# LLMS Adaptive Beamforming Algorithm Implemented with Finite Precision

Jalal Abdulsayed SRAR, Kah-Seng CHUNG and Ali MANSOUR

***Abstract* — This paper studies the influence of the use of finite wordlength on the operation of the LLMS adaptive beamforming algorithm. The convergence behavior of LLMS algorithm, based on the minimum mean square error (MSE), is analyzed for operation with finite precision. Computer simulation results verify that a wordlength of eight bits is sufficient for the LLMS algorithm to achieve performance close to that provided by full precision. Based on the simulation results, it is shown that the LLMS algorithm outperforms least mean square (LMS) in addition to other earlier algorithms, such as, modified robust variable step size (MRVSS) and constrained stability LMS (CSLMS).**

**Keywords — LLMS algorithm, array beamforming, fixed-point arithmetc.**

## I. Introduction

SPATIAL division multiple access is gaining popularity as a mean for enhancing greater capacity in wireless communications to meet the ever growing demand for higher data rates and wider coverage without a corresponding increase in spectrum allocation. Adaptive or smart antennas are used to exploit the spatial domain for minimizing interferences. These antennas are required to have fast convergence with low mean square error (MSE), good channel tracking and low complexity.

Various adaptive algorithms, including least mean square (LMS), variable step size LMS (VSSLMS) [1], constrained stability LMS (CSLMS) [2] and modified robust variable step size LMS (MRVSS) [3] have been introduced for use in adaptive beamforming. These LMS based adaptive algorithms make use of adaptive step size to improve the convergence speed of the conventional LMS algorithm and tracking ability in time varying environments.

Alternatively, schemes involving the use of a combination of different algorithms have gained research interests in an attempt to improve system performance [4]. Examples of these schemes are presented in [5, 6]. With these schemes, the final output is obtained by combining through a mixing factor the outputs of two or more independent algorithm stages operating in parallel [7]. However, proper choices of the step sizes and mixing parameters used are dependent on the characteristics of each individual algorithm as well as variations in the input signal and noise [5, 7]. Also, the scheme described in [8], makes use of a RLS algorithm stage to achieve fast convergence when operating on the first frame containing the training sequences. This is then followed by an LMS algorithm stage which operates on the rest of the data.

More recently, a different approach is presented in [9] to combine the use of two LMS algorithm stages. This new scheme is called the LLMS algorithm, which overcomes many of the drawbacks associated with previous use of combined algorithms. The new LLMS algorithm is able to converge rapidly to properly track the input signal in time varying environments. Its complexity is only slightly more than double that of a conventional LMS algorithm. As shown in [9], the LLMS algorithm consists of two LMS algorithm stages interconnected via an array image vector ($\mathcal{F}$). The results published in [9] show that the LLMS algorithm, when used in a beamforming application, converges faster than either the LMS, CSLMS or MRVSS algorithms. Also, the convergence is less sensitive to variations in the input signal to noise ratio (SNR). Furthermore, the LLMS algorithm can operate with a noisy reference signal. As described in [9], the LLMS algorithm can operate in either fixed or adaptive beamforming mode, where the former is referred to $\text{LLMS}_1$ and the latter as LLMS in this paper. For $\text{LLMS}_1$, the elements of $\mathcal{F}$ are pre-calculated according to the direction of arrival of the desired signal, while LLMS adapts automatically the elements of $\mathcal{F}$ to track the target.

In this paper, we extend our study in [9] by considering the effect on the operation of the LLMS algorithm due to the use of finite precision arithmetic. This leads to the determination of the minimum wordlength, in terms of binary bits, required to achieve a minimum degradation in performance when compared with a full precision implementation.

After the description of the signal model in Section II, mathematical expressions are derived in Section III for the estimation of the quantization error associated with the overall error signal, $e_{\text{LLMS}}$. Computer simulation results obtained for an 8-element array are presented in Section IV. Finally, in Section V, we conclude the paper.

