Adaptive Array Beamforming using a Combined LMS-LMS Algorithm

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Abstract—A new adaptive algorithm, called LLMS, which employs two Least Mean Square (LMS) sections in tandem, is proposed for different applications of array beamforming. [12]The convergence of the LLMS algorithm is analyzed, in terms of mean square error, in the presence of Additive White Gaussian Noise (AWGN) for two different operation modes; normal referencing and self-referencing. Computer simulation results show that the convergence performance of LLMS is superior to the conventional LMS algorithms as well some of the more recent LMS based algorithms, such as constrained-stability LMS (CSLMS), and Modified Robust Variable Step Size LMS (MRVSS) algorithms. It is shown that the convergence of LLMS is quite insensitive to variations in both the input signal-to-noise ratio and the step size used. Also, the operation of the proposed algorithm remains stable even when its reference signal is corrupted by AWGN noise. Furthermore, the fidelity of the signal at the output of the LLMS beamformer is demonstrated through the Error Vector Magnitude (EVM) and the scatter plot obtained.

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1. INTRODUCTION

In recent years, adaptive or smart antennas have become a key component for various wireless applications, such as radar, sonar and cellular mobile communications [1]. Its use could lead to an increase in the detection range of radar and sonar systems, and the capacity of mobile radio communication systems. These antennas are used as spatial filters for receiving the desired signals coming from specific direction or directions while minimizing the reception of unwanted signals emanating from other directions. Beamforming is central to all antenna arrays, and a summary of beamforming techniques is presented in [2]. An overview of signal processing techniques used for adaptive antenna array beamforming is described in [3].

Because of its simplicity and robustness, the LMS algorithm has become one of the most popular adaptive signal processing techniques adopted in many applications including antenna array beamforming. Moreover, there is always a tradeoff between the speed of convergence of the LMS algorithm and its residual error floor when a given adaptation step size is used. Over the last three decades, several improvements have been proposed to speed up the convergence of the LMS algorithm. These include NLMS (normalized-LMS) [4, 5], transform domain algorithms [6], and recently the constrained-stability LMS (CSLMS) algorithm [7] and the Modified Robust Variable Step Size LMS (MRVSS) algorithm [8]. The CSLMS algorithm has been proposed for use in speech signals [7]. Because of its improved performance over other published LMS algorithms, it is included in this paper for performance comparison with the proposed LLMS scheme. In [9], a variable-length LMS algorithm that can accelerate the initial convergence of either the conventional LMS or the NLMS algorithm at the expense of an increase in computational complexity is described .

Yet another approach of attempting to speed up the convergence of LMS, without having to sacrifice too much of its error floor performance is through the use of a Variable Step Size LMS (VSSLMS) algorithm. All the published VSSLMS algorithms [9-13] make use of an initial large adaptation step size to speed up the convergence. Upon approaching the steady state, smaller step sizes are then introduced to decrease the level of adjustment, hence maintaining a lower error floor. More recently, the MRVSS algorithm, a modified version of the VSSLMS algorithm, has been proposed to improve both the anti-noise and tracking ability of the Robust VSSLMS algorithm (RVSS) presented in [12]. This algorithm is also used as a reference for performance comparison with LLMS proposed in this paper.

All the above previously published algorithms require an accurate reference signal for their proper operation. In some cases, several operating parameters are also required to be specified. For example, in the case of MRVSS, the algorithm makes use of twelve predefined parameters. As a result, the performance of such algorithm becomes highly dependent on the input signal [14]. Furthermore, the

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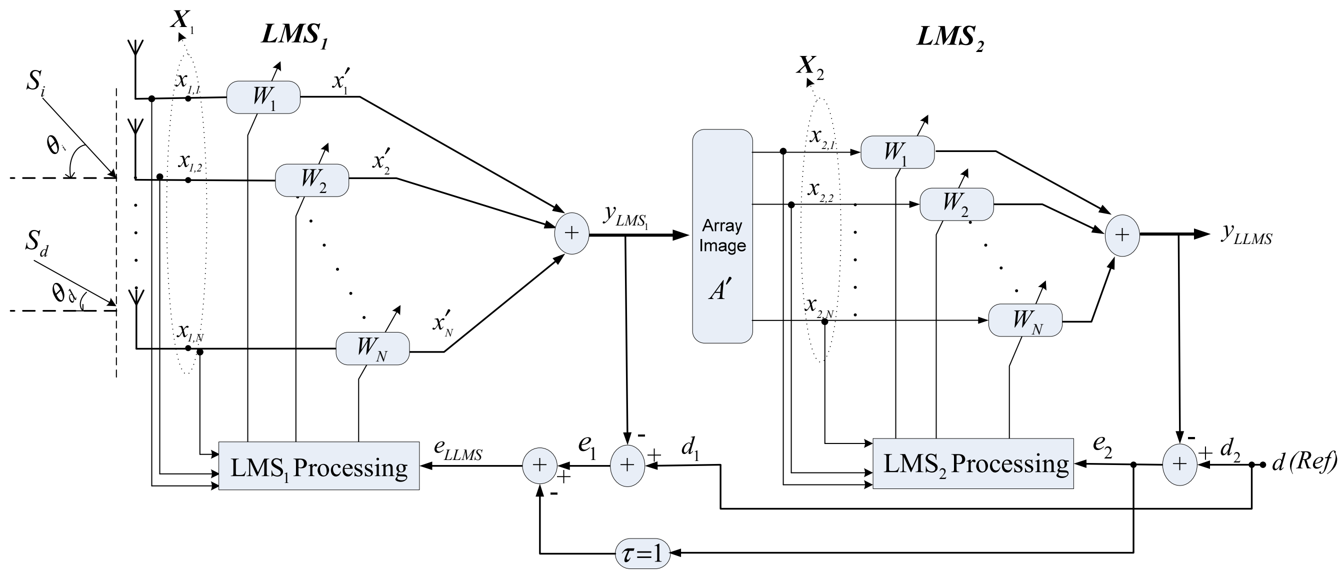


