within finite time and to stay within some pre-defined distance to it. The system's design proposed is as minimal as possible and reflects the basal reflex arc as observed in biological systems. The dynamics is due to a multiplicatively modified random walk. In particular only one simple, omni-directional sensor is used so that the robot does not receive any directional information about the target. The mobile robot shows a reliable and fast homing behavior towards a defined area and stays in some given neighborhood of it. The computational effort needed is seen to be minimal.

**Keywords**: mobile simple robot, system's control, stochastic approach, low-dimensional control, simple and fast algorithm.

## Introduction

The motion of simple animals, such as protozoa, bacteria, up to insects, is commonly regarded as a kind of random walk. Correspondingly, diffusion-reaction like processes have been considered in order to describe their fundamental motion patterns up to the emergence of grouping behavior (for further reading see [6]). Moreover, the assumption of random movement has made thermodynamic considerations a natural tool for analyzing systemic properties. The assumption of random movement certainly is reasonable, when considering a mobile system having a large number of degrees of freedoms, but also smaller systems with quasi-periodic or chaotic behavior [3].

We followed this idea of a random walk as a simple model for the motion of an *agent*. As a technical example, one may think about a simple **mobile robot**, which can move in a simple environment, for example an infinite, smooth plane, having a motor **E** of an appropriate number of degrees of freedom. According to classical mechanics, its spatial state is given by its spatial coordinates $\mathbf{Q} \in \mathfrak{R}^{\mathbf{d}}$, $d = 2,3$ and its momentum $\mathbf{P} \in \mathfrak{R}^{\mathbf{d}}$. The action of its motor is to change its spatial state due to

$$F:\begin{pmatrix} Q \\ P \end{pmatrix} \rightarrow \begin{pmatrix} Q' \\ P' \end{pmatrix} := \begin{pmatrix} Q + \tau P \\ D(\alpha)P \end{pmatrix},$$

where $\tau \in \mathfrak{R}$ is some scaling constant and $\mathbf{D}(\alpha)$ denotes a rotation of the momentum around some randomly chosen angle $\alpha \in [\, 0, 2\,\pi\,]$. The corresponding dynamics simply is a random walk in $\mathfrak{R}^{\mathbf{d}}$.

As the *agent*'s sensory pole **S**, we considered an *omni-sensor*, which is sensitive to light (of some frequency). According to the intensity measured, the sensor will produce some electrical signal $\boldsymbol{\nu}$, which is supposed to be

transferred to the motor along the pathway $\mathbf{S} \rightarrow \mathbf{E}$. Note that accordingly the sensory signal induced does not contain any directional information.
The motor can be thought to be either autonomous or dependent: if autonomous, its activity does not depend on the signal coming from the sensor (det $\mathbf{D}(\alpha)$ is independent of the sensory signal $\boldsymbol{\nu}$). But if the motor is dependent, det $\mathbf{D}(\alpha)$ may be some function of $\boldsymbol{\nu}$. For simplicity, it is assumed that the absolute value of the robot's velocity is constant, i.e. independent of $\boldsymbol{\nu}$, and the effector action exclusively consists in changing the direction of the momentum randomly.

As our key assumption, we assume that the *agent* has an internal component **I** whose states are called the *internal states* of the *agent*. The role of internal state, *essential variables*, was already mentioned by R.W. Ashby [1,2]. A simple model for the adaptive regulation of cells by modulation of sensitivity was analyzed in [7]. More general considerations of the biological background can be found in [8].

Receptive signals are supposed to affect the *agent*'s internal state according to some function *g*, so that for each position $\mathbf{Q} \in \mathfrak{R}^{\mathbf{d}}$ corresponds an internal state $x = g(Q) \in X$. Let $Q'$ denote the agent's next spatial position due to the dynamics defined above. Then the internal state $x'$ corresponding to this new position is a function of the coordinates $(Q, P)$. As such the evolution of the *agent*'s internal state is related to its spatial movement.

Further, we assume that there exists a set $Y \subset X$ of "essential" internal states, which is called the "homeostatic range" of the *agent*. The homeostasis condition is that during its dynamics, the internal state of the *agent* has to be kept close to this homeostatic range. As the distance measure, define the distance between the internal state $x \in X$ and the homeostatic range $Y \subset X$, i.e. $d(x, Y) = 0$ if and only if $x \in Y$, i.e. if the internal state is homeostatic. A direction $P$ is regarded as "GOOD", if the internal state related to the new position $Q'$ is closer to the homeostatic range than that related to the former position $Q$. This "weight" is formally defined as:

$$c(Q,P) := \begin{cases} +1 \text{ iff } d(x,Y) \geq d(x',Y) \\ -1 \quad \text{else,} \end{cases}$$

