

Blind multiuser separation of instantaneous mixture algorithm based on geometrical concepts

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Abstract

In this paper, we propose a new and simple algorithm for blind separation of sources based on geometrical concepts. This algorithm deals with the instantaneous mixtures and does not require the estimation of high-order statistics (HOS). The proposed algorithm can separate sources that belong to two different categories: (1) Signals with uniform or close to uniform probability density function (PDF). (2) Unimodal PDF (including Pascal and Gamma) as well as some real signals such as speech or music. Unfortunately, the actual version of the algorithm cannot deal with the mixing of signals from both categories. Finally, the experimental results show good performances and that the separation can be carried out in a very short time (a few seconds, using Matematica as the programming language and our ultra 30 Sun station). © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Blind separation of sources; ICA; PDF; Geometrical methods; Decorrelation; Cholesky factorization; Orthogonalization and whitening process

1. Introduction

Because it can be applied in different situations and has many applications [21], the blind separation of sources problem (BSS) has been a topic of interest in signal processing since 1984 [13]. In the past decade, it has been studied by many authors, and many algorithms have been proposed. The BSS involves retrieving unknown sources $X(n)$ by only observing a mixture $Y(n)$ of them [8,14] (see Fig. 1). In general, authors assume that the sources are non-Gaussian signals (at most, one of the sources can be a Gaussian signal) and statistically independent of one another. Therefore, concepts of independent component analysis (ICA) [6] have been widely used and developed to solve the BSS.

Conventionally, researchers consider three models of the transmission channel: a linear model such as an instantaneous mixture (i.e., memory-free channel) or a convolutive mixture (i.e., memory channel) or

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Nomenclature

p	number of sources and the number of sensors
n	time
$X(n) = (x_i)$	vector of the sources (x_i is the i th source)
$Y(n)$	vector of the observations or the mixing signals
$Z(n)$	vector of the orthogonal signals
$S(n)$	estimated signals
\mathbf{M}	channel effect or the mixing matrix
\mathbf{W}	separation matrix
$\mathbf{G} = \mathbf{W}\mathbf{M}$	global matrix
\mathbf{K}	orthogonalization matrix
\mathbf{I}_p	$p \times p$ identity matrix
$\mathbf{R}(\theta)$	rotation matrix
\mathbf{R}_Z	covariance matrix of $Z(n)$
\mathbf{R}_Y	covariance matrix of the mixing signals $Y(n)$
$\dot{g}(u)$	derivative of $g(u)$
$\text{Sign}(x)$	function sign: $\text{Sign}(x) = x/ x $ if $x \neq 0$ and $\text{Sign}(x) = 1$ for $x = 0$

a nonlinear channel. The last model has been described very little in general, however some algorithms have been proposed for specific nonlinear models [17,26,28,35]. Concerning the instantaneous or the convolutive model, different criteria and approaches have been proposed [1,4,5,10,19,22,24], most of which are based on HOS.

In the general case, when the sources are assumed to be non-Gaussian and independent signals, it has been proved that the second-order statistics (SOS) are not sufficient to separate the instantaneous mixture (i.e., the channel effect can be represented as the product of the source vector X and a scalar full-rank matrix \mathbf{M}) [8,23]. However, many algorithms based only on SOS have been proposed to deal with special signals: correlated signals with time [3,7,29,36], colored sources [9], and nonstationary signals [25]. Without using statistical information (moments or cumulants), some researchers proposed a very simple geometrical approach to deal with special signals (as binary or n -valued) [33]. This algorithm was modified to separate analog signals [32,34]. The idea is based on using the independence concept from the geometrical point of view. In fact, the scatter plot of two independent signals (i.e., $x_2(n)$ against $x_1(n)$ for every n) is rectangular. On the other hand, the mixing effect induces a geometrical transformation of that rectangular plot to a parallelogram (see Fig. 2).

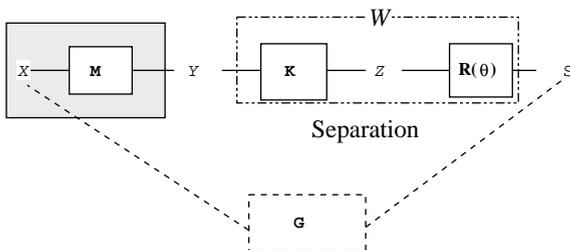


Fig. 1. Channel model.