Figure 1 – The proposed LLMS algorithm with an external reference signal

computational complexity of MRVSS involves 9N complex multiplications and 4N complex additions [15], while the CSLMS requires (3N+1) complex multiplications, one complex division and (4N+3) complex additions, where N is the number of antenna array elements.

In an attempt to achieve fast convergence in conjunction with less complexity, better performance, and a lower requirement for an accurate reference, a new algorithm, called LLMS, which employs two LMS sections in tandem, is proposed for adaptive array beamforming. A block diagram of the proposed scheme is shown in Fig. 1. It involves 4N+1 complex multiplications and 2N complex additions.

With the proposed LLMS scheme, as shown in Fig. 1, the intermediate output, y_{LMS1}, yielded from the first LMS section, LMS_1, is multiplied by the image array factor ($\boldsymbol{A'}$) of the desired signal. The resultant "filtered" signal is further processed by the second LMS section, LMS_2. For the adaptation process, the error signal of LMS_2, e_2, is fed back to combine with that of LMS_1, to form the overall error signal, e_{LLMS}, for updating the tap weights of LMS_1. As shown in Fig. 1, a common external reference signal is used for both the two LMS sections, i.e., d_1 and d_2. Moreover, this external reference signal may be replaced by y_{LMS1} in place of d_2, and y_{LLMS} for d_1 to produce a self-referenced version of the LLMS scheme, as described in Section II B.

The rest of the paper is organized as follows. In section II, the convergence of LLMS is analyzed in the presence of an external reference signal. This is then followed by an analysis involving the use of the estimated outputs, y_{LMS1} and y_{LLMS} in place of the external reference. The latter is referred to as self-referencing, from hereon. Results obtained from computer simulations for an eight element array are presented in Section III. Finally, Section IV concludes the paper.

2. CONVERGENCE OF THE PROPOSED *LLMS* ALGORITHM

The convergence of the proposed LLMS algorithm has been analyzed with the following assumptions:

(i) The propagation environment is stationary.

(ii) The components of the signal vector $\boldsymbol{X}_I(j)$ should be independent identically distributed (iid).

(iii) All signals are zero mean and stationary at least to the second order.

Analysis with an external reference

First, we consider the case when an external reference signal is used. From Fig. 1, the error signal for updating $LLMS_1$ at the j^{th} iteration is given by

$$e_{LLMS}(j) = e_1(j) - e_2(j-1) \tag{1}$$

with $e_1(j) = d_1(j) - \boldsymbol{W}_1^H(j)\boldsymbol{X}_I(j)$

and $e_2(j) = d_2(j) - \boldsymbol{W}_2^H(j)\boldsymbol{X}_2(j)$

where $\boldsymbol{X}_i(\cdot)$ and $\boldsymbol{W}_i(\cdot)$ represent the input signal and weight vectors, respectively of the i^{th} LMS section. $(\cdot)^H$ denotes the Hermitian matrix of $(\cdot)$.

The input signal of LMS_2 is derived from the LMS_1, such that

$$X_2(j) = A' y_{LMS1}(j) = A' W_1^H(j) X_I(j)$$

where A' is the image of the array factor of the desired signal.

The weight vector $W_i(\cdot)$ for the i^{th} LMS section is updated according to [16],

$$W_i(j+1) = W_i(j) + \mu_i e_i(j) X_i(j), \qquad 0 < \mu_i < \mu_0 \tag{2}$$

where $i = 1$ for LMS_1 and 2 for LMS_2; μ_i is the step size, and μ_0 is a positive number that depends on the input signal statistics.

Now, the convergence performance of the LLMS algorithm can be analyzed in terms of the expected value of e_{LLMS}^2, such that

$$\begin{aligned}
\xi(j) &\triangleq E\left[\left|e_{LLMS}(j)\right|^2\right] = E\left[\left|e_1(j) - e_2(j-1)\right|^2\right] \\
&= E\left[\left|d_1(j) - W_1^H(j) X_I(j) - e_2(j-1)\right|^2\right] \\
&= E\left[\left|D(j)\right|^2\right] + W_1^H(j) Q(j) W_1(j) \\
&\quad - E\left[D(j) X_I^H(j) W_1(j) + D^*(j) W_1^H(j) X_I(j)\right]
\end{aligned} \tag{3}$$

where $|\bullet|$ signifies modulus; $D(j) = d_1(j) - e_2(j-1)$, and Q is the correlation matrix of the input signals given by [17] as

$$Q(j) = E\left[X_I(j) X_I^H(j)\right] \tag{4}$$

Consider the first term on the RHS of (3). It can be expressed as

$$\begin{aligned}
E\left[\left|D(j)\right|^2\right] &= E\left[\left|d_1(j) - e_2(j-1)\right|^2\right] \\
&= E\left[\left|d_1(j)\right|^2\right] + E\left[\left|e_2(j-1)\right|^2\right] \\
&\quad - E\left[d_1(j) e_2^*(j-1) + d_1^*(j) e_2(j-1)\right]
\end{aligned} \tag{5}$$

where * stands for conjugate operator.