## II. Signal Model

Consider a uniform linear array (ULA) consisting of $N$ omni-directional antenna elements with inter-element spacing $\mathcal{D}$. Let $s_d(t)$, and $s_i(t)$ be the desired and interfering signals, respectively, which arrive at the array from different directions at a distance. Then, the input

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signals of the array, in the time domain, can be expressed as

$$\begin{aligned}\boldsymbol{x}_1(t) &= [x_1(t),\ x_2(t),\ ...,x_N(t)]^T \\ &= \boldsymbol{A}_d s_d(t) + \boldsymbol{A}_i s_i(t) + \boldsymbol{n}(t).\end{aligned} \tag{1}$$

where $\boldsymbol{A}_d$ and $\boldsymbol{A}_i$ are the $[N \times 1]$ complex array vectors for the desired signal and the cochannel interference, respectively as explained in [9]. $\boldsymbol{n}(t)$ is the additive white Gaussian noise. The output of the LLMS beamformer can be expressed as,

$$y_{\text{LLMS}}(j) = \boldsymbol{w}^H_{\text{LLMS}}(j)\boldsymbol{x}_1(j)$$

where $\boldsymbol{w}^H_{\text{LLMS}} = \boldsymbol{w}^H_2 \mathcal{F} \boldsymbol{w}^H_1$ with $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ being the weight vectors for the first and second LMS algorithms stages, respectively. $(\bullet)^H$ denotes the Hermitian matrix of $(\bullet)$.

## III. Error Convergence

The structure of the LLMS algorithm as depicted in Fig. 1 of [9] shows that the overall error signal, $e_{\text{LLMS}}$, at the $j^{th}$ iteration is given by

$$e_{\text{LLMS}}(j) = e_1(j) - e_2(j) \tag{2}$$

with

$$e_1(j) = d_1(j) - \boldsymbol{w}_1^H(j)\boldsymbol{x}_1(j) \tag{3}$$

$$e_2(j) = \varepsilon\left(d_2(j) - \boldsymbol{w}_2^H(j-1)\boldsymbol{x}_2(j-1)\right) \tag{4}$$

where $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are the input signals of the first and second LMS algorithm stages, respectively. $\varepsilon$ is a constant used to determine the amount of feedback, $d_1(j) = d(j)$, $d_2(j) = d(j-1)$, and $d(j)$ is the external reference signal.

By substituting $e_1(j)$ and $e_2(j)$ from (3) and (4) in (2), we obtain

$$\begin{aligned}e_{\text{LLMS}}(j) &= d(j) - \varepsilon d(j-1) - \boldsymbol{w}_1^H(j)\boldsymbol{x}_1(j) \\ &\quad + \varepsilon \boldsymbol{w}_2^H(j-1)\boldsymbol{x}_2(j-1)\end{aligned} \tag{5}$$

Now, with the LLMS algorithm being implemented using finite precision arithmetic, the results of the various mathematical calculations will be affected by round-off and truncation errors. The influence of these errors on the operation of the LLMS algorithm is analyzed as follows.

First, the signal terms expressed in finite precision are represented by primed symbols to differentiate them from their corresponding counterparts in full precision. For example, the input signal and weight vectors in finite precision can be expressed as

$$\boldsymbol{x}'(j) = \boldsymbol{x}(j) + \boldsymbol{\alpha}(j) \tag{6}$$

$$\boldsymbol{w}'(j) = \boldsymbol{w}(j) + \boldsymbol{\rho}(j) \tag{7}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\rho}$ are the corresponding quantization error vectors, which are assumed to be mutually independent and independent of $\boldsymbol{x}$ and $\boldsymbol{w}$, respectively, and both are white sequences with zero mean and their coefficients ($\alpha_i$) and ($\rho_j$) are uniformly distributed and have $\sigma_q^2$ as variance. For a signal of ±1 V amplitude range represented by an $N_b$–bit wordlength, the resulting variance of the quantization error is given by [10]

$$\sigma_q^2 = 2^{2(1-N_b)}/12 \tag{8}$$

Substituting (6) and (7) in (5), we obtain the overall error signal in finite precision, $e'_{\text{LLMS}}$, as

$$\begin{aligned}e'_{\text{LLMS}}(j) &= d(j) - \varepsilon d(j-1) - \eta_1(j) + \varepsilon\eta_2(j-1) \\ &\quad - \boldsymbol{w}_1^H(j)\boldsymbol{x}_1(j) + \varepsilon \boldsymbol{w}_2^H(j-1)\boldsymbol{x}_2(j-1) \\ &\quad - \boldsymbol{\rho}_1^H(j)\boldsymbol{x}_1(j) + \varepsilon \boldsymbol{\rho}_2^H(j-1)\boldsymbol{x}_2(j-1) \\ &\quad - \boldsymbol{w}_1^H(j)\boldsymbol{\alpha}_1(j) + \varepsilon \boldsymbol{w}_2^H(j-1)\boldsymbol{\alpha}_2(j-1)\end{aligned} \tag{9}$$