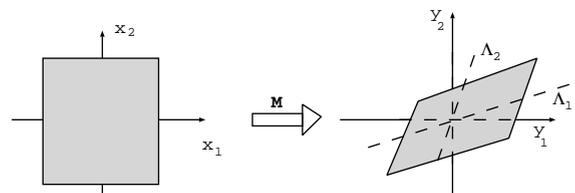


Fig. 2. Geometrical concepts.

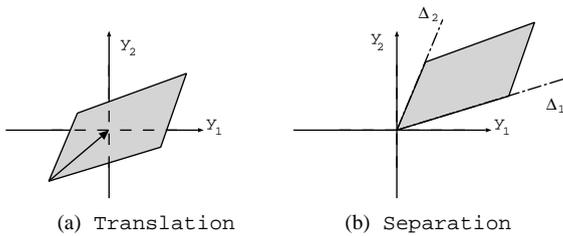


Fig. 3. Previous algorithm. (a) Translation (b) separation.

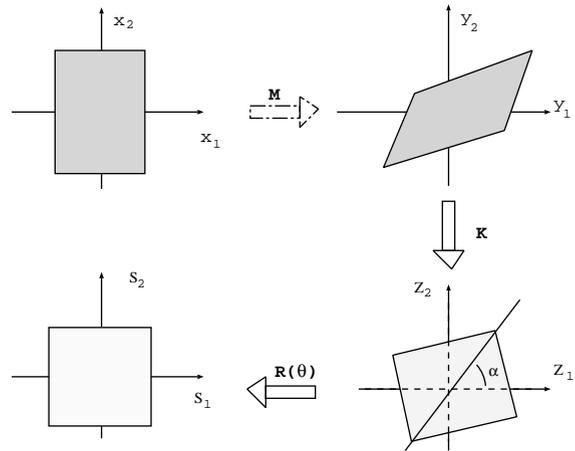


Fig. 4. The new algorithm.

The algorithm proposed in [31,32] consists of two steps (see Fig. 3):

- Translation of the parallelogram to the first quadrant.
- Determining the slopes of the edges Δ_1 and Δ_2 of the parallelogram. The researchers proved that the slopes are equal to $\min_i(x_i(n)/x_j(n))$, for $i \neq j$. They showed how the mixing parameters can be easily obtained from the slopes.

The performances and the limitations of the algorithm were discussed in [31]. The main problem with the algorithm is the necessity of the translation step. On the other hand, it becomes very unlikely to determine the edges of the parallelogram in the case of nonuniform PDF signals. If there are more than two sources, their algorithm cannot be easily generalized.

Recently, another geometrical algorithm was proposed to deal only with speech signals or signals with PDF close to Gamma PDF [30]. For such signals, it is very difficult to determine the edges of the parallelogram. Therefore, the authors suggest an algorithm for determining the slopes of the principle axes of symmetry A_1 and A_2 of the parallelogram (see Fig. 2). To achieve this goal, the authors divide the mixture plan (the scatter plot) into many clusters and they chose the direction of the principle axes of symmetry A_1 and A_2 as the direction of the more popular cluster (i.e., the cluster which contains the maximum number of observation samples). One should note that the performance of this new approach depends on the estimation of the slopes of axes A_1 and A_2 . The latter estimate depends on the number of the clusters and the decision to select the right one (if the number of clusters is small then the decision can be made easily but the estimation of the slopes becomes biased, and vice versa).

2. Main idea

In the following, we consider that the mixture is an instantaneous mixture with the same known number p of sources and sensors.¹ It is known that the separation of the instantaneous mixture can be achieved up

¹ The number of sources can be easily determined as the number of the strongest eigenvalue of the covariance matrix of the mixing signals [11,27].

to a permutation and a scale factor [6,21] (i.e., the estimated sources $S(n)$ are equal to the sources up to a permutation and a scale factor). Therefore, the global matrix $\mathbf{G} = \mathbf{W}\mathbf{M}$ (\mathbf{W} is the demixing matrix or the separating matrix) should satisfy the following:

$$\mathbf{G} = \mathbf{P}\mathbf{D}, \quad (1)$$

where \mathbf{P} is any permutation and \mathbf{D} is any full-rank diagonal matrix.