With $d_1(j)$ and $e_2(j-1)$ being zero mean and uncorrelated based on the assumptions (ii), (ii) and (iii), the last RHS term of (5) is therefore equal to zero. This gives

$$E\left[\left|D(j)\right|^2\right] = E\left[\left|d_1(j)\right|^2\right] + E\left[\left|e_2(j-1)\right|^2\right] \tag{6}$$

From (1), the last RHS term of (6) becomes

$$\begin{aligned}
E\left[\left|e_2(j-1)\right|^2\right] &= E\left[\left|d_2(j-1)\right|^2\right] + E\left[\left|y_{LLMS}(j-1)\right|^2\right] \\
&\quad - E\left[d_2^*(j-1) y_{LLMS}(j-1) + d_2(j-1) y_{LLMS}^*(j-1)\right]
\end{aligned} \tag{7}$$

Assume $d_2(j) = d_1(j)$, and $y_{LLMS} = W_{LLMS}^H X_I$ where $W_{LLMS}^H = W_2^H A' W_1^H$, (7) can be rewritten as

$$\begin{aligned}
E\left[\left|e_2(j-1)\right|^2\right] &= E\left[\left|d_2(j-1)\right|^2\right] \\
&\quad - W_{LLMS}^H(j-1) Z(j-1) \\
&\quad - Z^H(j-1) W_{LLMS}(j-1) \\
&\quad + W_{LLMS}^H(j-1) Q(j-1) W_{LLMS}(j-1)
\end{aligned} \tag{8}$$

where $Z(j)$ corresponds to the input signal cross-correlation vector given by [17] as

$$Z(j) = E\left[X_I(j) d_2^*(j)\right] \tag{9}$$

Substituting (8) in (6), the first term on the RHS of (3) becomes

$$\begin{aligned}
E\left[\left|D(j)\right|^2\right] &= E\left[\left|d_1(j)\right|^2\right] + E\left[\left|d_2(j-1)\right|^2\right] \\
&\quad - W_{LLMS}^H(j-1) Z(j-1) - Z^H(j-1) W_{LLMS}(j-1) \\
&\quad + W_{LLMS}^H(j-1) Q(j-1) W_{LLMS}(j-1)
\end{aligned} \tag{10}$$

The last RHS term of (3) may be written as

$$\begin{aligned}
&E\left[D(j) X_I^H(j) W_1(j) + D^*(j) W_1^H(j) X_I(j)\right] \\
&= +Z^H(j) W_1(j) + W_1^H(j) Z(j) \\
&\quad - E\begin{bmatrix} e_2(j-1) X_I^H(j) W_1(j) \\ + e_2^*(j-1) W_1^H(j) X_I(j) \end{bmatrix}
\end{aligned} \tag{11}$$

Applying the assumptions (ii), (iii) and (iv), we obtain

$$\begin{aligned}
&E\left[D(j) X_I^H(j) W_1(j) + D^*(j) W_1^H(j) X_I(j)\right] \\
&= Z^H(j) W_1(j) + W_1^H(j) Z(j)
\end{aligned} \tag{12}$$

As a result, the mean square error ξ as specified by (3) can be rewritten to include the results of (10) and (12) to become

$$\begin{aligned}
\xi(j) &= E\left[\left|d_1(j)\right|^2\right] + E\left[\left|d_2(j-1)\right|^2\right] \\
&\quad + W_{LLMS}^H(j-1) Q(j-1) W_{LLMS}(j-1) - Z^H(j) W_1(j) \\
&\quad - W_{LLMS}^H(j-1) Z(j-1) - Z^H(j-1) W_{LLMS}(j-1) \\
&\quad - W_1^H(j) Z(j) + W_1^H(j) Q(j) W_1(j)
\end{aligned} \tag{13}$$

Differentiating (13) with respect to the weight vector $W_1^H(j)$ then yields the gradient vector $\nabla(\xi)$ so that

$$\nabla(\xi) = -Z(j) + Q(j) W_{opt1}(j) \tag{14}$$

By equating $\nabla(\xi)$ to zero, we obtain the optimal weight vector as

$$W_{opt1}(j) = Q^{-1}(j) Z(j) \tag{15}$$

This represents the Wiener-Hopf equation in matrix form. Therefore, the minimum MSE can be obtained from (15) and (13) to give

$$\xi_{\min} = E\left[\left|d_1(j)\right|^2\right] + E\left[\left|d_2(j-1)\right|^2\right] - \boldsymbol{Z}^H(j)\boldsymbol{W}_{opt1}(j) - \boldsymbol{Z}^H(j-1)\boldsymbol{W}_{LLMS}(j-1) + \boldsymbol{W}_{LLMS}^H(j-1)\boldsymbol{Z}(j-1)\left\{-1+\boldsymbol{A}'^H\boldsymbol{W}_2(j-1)\right\} \quad (16)$$

Based on (15) and (16), (13) becomes

$$\xi = \xi_{\min} + \left(\boldsymbol{W}_1 - \boldsymbol{W}_{opt1}\right)^H \boldsymbol{Q}\left(\boldsymbol{W}_1 - \boldsymbol{W}_{opt1}\right) \quad (17)$$

The error values of (17) are plotted as the theoretical curve in Fig. 2b.