where $\eta_i = \boldsymbol{\rho}_i^H \boldsymbol{\alpha}_i = \sum_{j=1}^{N} \rho_{ij}^* \alpha_{ij}$, with $i = 1$ or 2 to indicate the association with the first or second LMS stage, respectively. $\alpha_{ij}$ is the $j^{th}$ coefficient of $\alpha_i$. With $\boldsymbol{\alpha}_i$ and $\boldsymbol{\rho}_i$ assumed to be white sequences of zero mean and their respective coefficients, $\alpha_i$ and $\rho_i$, being uniformly distributed, it can be shown that $\sigma_\eta^2 = N\sigma_q^4$.

As the first four terms on the right hand side of (9) are the same as those for $e_{\text{LLMS}}$ given in (5), we can rewrite (9) as

$$\begin{aligned}e'_{\text{LLMS}}(j) &= e_{\text{LLMS}}(j) - \boldsymbol{\rho}_1^H(j)\boldsymbol{x}_1(j) \\ &\quad + \varepsilon \boldsymbol{\rho}_2^H(j-1)\boldsymbol{x}_2(j-1) - \boldsymbol{w}_1^H(j)\boldsymbol{\alpha}_1(j) \\ &\quad + \varepsilon \boldsymbol{w}_2^H(j-1)\boldsymbol{\alpha}_2(j-1) - \eta_1(j) + \varepsilon\eta_2(j-1)\end{aligned} \tag{10}$$

Rearranging (10), so that

$$e'_{\text{LLMS}}(j) = e_{\text{LLMS}}(j) - e_{\text{Q}}(j) \tag{11}$$

where

$$\begin{aligned}e_{\text{Q}}(j) &= \boldsymbol{\rho}_1^H(j)\boldsymbol{x}_1(j) - \varepsilon \boldsymbol{\rho}_2^H(j-1)x_2(j-1) + \boldsymbol{w}_1^H(j)\boldsymbol{\alpha}_1(j) \\ &\quad - \varepsilon \boldsymbol{w}_2^H(j-1)\boldsymbol{\alpha}_2(j-1) + \eta_1(j) - \varepsilon\eta_2(j-1)\end{aligned}$$

Convergence in mean-square error, $\xi'$, for the finite precision case can be analyzed by observing the expected value of $e'_{\text{LLMS}}$, such that

$$\begin{aligned}\xi'(j) &= \text{E}\left[\left|e'_{\text{LLMS}}(j)\right|^2\right] = \text{E}\left[\left|e_{\text{LLMS}}(j) - e_{\text{Q}}(j)\right|^2\right] \\ &= \text{E}\left[\left|e_{\text{LLMS}}(j)\right|^2\right] + \text{E}\left[\left|e_{\text{Q}}(j)\right|^2\right] \\ &\quad - \text{E}\left[e^*_{\text{LLMS}}(j)e_{\text{Q}}(j)\right] - \text{E}\left[e_{\text{LLMS}}(j)e^*_{\text{Q}}(j)\right]\end{aligned} \tag{12}$$

Following the same analysis in [9], the first term on the RHS of (12) is given by

$$\begin{aligned}\xi(j) &= \text{E}\left[\left|d(j)\right|^2\right] + \varepsilon^2 \text{E}\left[\left|d(j-1)\right|^2\right] \\ &\quad + \varepsilon^2 \boldsymbol{w}^H_{\text{LLMS}}(j-1)\boldsymbol{Q}(j-1)\boldsymbol{w}_{\text{LLMS}}(j-1) - \boldsymbol{z}^H(j)\boldsymbol{w}_1(j) \\ &\quad - \varepsilon^2 \boldsymbol{w}^H_{\text{LLMS}}(j-1)\boldsymbol{z}(j-1) - \varepsilon^2 \boldsymbol{z}^H(j-1)\boldsymbol{w}_{\text{LLMS}}(j-1) \\ &\quad - \boldsymbol{w}_1^H(j)\boldsymbol{z}(j) + \boldsymbol{w}_1^H(j)\boldsymbol{Q}(j)\boldsymbol{w}_1(j)\end{aligned} \tag{13}$$

where $\boldsymbol{Q}$ is the correlation matrix of the input signals, such that $\boldsymbol{Q}(j) = \text{E}\left[\boldsymbol{x}_1(j)\boldsymbol{x}_1^H(j)\right]$, and $\boldsymbol{z}(j)$ is the input signal cross-correlation vector, given by $\boldsymbol{z}(j) = \text{E}\left[\boldsymbol{x}(j)d^*(j)\right]$.