The instantaneous mixture can be considered a geometrical transformation in the observation space (or plane in the case of two sources) that it transforms, in the case of two sources for example, the scatter plot of the sources from rectangular to parallelogram. To separate the signals or to cancel the mixing effect, we propose a new algorithm consisting of two steps; see Figs. 1 and 4. To better explain our idea, we first describe the algorithm in the case of two signals, after which we discuss the general case.

2.1. Transformation

Orthogonal signals Z can be easily generated from the mixing signals Y by Cholesky decomposition [12]. By achieving this, the mixing parallelogram is transformed into a square. Let $\mathbf{R}_Y = \mathbf{E}Y Y^T$ be the covariance matrix of $Y(n)$. Using Cholesky decomposition, one can obtain a square root \mathbf{L} of \mathbf{R}_Y such that $\mathbf{R}_Y = \mathbf{L}\mathbf{L}^T$. When the sources are statistically independent and the number of sources is equal to the number of sensors, the mixing covariance matrix \mathbf{R}_Y becomes a full-rank matrix as well as its square root \mathbf{L} . Let $\mathbf{K} = \mathbf{L}^{-1}$. Finally, Z can be obtained as $Z(n) = \mathbf{K}Y(n)$. We should mention that the covariance matrix \mathbf{R}_Z of Z is equal to the identity matrix $\mathbf{R}_Z = \mathbf{E}Z Z^T = \mathbf{K}\mathbf{E}Y Y^T\mathbf{K}^T = \mathbf{L}^{-1}\mathbf{R}_Y\mathbf{L}^{-T} = \mathbf{I}_p$; here, \mathbf{I}_p is a $p \times p$ identity matrix.

2.2. Rotation

The estimated signals $S(n)$ can be obtained from the orthogonal mixing signals $Z(n)$ via rotation by an angle θ as $S(n) = \mathbf{R}(\theta)Z(n)$; here

$$\mathbf{R}(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}.$$

Let α be the angle between the first diagonal and the horizontal axis (see Fig. 4). Angle α can be estimated from the coordinate of the farthest point from the origin. It is clear that the determination of α is not unique and this is due principally to the permutation indeterminacy; see Eq. (1). Taking into account the fact that all the physical signals are bounded and using the scale factor indeterminacy (1), one can consider that the sources have the same maximum amplitude or that the scatter plot of the sources is square. Finally, to separate the signals, one should rotate the signals Z by an angle θ .

$$\theta_U = \pi/4 - \alpha. \quad (2)$$

We should mention that θ_U can be obtained using the previous equation when the sources have uniform or close to uniform PDF (as well as signals with a histogram similar to a rectangle as sinusoidal signals). In the case of the signals with unimodal PDF the same as the symmetrical Gamma PDF and similar to symmetrical Gamma (as Cauchy PDF or Laplace's PDF) or close to symmetrical Gamma (as the speech signals), the scatter plot of these signals is more likely to be a kind of cross with a disc shape than a rectangle (see Fig. 5). In this case the farthest point (i.e., the second step of our algorithm) will correspond to one of the two axes of the cross. Therefore θ should be evaluated as:

$$\theta_G = -\alpha. \quad (3)$$

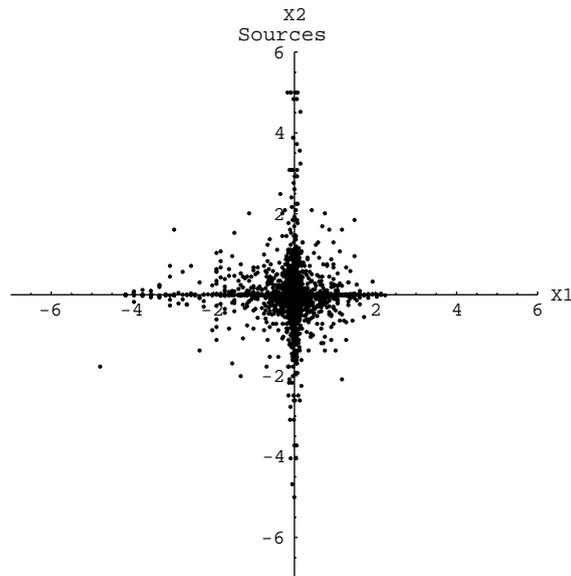


Fig. 5. The scatter plot of two speech signals.