Now, define

$$\boldsymbol{V}_1 \triangleq \left(\boldsymbol{W}_1 - \boldsymbol{W}_{opt1}\right) \quad (18)$$

so that (17) can be written as

$$\xi = \xi_{\min} + \boldsymbol{V}_I^H \boldsymbol{Q}\boldsymbol{V}_I \quad (19)$$

Differentiating (19) with respect to $\boldsymbol{V}_I^H$ will yield another form for the gradient [18], such that

$$\nabla(\xi) = \boldsymbol{Q}\boldsymbol{V}_I \quad (20)$$

Using eigenvalue decomposition (EVD) of $\boldsymbol{Q}$ in (20) yields

$$\boldsymbol{Q} = \boldsymbol{q}_I \Lambda_1 \boldsymbol{q}_I^{-1} = \boldsymbol{q}_I \Lambda_1 \boldsymbol{q}_I^H \quad (21)$$

where Λ_1 is the diagonal matrix of eigenvalues of $\boldsymbol{Q}$ for an N element array, i.e.,

$$\Lambda_1 = diag[E_1, E_2, \cdots, E_N] \quad (22)$$

For steepest descent, the weight vector is updated according to

$$\boldsymbol{W}_1(j+1) = \boldsymbol{W}_1(j) + \mu_1(-\nabla(\xi(j))) \quad (23)$$

where μ_1 is the convergence constant that controls the stability and the rate of adaptation of the weight vector, and $\nabla(j)$ is the gradient at the j^{th} iteration.

We may rewrite (23) in the form of a linear homogeneous vector difference equation using (18), (20) and (21) to give

$$\boldsymbol{V}_1(j+1) = \boldsymbol{V}_1(j) - \mu_1 \boldsymbol{Q}_1 \boldsymbol{V}_1(j) \quad (24)$$

Alternatively, (24) can be written as

$$\begin{aligned}\boldsymbol{V}_1(j) &= \left(\boldsymbol{q}_I \boldsymbol{q}_I^H - \mu_1 \boldsymbol{q}_I \Lambda_1 \boldsymbol{q}_I^H\right)\boldsymbol{V}_1(j-1) \\ &= \boldsymbol{q}_I\left(\boldsymbol{I} - \mu_1\Lambda_1\right)\boldsymbol{q}_I^H \boldsymbol{V}_1(j-1) \\ &= \boldsymbol{q}_I\left(\boldsymbol{I} - \mu_1\Lambda_1\right)^j \boldsymbol{q}_I^H \boldsymbol{V}_1(0)\end{aligned} \quad (25)$$

By substituting (25) in (19), the MSE at the j^{th} iteration is given by

$$\xi(j) = \xi_{\min} + \boldsymbol{V}_1^H(0)\boldsymbol{q}_I\left|\left(\boldsymbol{I} - \mu_1\Lambda_1\right)^j\right|^2 \boldsymbol{q}_I^H \boldsymbol{V}_1(0) \quad (26)$$

From (26), the asymptotic value of ξ becomes

$$\lim_{j\to\infty}\left(\boldsymbol{I} - \mu_1\Lambda_1\right)^j = 0 \quad (27)$$

With the term $\left(\boldsymbol{I} - \mu_1\Lambda_1\right)$ converging, as discussed in section III, we finally obtain

$$\lim_{j\to\infty}\xi(j) = \xi_{\min} \quad (28)$$

Analysis of the self-referencing scheme

Next, consider the case when the external reference is being replaced by internally generated signals, such that

$$d_1(j) = y_{LLMS}(j-1), \quad \text{and} \quad d_2(j) = y_{LMS1}(j) \quad (29)$$

As a result of these changes, and note that the error signal $e_2 = d_2 - y_{LLMS}$, we can redefine $D(j)$ in (3) as

$$\begin{aligned}d(j) &\triangleq D(j) \\ &= 2y_{LLMS}(j-1) - y_{LMS1}(j-1)\end{aligned} \quad (30)$$

Based on the definition of (30), we reanalyze the MSE as defined in (3) to yield

$$\begin{aligned}\xi(j) = {} & E\left[\left|d(j)\right|^2\right] - \boldsymbol{Z}'^H(j)\boldsymbol{W}_1(j) \\ & - \boldsymbol{W}_1^H(j)\boldsymbol{Z}'(j) + \boldsymbol{W}_1^H(j)\boldsymbol{Q}(j)\boldsymbol{W}_1(j)\end{aligned} \quad (31)$$

where $\boldsymbol{Z}'(j)$ corresponds to the input signal cross-correlation vector given by

$$\boldsymbol{Z}'(j) = E\left[\boldsymbol{X}_I(j)d^*(j)\right] \quad (32)$$

The error values obtained from (31) are plotted as the theoretical curve in Fig. 4.