Now, the second term on the RHS of (12) can be analyzed separately using the relationships of (6) and (7), such that

$$\text{E}\left[\left|e_{\text{Q}}(j)\right|^2\right] = \text{E}\left[\left(-\eta_1(j) + \varepsilon\eta_2(j-1)\right)^2\right] \tag{14}$$

By analyzing the third and fourth terms on the RHS of (12), it can be shown that

$$\text{E}\left[e^*_{\text{LLMS}}(j)e_{\text{Q}}(j)\right] = \text{E}\left[e^*_{\text{LLMS}}(j)\eta_1(j)\right] \tag{15}$$

$$\mathrm{E}\left[e_{\mathrm{LLMS}}(j)e_{\mathrm{Q}}^{*}(j)\right]=\mathrm{E}\left[e_{\mathrm{LLMS}}(j)\eta_{1}^{*}(j)\right] \qquad (16)$$

Substituting the results from (13), (14), (15) and (16) into (12), and based on the fact that $\alpha_i$ and $\rho_i$ are zero-mean and mutually independent, we obtain

$$\xi'(j)=\xi(j)+\mathrm{E}\left[\left(-\eta_{1}(j)+\varepsilon\eta_{2}(j-1)\right)^{2}\right]- \mathrm{E}\left[e_{\mathrm{LLMS}}^{*}(j)\eta_{1}(j)\right]-\mathrm{E}\left[e_{\mathrm{LLMS}}(j)\eta_{1}^{*}(j)\right] \qquad (17)$$

Also, the correlation coefficient between $e_{\mathrm{LLMS}}$ and $\eta_1$ is given by

$$\rho_{e_{\mathrm{LLMS}}(j)\eta_1(j)}=\frac{\mathrm{E}\left[e_{\mathrm{LLMS}}(j)\eta_1(j)\right]-\mathrm{E}\left[e_{\mathrm{LLMS}}(j)\right]\mathrm{E}\left[\eta_1(j)\right]}{\sigma_{\eta_1(j)}\sigma_{e_{\mathrm{LLMS}}(j)}} \qquad (18)$$

where $\sigma_{e_{\mathrm{LLMS}}}$ is the standard deviation of the overall LLMS algorithm error signal.

Using the relationship of (18), the second and third terms of (17) can be estimated as

$$\mathrm{E}\left[e_{\mathrm{LLMS}}^{*}(j)\eta_{1}(j)\right]=\sigma_{\eta_1(j)}\sigma_{e_{\mathrm{LLMS}}^{*}(j)}\rho_{\eta_1(j)e_{\mathrm{LLMS}}^{*}(j)} \qquad (19)$$

Similarly, the fourth RHS term of (17) can be estimated as

$$\mathrm{E}\left[e_{\mathrm{LLMS}}(j)\eta_{1}^{*}(j)\right]=\sigma_{\eta_1^{*}(j)}\sigma_{e_{\mathrm{LLMS}}(j)}\rho_{\eta_1^{*}(j)e_{\mathrm{LLMS}}(j)} \qquad (20)$$

Substituting (19) and (20) into (17), and observing that $\sigma_{e_{\mathrm{LLMS}}^{*}(j)}=\sigma_{e_{\mathrm{LLMS}}(j)}$ and $\sigma_{\eta_1(j)}=\sigma_{\eta_1^{*}(j)}$, we obtain

$$\xi'(j)=\xi(j)+\mathrm{E}\left[\left(-\eta_{1}(j)+\varepsilon\eta_{2}(j-1)\right)^{2}\right] -\sigma_{\eta_1(j)}\sigma_{e_{\mathrm{LLMS}}(j)}\left(\rho_{\eta_1(j)e_{\mathrm{LLMS}}^{*}(j)}+\rho_{\eta_1^{*}(j)e_{\mathrm{LLMS}}(j)}\right) \qquad (21)$$

Equation (21) indicates that the mean-square error of the overall error signal, $\xi'$, associated with the use of finite precision, converges to a value different from that for full precision. The difference in value is governed by the truncation and round-off errors of the two algorithm stages as well as the feedback gain constant, $\varepsilon$.