3. Separation of multi-sources

Here, we explain how the actual algorithm can be generalized for more than two sources. First of all, the case of three sources is discussed. Subsequently, we briefly emphasize the problems and the generalization process for the case of more than three sources.

3.1. Three sources

Similar to the case of two sources, one should distinguish between the signals with similar or close-to-uniform PDF and the speech signals or the signals with the Laplacian PDF or Gamma PDF. To Clarify, let us first discuss the case of uniform PDF signals. In the 3D space of the sources, the scatter plot of the independent sources $X(n)$ becomes a rectangular parallelepiped (similar to a box) such that its edges are parallel to the base axes, and its center is at the origin of the axes denoted by the point “O”. On the other hand, the scatter plot of the mixture signals $Y(n)$ becomes a parallelepiped.

Using the same approach as that in Section 2.1, one can generate three orthogonal signals $Z(n)$ so that their scatter plot is a cube C_o and the center of the cube is the origin of the axes. To separate the sources up to the permutation and scale factors, one should rotate the cube such that its edges are parallel to the base axes and its four principal diagonals are defined by their parametric equations as: $D_1: x = y = z = t$, $D_2: -x = y = z = t$, $D_3: x = -y = z = t$ and $D_4: x = y = -z = -t$. One vertex, say point “A”, of the cube can be easily determined using the “farthest point from the origin” strategy. One can easily rotate the cube C_o such that point “A” becomes a point on the straight line D_1 ; see Appendix C. In this case, one principal diagonal of the rotated cube C_r becomes colinear to D_1 . The position of the cube C_r can be determined up to one degree of freedom (i.e., a rotation by any angle ψ around the fixed principal diagonal or the straight line D_1). Finally, one can find another vertex “B” of the cube C_r (see Appendix D), which is then rotated around the straight line D_1 (using the rotation matrix of Appendix E) by an angle ψ (see Appendix F) such that the edges of the final rotated cube will be parallel to the axes or the signals will become independent.

In the case of speech signals, signals with Laplace PDF, Gamma Pdf or close to such PDF, the scatter plot of three independent zero-mean signals becomes close to star form.² Using the same approach as that in Section 2.1, one can generate three orthogonal signals $Z(n)$ such that their scatter plot is similar to the shape of the scatter plot of the sources except that it is rotated in 3D space. As in the case of two sources, using the “farthest point from the origin” strategy one can find a radiating point “ A ” (i.e., one symmetrical axis \vec{oA}) of the scatter plot of $Z(n)$. Therefore the scatter plot of $Z(n)$ can be rotated in 3D space (using the rotation matrix from Appendix C) such that axis \vec{oA} becomes colinear with the vertical axis \vec{oZ} and the obtained scatter plot is denoted by SP_r . After rotation, a second symmetrical axis can be found based on the fact that the “farthest point B ” in the horizontal plan oxy is a radiating point of the rotated scatter plot SP_r . Finally, the separation is obtained after the rotation of the scatter plot SP_r around the vertical axis \vec{oZ} such that \vec{oB} becomes colinear to the horizontal axis \vec{oX} using the following Givens matrix:

$$R(\alpha) = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (4)$$

Here, $\alpha = (1 - \text{Sign}(X_B))\pi/2 + \arctan(Y_B/X_B)$, and X_B and Y_B are the coordinates of B .