By following the same analyzing steps of (5) to (31), it can be shown that the proposed LLMS algorithm will converge under the condition of self-referencing.

3. SIMULATIONS

The performance of the proposed LLMS algorithm has been studied by means of MATLAB simulation. For comparison purposes, results obtained with the conventional LMS, CSLMS and MRVSS algorithms are also presented. For the simulations, the following parameters are used:

- A linear array consisting of 8 isotropic elements.
- A BPSK signal arriving at an angle of 0°, or if specified at 10°.
- An AWGN channel.
- All weight vectors are initially set to zero.
- Unless otherwise specified, $\mu_1 = \mu_2 = 0.05$.
- An interference BPSK signal arrives at $\theta_i = 45^\circ$ with the same amplitude as the desired signal.

To facilitate the comparison with the published algorithms; CSLMS in [7] and MRVSS in [8], a brief description of the weight adaptation of these algorithms is given here.

The weight adaptation of the CSLMS algorithm is as follow:

$$\boldsymbol{W}(j+1)=\boldsymbol{W}(j)+\frac{\mu}{\|\delta\boldsymbol{W}(j)\|^2+\varepsilon}\delta\boldsymbol{X}(j)\left(\delta e^{[j]}(j)\right)^* \quad (33)$$

where ε is a small constant and is adjusted to yield the best possible performance in the operating–environment under consideration in this paper, and

$$\delta\boldsymbol{W}(j)=\boldsymbol{W}(j)-\boldsymbol{W}(j-1),$$

$$\delta\boldsymbol{X}(j)=\boldsymbol{X}(j)-\boldsymbol{X}(j-1),$$

$$\delta e^{[j]}(j)=e^{[j]}(j)-e^{[j]}(j-1),$$

and $e^{[k]}(j)=d(j)-\boldsymbol{W}^H(k)\boldsymbol{X}(j)$.

As for the MRVSS algorithm, the step size, μ, is updated as

$$\mu(j+1)=\begin{cases}\mu_{\max} \text{ ; if } \mu(j+1)>\mu_{\max}\\ \mu_{\min} \text{ ; if } \mu(j+1)<\mu_{\min}\\ \alpha\mu(j)+\gamma P^2(j)\end{cases}$$

with $P(j+1)=(1-\beta(j))P(j)+\beta(j)e(j)e(j-1)$

$$\text{and } \beta(j+1)=\begin{cases}\beta_{\max} \text{ ; if } \beta(j+1)>\beta_{\max}\\ \beta_{\min} \text{ ; if } \beta(j+1)<\beta_{\min}\\ \eta\beta(j)+\upsilon P^2(j)\end{cases}$$

where $\alpha>0, \eta>1, (\gamma,\upsilon)>0$, and $P(j)$ is the time averaged over two consecutive values of the error correlation. β is the time average of the error square signal with its upper and lower bounds as $\beta_{\max}$ and $\beta_{\min}$, respectively. $\mu_{\max}$ and $\mu_{\min}$ are the upper and lower bounds of μ respectively.

Table 1 tabulates the values of the various constants adopted for the simulations using the four different adaptive algorithms. Some of these values are given in [8, 11, 12].

Often, performance comparison between different adaptive beamforming schemes is made in terms of the convergence errors and resultant beam patterns. Moreover, for a digitally modulated signal, it is also convenient to make use of the Error Vector Magnitude (EVM) as an accurate measure of any distortion introduced by the adaptive scheme on the received signal at a given signal-to-noise ratio (*SNR*). It is shown in [19] that EVM is more sensitive to variations in *SNR* variations than Bit Error Rate (BER). EVM is defined as [20]

$$EVM_{RMS}=\sqrt{\frac{\frac{1}{K}\sum_{j=1}^{K}|S_r(j)-S_t(j)|^2}{P_o}} \quad (34)$$

where K is the number of symbols used, $S_r(j)$ is the normalized jth output of the beamformer, and $S_t(j)$ is the jth transmit symbol. P_o is the normalized transmit symbol power.

Table 1. Values of the Constants Uesd in Simulation

Algorithm	Value(s) of the different constants
LMS	$\mu=0.05$
LLMS	$\mu_1=\mu_2=0.05$
CSLMS	$\varepsilon=0.05$
MRVSS	$\alpha=0.97$, $\gamma=4.8e-4$, $\eta=0.97$, $\upsilon=5e-4$ $\mu_{\max}=0.2$, $\mu_{\min}=1e-4$, $\beta_{\max}=1$, $\beta_{\min}=0$

Performance with an external reference

First, the performances of the LLMS, CSLMS, MRVSS and LMS schemes have been studied in the presence of an external reference signal. The convergence performances of these schemes are compared based on the ensemble average squared error $\left(\tilde{e}^2\right)$ obtained from 100 individual simulation runs. The results obtained for different values of input *SNR*, and step size, μ_1 and μ_2, are presented.