## IV. PERFORMANCE EVALUATION BY COMPUTER SIMULATIONS

In this section, we examine the influence on the performance of the LLMS algorithm when it is implemented using finite precision arithmetic. The performance evaluation includes the two modes of operation of the algorithm, namely operating with an adaptive array vector $\mathcal{F}$ (LLMS algorithm), as well as with a fixed prescribed $\mathcal{F}$ ($\mathrm{LLMS}_1$ algorithm).

Consider a uniform linear array consisting of eight isotropic antenna elements spaced half a wavelength apart. The adaptation process of the algorithm operating with finite numerical precision is simulated with $\varepsilon=1$. In order to make full use of a given wordlength, the input signal vector is normalized with the largest component taking on an amplitude range of $\pm 1$ Volt. Furthermore, the inputs to every arithmetic function, such as multiplication and addition, are expressed with the same specified numerical precision. The result of each arithmetical operation is then rounded to the specified wordlength.

### *A. MSE Performance*

In order to determine an acceptable numerical precision required for implementing the LLMS algorithm, we examine, by means of computer simulation, the convergence behavior of the algorithm operating with different wordlengths in a noise-free condition. In each case, the mean square value of the overall error signal, $e'_{\mathrm{LLMS}}$, i.e., MSE, is obtained after 1024 iteration to ensure complete convergence of the algorithm. The values of the step sizes adopted for simulating the LLMS and $\mathrm{LLMS}_1$ algorithms are as tabulated in Table 1 for operation with wordlengths, Nb, ranging from 6 to 12 bits. In the case of $N_b=6$ bits, slightly different step size values are used to obtain acceptable MSE values. The resultant residue MSE values obtained when the $\mathrm{LLMS}_1$ and LLMS algorithms are operating with wordlengths Nb ranging from 6 to 12 bits are plotted in Fig. 1. It is observed that a minimum numerical precision equivalent to a wordlength of 8 bits is considered adequate to yield an acceptable MSE performance for the $\mathrm{LLMS}_1$ and LLMS algorithms.

TABLE 1: VALUES OF THE CONSTANTS ADOPTED FOR OPERATION WITH THE WORDLENGTH, $N_B$, IN A NOISE FREE CHANNEL.

| *Algorithm* | *In the range of 7 to 12 bits* | *With 6-bit* |
|---|---|---|
| $\mathrm{LLMS}_1$ | $\mu_1=0.2,\ \mu_2=0.03$ | $\mu_1=0.2,\ \mu_2=0.05$ |
| LLMS | $\mu_1=0.2,\ \mu_2=0.1$ | $\mu_1=0.2,\ \mu_2=0.1$ |

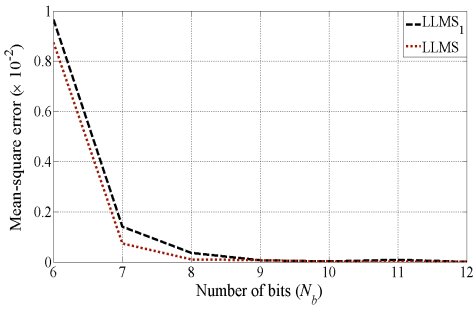


Fig. 1. The variations of residual MSE as a function of the wordlength used to implement the $\mathrm{LLMS}_1$ and LLMS algorithms in a noise free channel.

The theoretical overall error signals, $e'_{\mathrm{LLMS}}$, for the $\mathrm{LLMS}_1$ and LLMS algorithms have been computed from (9) for a wordlength of 8 bits. These are plotted in Fig. 2. For comparison, the overall error signals, $e_{\mathrm{LLMS}}$, computed from (5) with full numerical precision are also plotted in Fig. 3 for the two versions of the LLMS algorithm. It is observed that the values of $e'_{\mathrm{LLMS}}$ are only slightly larger than the corresponding $e_{\mathrm{LLMS}}$ values during their transition to convergence. The difference becomes insignificant beyond 30 iterations. This confirms the asymptotic behaviour of the LLMS algorithm operating with finite numerical precision, as predicted by (21).

Next, the rates of convergence of the $\mathrm{LLMS}_1$ and LLMS algorithms have been simulated using an 8-bit wordlength. The resulting curves are plotted as shown in Fig. 3. The simulated results compare well with the theoretical curve presented in Fig. 2 for the $\mathrm{LLMS}_1$ and LLMS algorithms. It is observed from Fig. 3 that the $\mathrm{LLMS}_1$ and LLMS algorithms converge much quicker than the other three algorithms, namely, LMS, MRVSS and CSLMS algorithms, which have also been implemented with the same numerical precision.