3.2. General case

The first step (Section 2.1) of the algorithm can be used without any modification. For the second step (Section 2.2), one should consider $p - 1$ angles. These angles can be determined from the projection of the farthest point, and the rotation can be conducted mainly using the Givens rotation matrix [12]. In other words, one can use an approach similar to that on the 3D case to generalize this approach for more than three sources. On the other hand, it should be mentioned that to the extent that we increase the space dimensions of the sources, the number of samples needed to achieve the separation increases rapidly. In the case of two sources, the size of signals needed to achieve the separation is less than 6000 samples and the minimum number is three to four hundred samples. However, in the case of three sources, the experimental study showed that the minimum number of samples is between 9000 and 20,000 samples. Good performances were obtained for more than 30,000 samples (in the case of speech or musical signals). Due to these limitations, we do not present here any further details of the experimental results concerning the generalization process. Actually, we are attempting to modify the second step of the proposed geometrical approach such that it can be easily applied for a general number of sources, and to improve its performance to achieve the separation using a small number of samples.

4. Experimental results

We conducted many experiments using different kinds of signals. All of the conducted experiments show good results. In this section, we present some of the experiment results obtained.

4.1. Two sources

Firstly, we consider two zero-mean signals with uniform PDF and unity variance. The mixing matrix is

$$\mathbf{M} = \begin{pmatrix} 4 & 1 \\ 2 & 5 \end{pmatrix}.$$

² This form is the generalization of the form in the case of two sources; see Fig. 5.

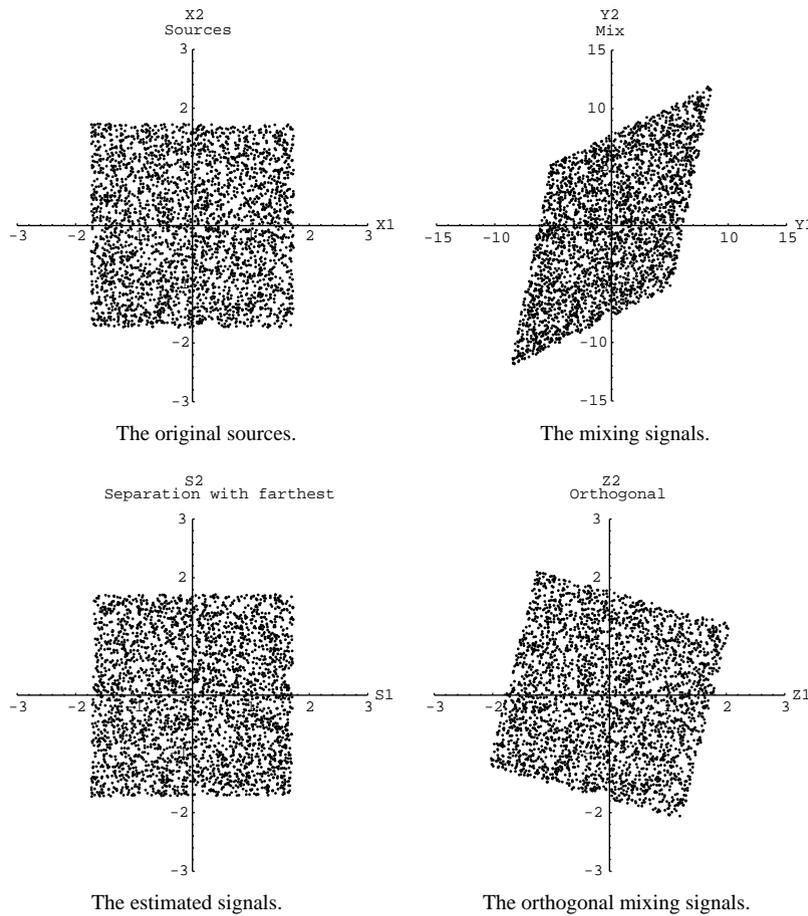


Fig. 6. The separation of two uniform PDF signals. (a) The original sources, (b) the mixing signals, (c) the estimated signals, (d) the orthogonal mixing signals.

The estimated signals have been obtained with a global matrix of

$$\mathbf{G} = \begin{pmatrix} 0.0110814 & -0.989114 \\ 0.989706 & -0.0061244 \end{pmatrix}.$$

We used 3000 samples (see Fig. 6). The experimental study shows satisfactory results even with a few hundred samples (in the case of uniform PDF, we obtained satisfactory results with 150 samples and very good results with more than one thousand samples).