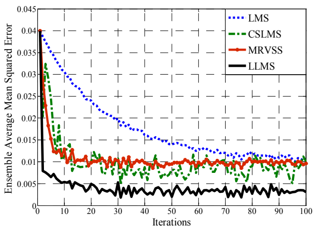
Figs. 2a – 2c show the convergence behaviors of the four adaptive schemes for SNR values of 5, 10, and 15 dB, respectively. For the proposed LLMS scheme, the theoretical convergence error calculated using (16) and (17) for SNR=10 dB is also shown in Fig. 2b. It is observed that under the given conditions, the proposed LLMS algorithm converges much faster than the other three schemes. Furthermore, the error floor of LLMS is less sensitive to the input SNR. As shown in Fig. 2b, there is close agreement between the simulated and theoretical error plots for the proposed of LLMS scheme. As for the CSLMS and MRVSS algorithms, they share the same performance for all the three *SNR* values considered.

Next, it can be shown that for ensuring convergence of the LLMS algorithm, the values of the step size used have to be within the following bounds:

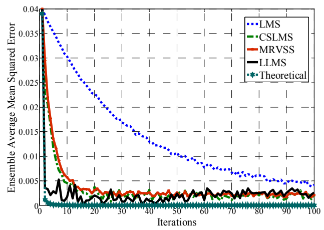
$$0<\mu_1<2/E_{\max} \quad (35)$$

$$\text{and} \quad 0<\mu_2<2/N\sigma_1^2 \quad (36)$$

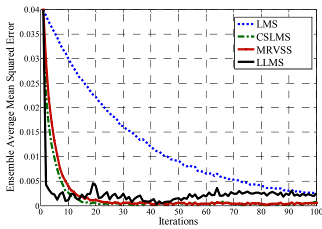
where $E_{\max}$ is the largest eigenvalue given in (22), and σ_1^2 is the variance of y_{LMS1}.



(a) SNR=5dB



(b) SNR=10dB



(c) SNR=15dB

Figure 2 – The convergence of LLMS, CSLMS, MRVSS and LMS with the parameters given in Table I, for three different values of input SNR.

For an 8-element array operating with an input SNR of 10 dB, we have $0 < \mu_1 < 0.8$ and $0 < \mu_2 < 0.726$. When the step sizes are chosen to be well within their limits, such as μ_2=0.05 or o.1 in conjunction with μ_1=0.1 or 0.005 respectively, Fig. 3 shows that LLMS converges within a few iterations to a low error floor. However, LLMS shows sign of instability when operating with step sizes close to their upper limits, as shown in the convergence behavior for the two cases with μ_1=0.005 and μ_2=0.6, and μ_1=0.799 and μ_2=0.05.

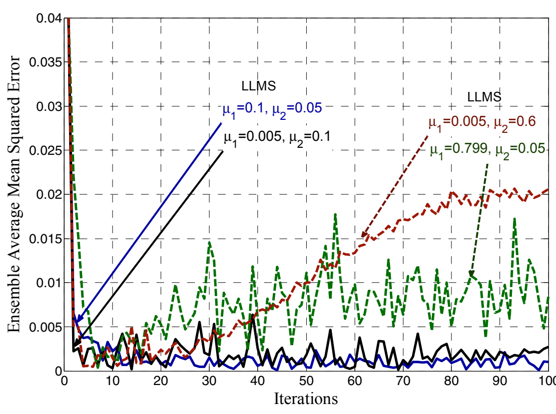


Figure 3 – The convergence of the LLMS algorithm at $SNR = 10\,dB$ for different combinations of step sizes.

Performance with self-referencing

As shown in Fig. 2 and Fig. 3, the LLMS algorithm can converge within ten iterations. Once this occurs, the intermediate output, y_{LMS1}, tends to resemble the desired signal $s_d(t)$, and may then be used in place of the external reference d_{LMS2} for the current iteration of the LMS_2 section. As the LMS_2 section converges, its output y_{LLMS} becomes the estimated $s_d(t)$. As a result, y_{LLMS} may be used to replace d_{LMS1} as the reference for the LMS_1 section. This feedforward and feedback arrangement enables the provision of self-referencing in LLMS, and allows the external reference signal to be discontinued after an initial four iterations. The ability of the LLMS algorithm to maintain operation with the internally generated reference signals is demonstrated in Fig. 4. On the other hand, it clearly shows that the traditional LMS, CSLMS, MRVSS algorithms are unable to converge without the use of an external reference signal. For comparison, the theoretical convergence errors calculated from (31) are also plotted in Fig. 4.

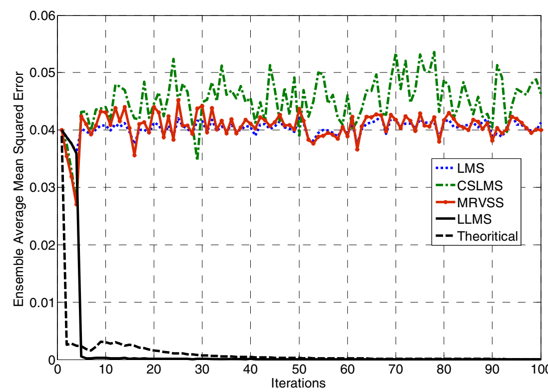


Figure 4 – The convergence of LLMS with self-referencing using the parameters given in Table I, for $SNR = 10\,dB$. An external reference is used for the initial four iterations.