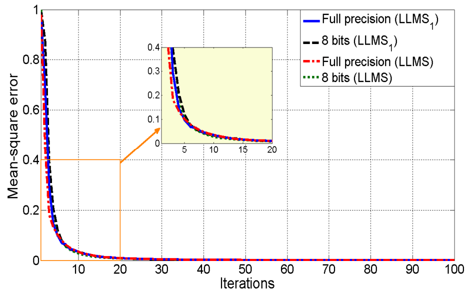


Fig. 2. The theoretical values of MSE of the LLMS and $LLMS_1$ algorithms obtained with full numerical precision and 8-bit precision.

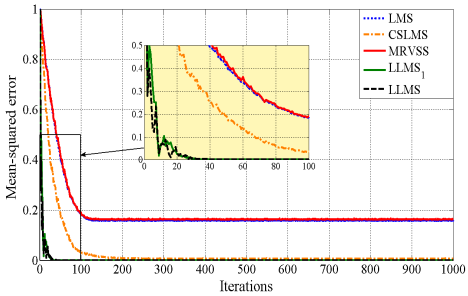


Fig. 3. The rates of convergence of the LLMS, $LLMS_1$, CSLMS, MRVSS, and LMS algorithms based on 8-bit precision.

### *B. EVM and Scatter Plots*

The influence of finite numerical precision on the fidelity of the received signal is investigated based on the error vector magnitude (EVM) [9]. The EVM values are calculated based on output signal obtained after 1024 iterations to ensure complete convergence of a given algorithm. Fig. 4 shows the EVM values obtained with the LLMS1, LLMS, LMS, MRVSS and CSLMS algorithms operating under different numerical precision, ranging from 6 to 12 bits. The values of the various parameters used to simulate the LMS, CSLMS and MRVSS algorithms are tabulated in Table 2. From the results of Fig. 4, it is observed that the proposed $LLMS_1$ and LLMS algorithms perform much better than the other three algorithms when operating under finite numerical precision.

TABLE 2: VALUES OF THE CONSTANTS ADOPTED, FOR THE LMS, CSLMS AND MRVSS ALGORITHMS OPERATING WITH DIFFERENT WORDLENGTHS.

| *Algorithm* | *Parameters for operation in a noise free channel* |
|---|---|
| LMS | $\mu = 0.02$ |
| CSLMS | $\varepsilon = 0.02,\ \ \mu = 0.02$ |
| MRVSS | $\beta_{\max} = 1,\ \beta_{\min} = 0,\ \upsilon = 5\times10^{-4},\ \mu_{\max} = 0.2,\ \mu_{\min} = 10^{-4}$<br>Initial $\mu = \mu_{\max},\ \alpha = 0.97,\ \gamma = 4.8\times10^{-4},\ \eta = 0.97$ |

## V. CONCLUSION

In this paper, the operation of the proposed LLMS algorithm under finite numerical precision has been examined. First, the convergence behavior of the LLMS algorithm, based on the minimum mean square error, has been analyzed for operation with finite numerical precision. It is shown, by means of computer simulation, that an eight element uniform linear array can be implemented with an 8-bit precision using the LLMS algorithm. The resulting performance of the array is close to that achieved when full numerical precision is used. Comparisons based on various performance measures, such as residual MSE, rate of convergence, and error vector magnitude, show that the LLMS and $LLMS_1$ algorithms outperform the other three previously published algorithms, namely, least mean square (LMS), modified robust variable step size (MRVSS) and constrained stability LMS (CSLMS).

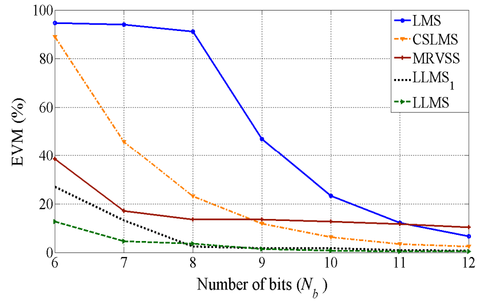


Fig. 4. The EVM values obtained with the $LLMS_1$, LLMS, LMS, CSLMS and MRVSS algorithms implemented with different wordlengths.

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