Secondly, the separation of two speech signals is presented. The mixing matrix is

$$\mathbf{M} = \begin{pmatrix} 4 & -2 \\ 3 & 8 \end{pmatrix}.$$

The separated signals have been estimated with a global matrix of

$$\mathbf{G} = \begin{pmatrix} 0.000513767 & -0.964096 \\ 0.99271 & 0.00539761 \end{pmatrix}.$$

We used the first 3000 samples of the signals (see Fig. 7). To separate some musical signals, we used up to 6000 samples and we obtained similarly good results.

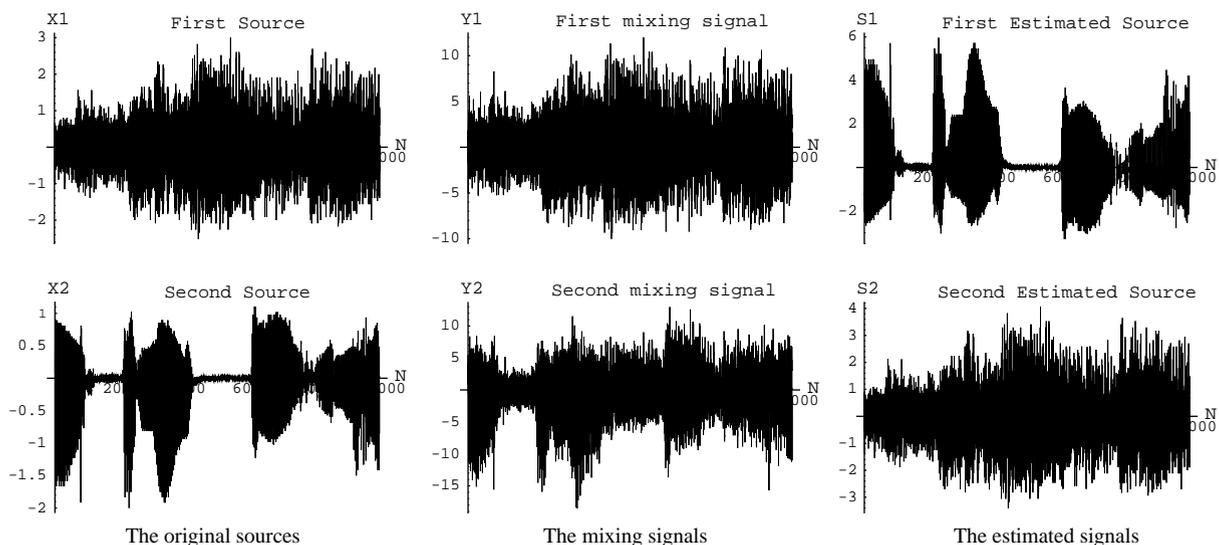


Fig. 7. The separation of two speech signals. (a) The original sources, (b) the mixing signals, (c) the estimated signals.

4.2. Three sources

For the separation of three zero-mean sources mixed with an instantaneous mixture and a random mixing matrix (full-rank), many experimental studies have been conducted. The sources are chosen from three different categories:

- Uniform or close-to-uniform PDF signals (for example sinusoidal signals).
- Laplacian or Gamma PDF or close to these PDF signals (see Appendix B).
- Speech or music signals are chosen from a wide data base (over 60 different signals of speech (from women and men) and music, registered under different conditions: e.g., normal room or anechoic room and different sampling frequencies)

For three zero-mean signals with uniform PDF, good performance results have been obtained with approximately 10,000 samples (see Fig. 8). In the experiment result shown in Fig. 8, the mixing matrix was

$$M = \begin{pmatrix} 5 & 1 & -2 \\ 0 & 7 & -1.5 \\ 2.1 & 3 & 6 \end{pmatrix}$$

and the separation was achieved with a global matrix of

$$G = \begin{pmatrix} -0.00191998 & -0.0189407 & 0.358797 \\ 0.59661 & 0.00140027 & -0.00694529 \\ -0.0249461 & 0.445696 & 0.00644428 \end{pmatrix}.$$