Performance with a noisy reference signal

The performances of LLMS, CSLMS, MRVSS and LMS have also been investigated when their reference signals used are corrupted by AWGN. This is done by examining the resultant mean square error ξ when the noise level in the reference signal is varied. Fig. 5 shows the ensemble average of the mean square error, $\overline{\xi}$, obtained from 100 individual simulation runs, as a function of the ratio of the rms noise level σ to the amplitude of the reference signal.

It is interesting to note that the conventional LMS, CSLMS and MRVSS algorithms are quite sensitive to the presence of noise in the reference signal. On the other hand, the LLMS algorithm becomes very tolerant to noisy reference signal. As shown in Fig. 5, the values of $\overline{\xi}$ associated with LLMS remain very small even when the rms noise level becomes as large as the reference signal.

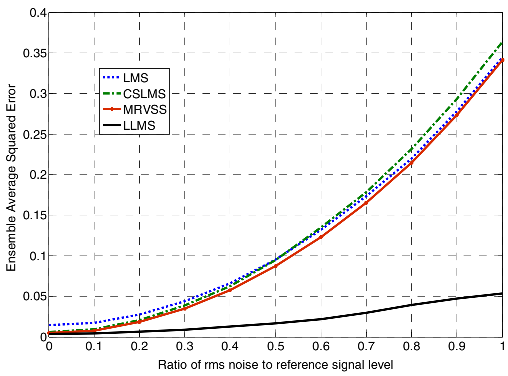


Figure 5 – The influence of noise in the reference signal on the mean square error ξ with $\mu = \mu_1 = \mu_2 = 0.05$

Tracking performance of LLMS

The ability of LLMS in tracking sudden interruptions in the input signal is investigated by examining the behavior of its error signal e^2_{LLMS}. For this study, the input signal is assumed to be periodically interrupted for 25 out of 100 iterations. The resulting tracking performance of LLMS is shown in Fig. 6, which shows that, the mean square error ξ increases very rapidly each time the input is switched on or off. This indicates the fast response of LLMS to sudden interruptions in the input signal. Unlike the responses for LMS, CSLMS and MRVSS, which are also included in Fig. 6 for comparison purpose, the mean square error ξ associated with LLMS remains low despite the interruption occurring in the input signal.

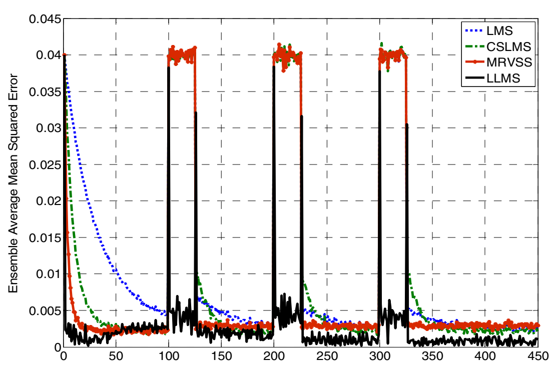


Figure 6 – Tracking performance comparison of LLMS, CSLMS, MRVSS and LMS with $\mu = \mu_1 = \mu_2 = 0.05$ and

$SNR = 10\,dB$

Beam pattern characteristics

Fig. 7 shows the beam patterns obtained with the LLMS, CSLMS, MRVSS and LMS algorithms at an input SNR of 10 dB and a signal-to-interference ratio SIR of 0 dB. In this case, the direction of arrival of the desired signal, i.e., $\theta_d = 10^o$ while the interference arrives at $\theta_i = 45^\circ$. It is assumed that an ideal reference is initially used for a given number of iterations. After that, LLMS switched to the self-referencing mode, while the other three algorithms reverted to using a random signal as the reference. In this way, it provides a fairer comparison between the different schemes, i.e., operating without an ideal reference signal. The results are shown for the number of iterations used in Figs. 7a, 7b and 7c show the results obtained when the external reference is used for the initial 5, 7, and 10 iterations, respectively. In Fig. 7d, all the algorithms make use of the external reference over the entire 100 iterations. As a consequence, all the algorithms have almost the same performance.

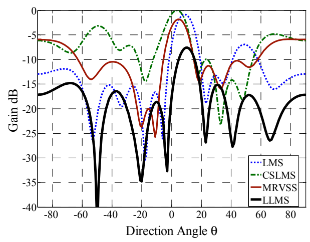
From Figs. 7a, 7b, and 7c, the following observations are made: i) LMS, CSLMS and MRVSS algorithms lose the direction of arrival of the desired signal when the external reference is removed after an initial period of operating with it, while LLMS algorithm maintains the maximum gain in this direction; ii) the difference between the gains at the desired and the interference directions for the LLMS algorithm is increased from 14 dB to 20 dB when the period of use of the external reference is extended from 5 to 10 iterations; and iii) this difference becomes almost the same when the external reference is initially applied for either 7 or 10 iterations. The latest observation confirms that the LLMS algorithm reached its steady state in 7 iterations.