Suppose that the actual position of the *agent* is $(Q,P)$. Then define the forward cone

$$K_+ := \left\{ p \in \mathfrak{R}^d : A < \langle p, P \rangle P^2 \right\},$$

where $\langle\ ,\ \rangle$ denotes the ordinary the scalar product on $\mathfrak{R}^{\mathbf{d}}$ and $0 < A < P\hat{}2$. Obviously, A is related to the opening "angle" of the cone. Analogously, K- is defined by the property - P^2 < $\langle$p, P$\rangle$ < - A. The modified model then is:

$$F_c : \begin{pmatrix} Q \\ P \end{pmatrix} \rightarrow \begin{pmatrix} Q' \\ P' \end{pmatrix} := \begin{pmatrix} Q + \tau P \\ c(Q,P) D(\alpha) P \end{pmatrix},$$

where $\tau$ has the same meaning as before, but the rotation angle $\alpha$ is randomly chosen from the interval
[- arccos($A/P^2$), arccos($A/P^2$)].

In words: the *agent* proceeds moving in its former direction, $P' \in K_+$ if this direction is GOOD, otherwise the movement of the robot is reversed, $P' \in K_-$. The random variable $\alpha$ represents the "internal noise" of the robots itself in that its velocity is only determined up to some extent, represented by A. If $A = P^2$, the dynamics is completely deterministic, i.e. the robot either maintains or precisely reverses its former direction.

The mapping (3) does not represent a diffusion process of Langevin type.To induce noise to a (deterministic) system, a *Langevin* force term is commonly *added*, which has to fulfill certain stochastic properties (vanishing average and δ-correlation). In contract, the modulation of the random walk in our approach is {\em multiplicative}. By comparing the two functions (1) and (2), one immediately sees that

$$F_c(Q,P) = c_*(Q,P).F(Q,P), \qquad c_*(Q,P) := \left(1 \quad c(Q,P)\right)$$

Therefore, $F_c$ can not be written as a random walk F to which an external driving force is superimposed. In fact, the dynamics of the *agent* may be regarded as a random dynamics in a gradient field. But this gradient field is internally defined, rather than externally: By the mapping g: U → X, the external signaling space U is mapped to an internal space X, on which the "force term" is defined. Therefore, this force can be regarded as being "generated" by the system itself.

## Numerical results

In the following, only some particular properties of the dynamics of the so-defined system will be considered. The model proposed represents an extension and generalization of the approach mentioned by O.E. Holland and C.R. Melhuich [5]. A mathematical analysis of the mapping including stability of the invariant set and the discussion of ergodicity are beyond the scope of this work. The main aim of this part is to very roughly compare the random walk F with the "weighted" random walk $F_c$.

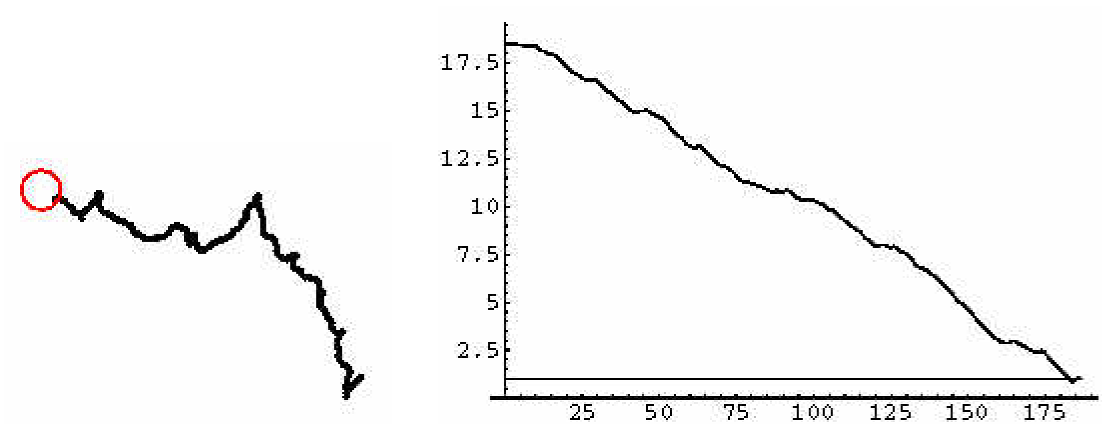


Figure 1: Phaseplot of the weighted random motion of the agent and the time-development of its spatial distance from the source, due to equation 3. The *agent*'s initial position is far away from the source.

The above figures display the spatial motion of an *agent* due to the mapping $F_c$, being subject to a signaling field emitted by some source, which is located inside the circle. The simulation was done for a very simple model: as the signaling source, we defined a light bulb of constant intensity, so that the signaling field emitted is proportional to the field of light intensity. Accordingly, the strength of the receptive signals was considered as a monotonously decreasing function of the spatial position of the agent. The action of its effector, i.e. the motor, was assumed to be autonomous, leaving the velocity unchanged, det $\mathbf{D}(\alpha) = 1$. Moreover, the action of the receptive signals on the internal state was assumed to be strictly monotonous, i.e. the mapping *g* was assumed to be strictly monotonous. As such, the disc displayed below reflects the homeostatic range of the agent in the spatial domain. In the simulations, the disc was assumed to have a finite extension, $0 < d < \infty$.

