Instantaneous MISO Separation of BPSK Sources

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Abstract. We present a new approach for signal separation from an undetermined instantaneous mixture of two BPSK (Binary Phase Shift Keying) signals in AWGN (Additive White Gaussian Noise) channel. The method uses frequency diversity of the mixture (frequency shift between carriers) and the fact that signals of interest are binary variables. We compare separation results of our method to a theoretical BER (Bit Error Rate) of unmixed signals, which reveal algorithm's performance for different communications scenarios.

1 Introduction

The problem of blind source separation (BSS) has been intensively studied in the literature and many effective solutions have been proposed in the case of instantaneous mixtures (memoryless channel) [1-4] and convolutive mixtures (channel effects can be considered as a linear filter) [5-9]. Most of the proposed algorithms deal with an undercomplete case (the number of sensors is equal or greater to the number of sources). For more sources than mixtures [10-14], the BSS problem is said to be overcomplete (undetermined) and is ill-posed.

In general, separation of overcomplete mixtures is still a real challenge for the scientific community. Even though the methods of identifying instantaneous mixing coefficients for n sources have been developed [15], they need at least 2 sensors. The same assumptions limit the method of separating undetermined mixtures proposed in [16] (two or more sensors).

In this contribution, the case of one sensor and two sources (undetermined problem) is addressed. The mixture is considered to be an instantaneous (memoryless channel) and the sources to be linearly modulated digital signals. Such a scenario can be found in a cellular phone reception, satellite transmissions, as well as in military communications (eg. signal interception, jamming or countermeasure). We present a new blind separation algorithm adopted to deal with BPSK signals, closely distributed in a frequency domain, as well as a method of identifying mixing coefficients. Experimental results reveal the performance of the proposed method.

J. Rosca et al. (Eds.): ICA 2006, LNCS 3889, pp. 862–867, 2006.

2 Signal Model

Let us consider a linear, instantaneous mixture $x(t)$ of two BPSK-type signals

$$x(t) = a_1 s_1(t) \mathrm{e}^{i(\omega_1 t + \varphi_1)} + a_2 s_2(t) \mathrm{e}^{i(\omega_2 t + \varphi_2)} \tag{1}$$

where a_k are unknown mixing coefficients, ω_k are carrier frequencies, φ_k are equivalent phases (sum of carrier and mixing coefficients phases), and $s_k(t)$ are equiprobable, i.i.d. random sequences during symbol period T_k[1]:

$$s_k(t) \in \{+1, -1\}, \quad \text{for} \quad t \in [l, (l+1)T_k] \tag{2}$$

We assume that source signals $s_k(t)$ are independent of each other and carrier frequencies ω_k are distinct and can be estimated [17, 18]. We consider scenario with small (compared to baud rates) frequency shifts, that any separation method based on signal filtering [19, 20] can't be applied. In a particular case where both signals are at the same frequency, the separation can be achieved using algorithm proposed in [21].

3 Theoretical Development

The basic idea of our algorithm consists of using frequency diversity of the mixture and the fact that signals of interest are binary variables. Assuming that carrier frequencies ω_k are already known, the mixing coefficients can be estimated using auxiliary signals defined as

$$Z_k(t) = x(t)\mathrm{e}^{-i\omega_k t} = a_k s_k(t) \mathrm{e}^{i\varphi_k} + a_l s_l(t) \mathrm{e}^{i((\omega_l - \omega_k)t + \varphi_l)} \tag{3}$$

for $k, l \in \{1, 2\}, k \neq l$, and mean values of its squares

$$\begin{aligned} \mathcal{E}\{Z_k^2(t)\} &= a_k^2 \mathcal{E}\{s_k^2(t)\} \mathrm{e}^{i2\varphi_k} + a_l^2 \mathcal{E}\{s_l^2(t)\} \mathcal{E}\left\{\mathrm{e}^{i2((\omega_l - \omega_k)t + \varphi_l)}\right\} \\ &+ 2a_k a_l \mathcal{E}\{s_k(t) s_l(t)\} \mathcal{E}\left\{\mathrm{e}^{i((\omega_l - \omega_k)t + \varphi_k + \varphi_l)}\right\} \end{aligned} \tag{4}$$

Using the fact that source signals are independent, and assuming that observation time is big enough, one has

$$\mathcal{E}\{s_k(t) s_l(t)\} = 0, \quad \text{and} \quad \mathcal{E}\left\{\mathrm{e}^{i2((\omega_l - \omega_k)t + \varphi_l)}\right\} = 0 \tag{5}$$

thus equation (4) becomes

$$\mathcal{E}\{Z_k^2(t)\} = a_k^2 \mathcal{E}\{s_k^2(t)\} \mathrm{e}^{i2\varphi_k} \tag{6}$$

For considered BPSK-type signals $\mathcal{E}\left\{s_k^2(t)\right\} = 1$, the mixing coefficients can be estimated as

$$\hat{a}_k = \sqrt{\left|\mathcal{E}\left\{Z_k^2(t)\right\}\right|}, \quad \text{and} \quad \hat{\varphi}_k = \tfrac{1}{2} \arg\left[\mathcal{E}\left\{Z_k^2(t)\right\}\right] \tag{7}$$

[1] The case of general, linear digital modulations ($s_k(t) \in \mathbb{C}$), as well as convolutive mixtures (channel effects taken into considerations) are our current topics of interest.