To study the effect of the number of samples on the performance of the algorithm, we conducted some experiments, where using the same mixing matrix and sources, we tried to separate the signals using different numbers of samples. The experimental study shows that the number of samples has a small impact on the performances as soon as we can estimate the farthest point belonging to a symmetrical axis of the scatter

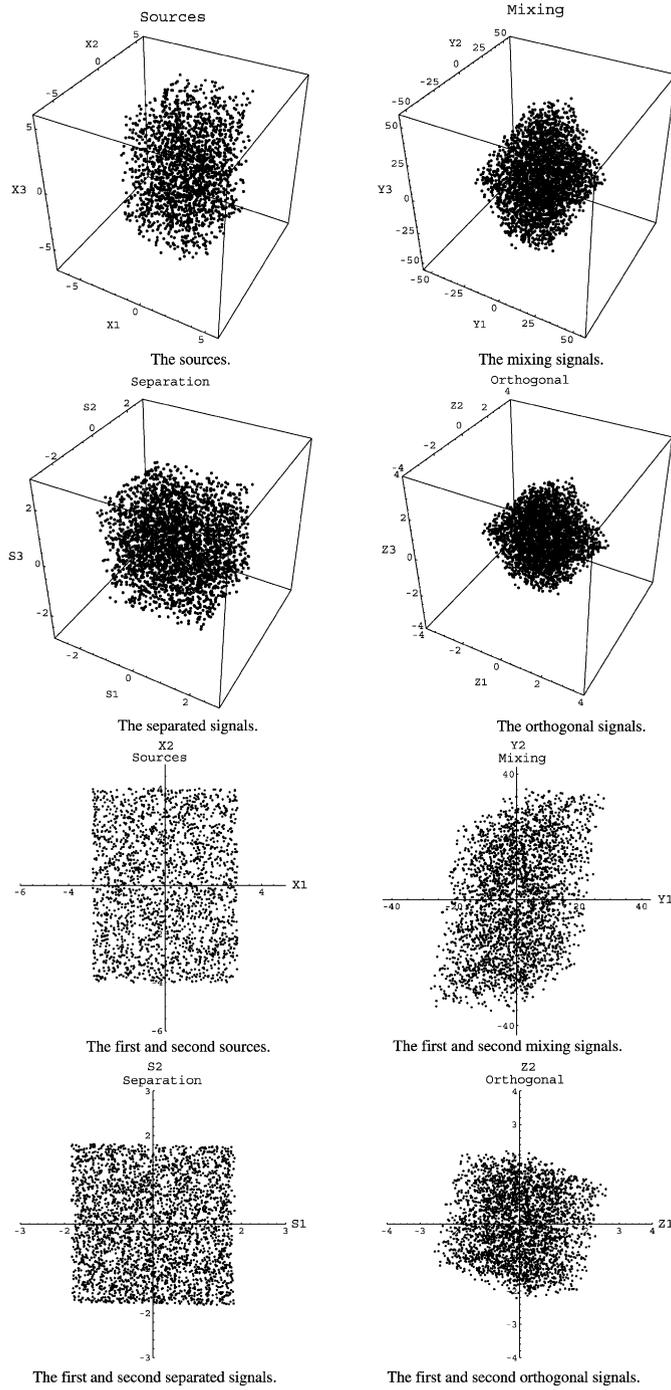


Fig. 8. The separation of three uniform PDF signals. (a) The sources, (b) the mixing signals, (c) the separated signals, (d) the orthogonal signals, (e) the first and second sources, (f) the first and second mixing signals, (g) the first and second separated signals, (h) the first and second orthogonal signals.