In contrast to a pure random walk, the trajectory due to $F_c$ can be seen composed by parts *of perturbed straight lines*, so that its motion becomes directed towards the target in the mean. First of all, it is clear that no real straight lines can occur because of the random choice of the rotation matrix. But apart from this, the segments have to be lines because of our definition of the weight function, which roughly says that a certain

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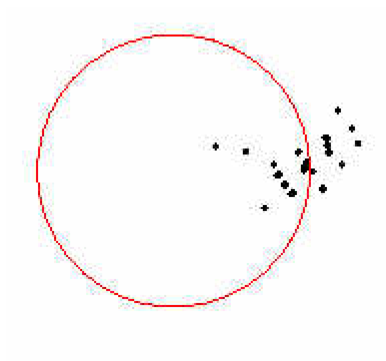
direction is maintained under certain conditions. The appearance of "straight" trajectories also can not be expected in pure random walks.



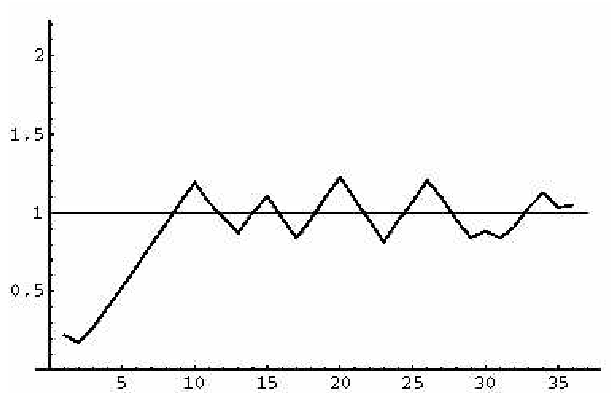



Figure 2: Phaseplot of the weighted random motion of the agent and the time-development of its spatial distance from the source, due to equation 3. The *agent*'s initial position is close to the source.

As well-known, purely diffusive processes in the plane have the mixing property according to which each trajectory of a purely diffusive system will meet any arbitrary small neighborhood of every point in the plane after a sufficient long time. Therefore, "homing behavior", i.e. reaching the disc, is trivially achieved by a random-walk dynamics. In fact, as apparent from the Figure 2, the agent reaches the source after a couple of iteration steps, its time development being due to $F_c$. For the same reason, a "purely" diffusive *agent* will leave every disc of finite radius d after some time $t \approx d^{-2}$. Mapping (3) can be seen not to be mixing, in the opposite: According to the dynamics defined by $F_c$, the *agent* will not escape from a finite neighborhood of the target, but will remain in some finite distance of the disc for all time. Actually, the spatial trajectory of the *agent* shows an oscillation around the border, its amplitude being dependent on the initial velocity of the agent and the time scale parameter. This, in fact, constitutes a major difference between the pure random walk F and our "weighted random walk" $F_c$.

## Conclusion

The model proposed represents a multiplicative modulation of a simple random walk: At each time-step, the direction is chosen randomly from a forward- or a backward cone according to the actual direction, due to the value of a weight function.

The forcing term is internal, rather than external, i.e. no external force field is superimposed. This "weighted" random walk exhibits dynamical aspects, which fundamentally differ from those of normal random walks:

1.) The dynamics of the model proposed is not "mixing", but establishes a motion, which converges to some given target.

2.) The mean direction is directed towards the target.

The computational effort needed is seen to be minimal and, in particular, does not include any "orientation", i.e. no direct directional information is present. This process of "weighted diffusion" may be important as a framework for describing and analyzing the motion pattern of simple animals.

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## References

1. R.W. Ashby, *Design for a brain*, John Wiley and Sons, 1952;
2. R.W. Ashby: *An Introduction to Cybernetics*. John Wiley, New York;
3. C. Beck, F. Schl\"ogl, *Thermodynamics of chaotic systems*, Cambridge Nonlinear Science Series **4**, Cambridge University Press, 1993;
4. C.W. Gardiner: *Handbook of Stochastic Methods for Physics, Chemistry, and the Natural Sciences*, Springer, New York, 1993;
5. O.E. Holland, C.R. Melhuish: *Getting the Most from the Least: Lessons for the Nanoscale from the Minimal mobile Agents*, ALIFE V, 1996;
6. J.D.Murray, *Mathematical Biology*, Springer Verlag, 1989;
7. S. Reimann: *Oscillation and pattern formation in a system of self-regulating cells*, Physica D, 114, (338-361), 1998;
8. S. Reimann, *Stability and equilibrated structure*, Proceeding of the 3-rd German Workshop on Artificial Life, (edd C. Wilke, S. Altmeyer, Th. Martinez), Harry Deutsch Verlag, 1998.