EVM and Scatter Plot

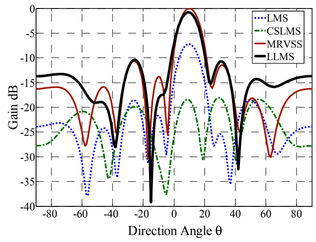
In this experiment, the rms EVM is computed, based on (34), for values of input SNR ranging from 0 – 30 dB in steps of 5 dB. The resulting EVM values, as shown in Fig. 8, have been calculated after each of the four different adaptive algorithms has converged. The superior performance of the proposed LLMS scheme is clearly demonstrated with its lower resultant EVM values compared with the other three schemes. This is particularly true at lower input SNR values. This further confirms the observation made from Fig. 2 showing that the operation of LLMS is quite insensitive to input SNR.

Next, the scatter plots of the BPSK signals recovered using the LMS, CSLMS, MRVSS, and LLMS adaptive Beamformer are shown in Figs. 9a – 9d, respectively.

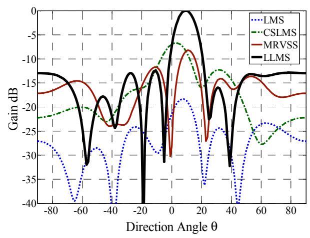
Each scatter plot is obtained for an input SNR of 10 dB using 100 signal samples after the algorithm has converged. Again, the scatter plot obtained with LLMS shows the least spreading, indicating its ability to retain the signal fidelity.



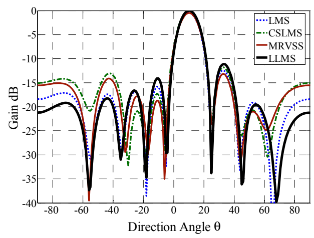
(a) 5 iterations



(b) 7 iterations



(c) 10 iterations



(d) 100 iterations

Figure 7 – The beams patterns achieved with the LLMS, CSLMS, MRVSS and LMS algorithms when the external reference is used for the intial 5, 7, 10, and 100 iterations for an input $SNR = 10\,dB$ and $SIR = 0$ dB. The parameters given in Table 1 are adopted.

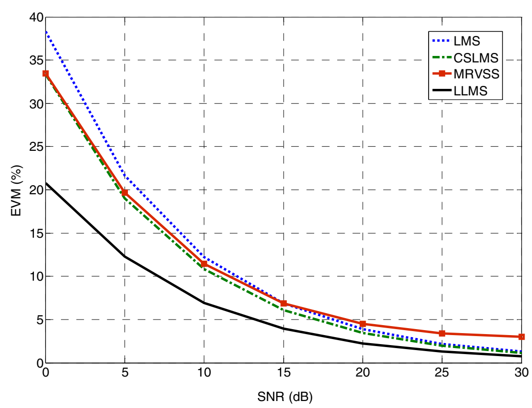
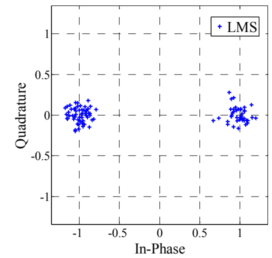


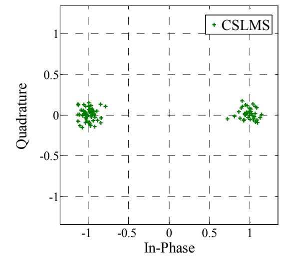
Figure 8 – The EVM values obtained with the LLMS, CSLMS, MRVSS and LMS algorithms for different input SNR.

4. CONCLUSIONS

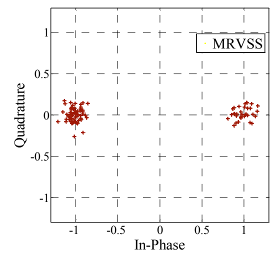
A new algorithm, called LLMS, which combines the use of two successive LMS sections, is presented for adaptive array beamforming. The convergence of LLMS has been analyzed assuming the use of an external reference signal. This is then extended to cover the case that makes use of self-referencing. The convergence behaviors of the LLMS algorithm with different step size combinations of μ_1 and μ_2 have been demonstrated by means of Matlab simulations under different input SNR conditions.



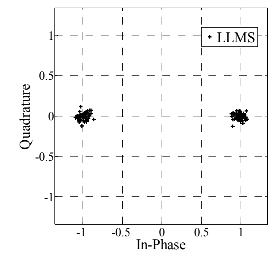
(a) LMS algorithm



(b) CSLMS algorithm



(c) MRVSS algorithm



(d) LLMS algorithm

Figure 9 – The scatter plots of BPSK signal obtained using 100 signal samples of LLMS, CSLMS, MRVSS and LMS algorithms under input $SNR = 10\,dB$ and $SIR = 0$ dB.

It is shown that the proposed LLMS algorithm can achieve rapid convergence, typically within a few iterations. Furthermore, the steady state MSE of LLMS is quite insensitive to input SNR. Also, unlike the conventional LMS, CSLMS and MRVSS algorithms, the proposed LLMS scheme is able to operate with noisy reference signal. Once the initial convergence is achieved, within a few iterations, the LLMS scheme can maintain its operation through self-referencing. Moreover, the resultant EVM and scatter plot of the proposed LLMS further demonstrate its superior performance over the other three LMS-based schemes.

The rapid convergence and robust operation of the proposed LLMS algorithm have been achieved with a complexity slightly larger than twice the LMS scheme. Moreover, its complexity is lower than the CSLMS and MRVSS algorithms, as well as our previously published RLMS scheme [21, 22].

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BIOGRAPHY

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