One should pay attention to the sign ambiguity which occurs when equivalent phases $|\varphi_k|$ are bigger than π, i.e.

$$\arg\left[\mathrm{e}^{i2(\varphi_k+m\pi)}\right] = \arg\left[\mathrm{e}^{i2\varphi_k}\right], \quad s_k(t)\mathrm{e}^{i(\omega_k t+\varphi_k+m\pi)} = (-1)^m s_k(t)\mathrm{e}^{i(\omega_k t+\varphi_k)}$$

for $m \in \mathbb{Z}$, thus the opposite sign signals can be observed (i.e. $\hat{s}_k = -s_k$). This ambiguity can be eliminated only during the modulation stage, e.g. applying differential modulation as DPSK (Differential Phase Shift Keying) instead of absolute BPSK type.

Once we estimated mixing parameters, the separation problem can be simplified to the solution of a linear system of equations. Let $\alpha = (\omega_1 - \omega_2)t + \hat{\varphi}_1 - \hat{\varphi}_2$, then one can find auxiliary signals $X_k(t)$ as

$$X_k(t) = x(t)\mathrm{e}^{-i(\omega_k t+\hat{\varphi}_k)} = a_k s_k(t) + a_l s_l(t)\mathrm{e}^{-i\alpha} \tag{8}$$

and the estimators of the original signals $s_k(t)$ by

$$\hat{s}_1(t) = \tfrac{1}{\hat{a}_1}\left[X_1(t) - \hat{a}_2 s_2(t)\mathrm{e}^{-i\alpha}\right], \quad \hat{s}_2(t) = \tfrac{1}{\hat{a}_2}\left[X_2(t) - \hat{a}_1 s_1(t)\mathrm{e}^{i\alpha}\right] \tag{9}$$

For BPSK-type signals ($s_k(t) = \pm 1$), previous equations become

$$\hat{s}_1(t) = \tfrac{1}{\hat{a}_1}\left[X_1(t) \pm \hat{a}_2\mathrm{e}^{-i\alpha}\right], \quad \hat{s}_2(t) = \tfrac{1}{\hat{a}_2}\left[X_2(t) \pm \hat{a}_1\mathrm{e}^{i\alpha}\right] \tag{10}$$

To eliminate the sign ambiguity, we propose a "solution selector" based on the minimization of the following instantaneous objective function

$$Q(t, \epsilon_1, \epsilon_2) = \left|x(t) - \hat{a}_1\hat{s}_1(t,\epsilon_1)\mathrm{e}^{i(\omega_1 t+\hat{\varphi}_1)} - \hat{a}_2\hat{s}_2(t,\epsilon_2)\mathrm{e}^{i(\omega_2 t+\hat{\varphi}_2)}\right|^2 \tag{11}$$

where

$$\hat{s}_k(t,\epsilon_k) = \Re\left\{\tfrac{1}{\hat{a}_k}\left[X_k(t) + \epsilon_k\hat{a}_l\mathrm{e}^{i((\omega_l-\omega_k)t+\hat{\varphi}_l-\hat{\varphi}_k)}\right]\right\} \tag{12}$$

and $(\epsilon_1, \epsilon_2) \in \{(+1,+1), (+1,-1), (-1,+1), (-1,-1)\}$.

4 Experimental Results

To corroborate the effectiveness of the proposed algorithm in various communications scenarios, extensive simulations were conducted on linear mixtures of two BPSK signals in AWGN channel. As a measure of performance, we have chosen the mean value of BERs calculated for each of the separated signals

$$\mathrm{BER} = \frac{N_{e_1} + N_{e_2}}{2N} \tag{13}$$

versus Signal to Noise Ratio (SNR) calculated as

$$\mathrm{SNR} = 10\log\left[\frac{P_{s_1} + P_{s_2}}{P_n}\right] \tag{14}$$

where N_{e_1} and N_{e_2} are numbers of erroneous symbols in the demodulated (separated) signals, N is a total number of symbols used in each trial ($N \approx 10^7$), P_{s_1} and P_{s_2} are powers of the source signals, and P_n is a power of the additive gaussian noise calculated in the sampling frequency band $[-F_s/2, +F_s/2]$ ($F_s = 8$ kHz). The initial phases (φ_k) were randomly chosen from the range $[-\pi/2, \pi/2]$ and the SNR was varying from 0 dB to 30 dB with a 2 dB step.

In all experiments, we have compared BERs of the separated signals with mean value of BERs (solid bold line) calculated for each original BPSK signal (assuming that all parameters needed for demodulation are known).

In the first experiment, we have verified the shape of BER curves for different ratios between amplitudes $\eta = \min\left[\frac{a_1}{a_2}, \frac{a_2}{a_1}\right]$. This ratio was fixed to be $\eta \in \{0.1, 0.4, 0.7, 1\}$. For each trial, signals were composed of 256 symbols, 5 samples per symbol, 40000 different realisations, and difference between carrier frequencies $|f_1 - f_2|$ was fixed to be 20 Hz at sampling frequency of 8 kHz. Corresponding results are presented in the figure 1, for known as well as estimated coefficients $(a_1, a_2, \varphi_1, \varphi_2)$. BERs for separated signals are the lines with markers, and the solid bold lines without markers correspond to the theoretical BERs for only one BPSK signal. One should pay attention to the following facts: the best results are obtained when $\eta \in \{0.4, 0.7\}$, the method used to estimate mixing coefficients plays an important role especially for small ratios between amplitudes ($\eta \approx 0.1$), the worst results are obtained for $a_1 = a_2$.

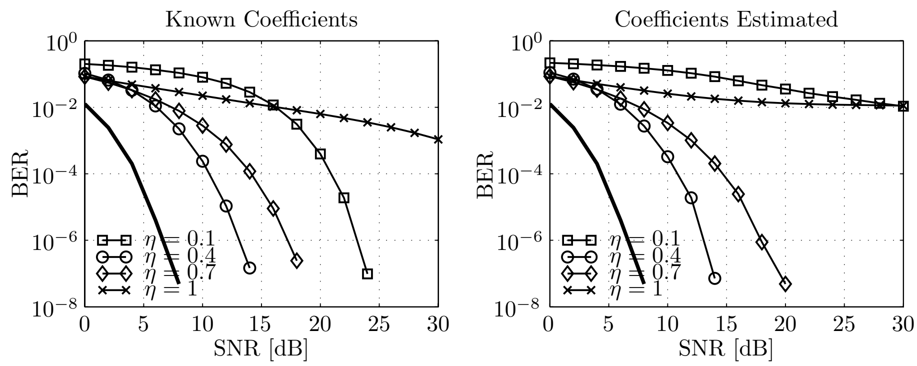

Fig. 1. BER versus SNR for different values of η

Other simulations were conducted to verify the behavior of the algorithm for different number of samples per symbol $N_{\mathrm{samp}} \in \{5, 10, 15, 20\}$ (or equivalently Baud Rates for fixed F_s). Simulation results are shown in the figure 2 for $\eta = 0.5$ and $|f_1 - f_2| = 20$ Hz. It is evident that regardless of method used to estimate the mixing coefficients, increasing N_{samp} for the same total number of transmitted symbols, improves the performance of the algorithm (which is also true for any demodulation-detection system working on only one signal [22]).

The influence of the frequency shifts ($|f_1 - f_2|$ changing from 20 to 200 Hz), as well as the total number of emitted symbols (N_{symb} changing from 256 to 2048)

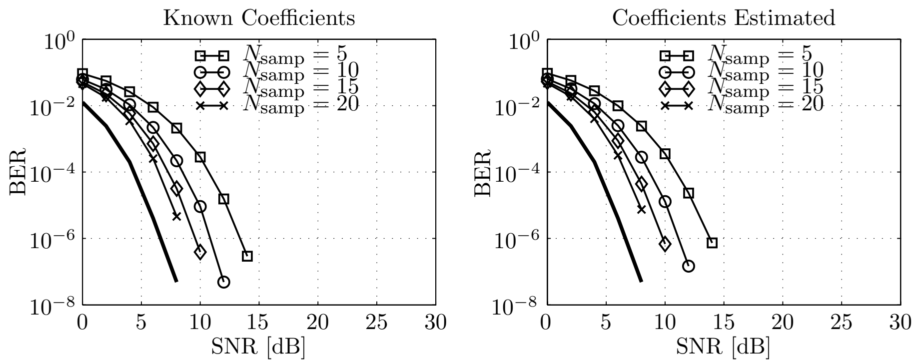

Fig. 2. BER versus SNR for different values of N_{samp}

on the performance of our algorithm has been also verified. Corresponding results reveal that our algorithm is invariant to signals' placement in the frequency domain (assuming that carrier frequencies are distinct and can be estimated), and to the number of available symbols (even when mixing coefficients have to be estimated).

5 Conclusion

Our new algorithm is targeted signal separation of undetermined instantaneous mixtures of two BPSK signals, closely distributed in a frequency domain. Using only one observation, we show a new solution for separation as well as for estimation of mixing coefficients. Experimental results reveal the robustness of our method to the number of samples per symbol (Baud Rates), total number of available symbols (possibility of working with small packets in "quasi real-time" applications), as well as to the shift between the carrier frequencies (even for overlapping bands). The separation algorithm is very robust to the ratio between amplitudes (excepted $a_1 = a_2$) and to the method used to estimate the mixing coefficients (excepted $\eta < 0.1$ or $\eta = 1$). All simulations have shown that experimental BERs are sufficiently close to the theoretical ones, which makes the proposed method of great interest in practice.

Methods of improving estimators of the mixing coefficients, as well as the possibility of using presented ideas to separate convolutive mixtures are currently investigated. Further researches will be conducted to generalise described methods for the mixtures of other types of linear modulations.

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