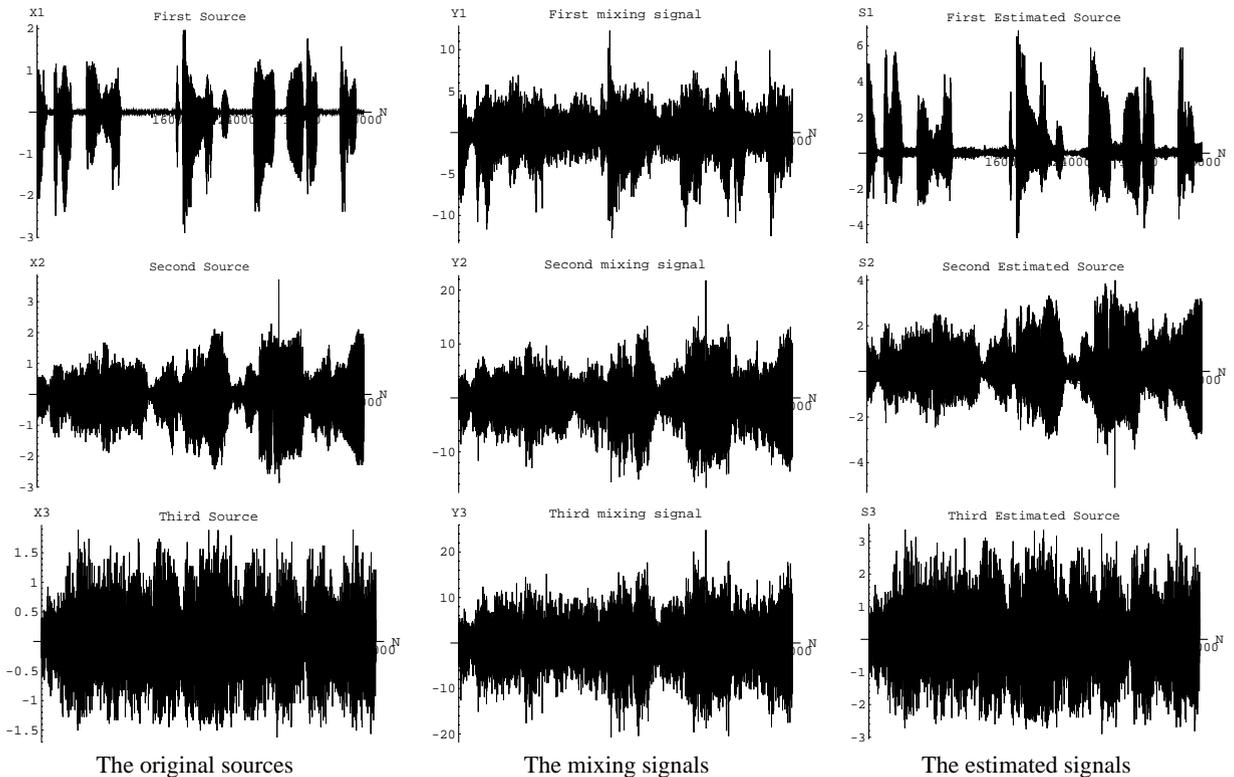


Fig. 9. The separation of two speech signals and a music signal. (a) The original sources, (b) the mixing signals, (c) the estimated signals.

plot. For example, for three zero-mean signals with different Laplacian PDFs, the obtained global matrix was

$$G = \begin{pmatrix} -0.0409554 & 1.43742 & 0.0503483 \\ -1.62211 & -0.0579908 & 0.176867 \\ 0.129741 & -0.0436741 & 2.20528 \end{pmatrix}$$

when we used 50,000 samples and

$$G = \begin{pmatrix} -0.0427005 & 1.46258 & 0.0509568 \\ -1.62352 & -0.0580412 & 0.177021 \\ 0.106611 & -0.0448919 & 2.22576 \end{pmatrix}$$

for 10000 samples. The mixing matrix was

$$M = \begin{pmatrix} 8 & 3.5 & -2 \\ 1 & 4 & -1.5 \\ 3.4 & 2.5 & 5 \end{pmatrix}.$$

Finally, many experiments were conducted to separate the instantaneous mixture of three speech or music signals. Good performances were obtained by applying the proposed algorithm for more than 20,000 samples. Fig. 9 shows the separation of two speech signals (for a woman and a man) and a song with a music signal using 40,000 samples. The mixing matrix was

$$M = \begin{pmatrix} 4 & -1 & 2.6 \\ 2.1 & 5.298 & 2.298 \\ 3.399 & 5.2 & 6 \end{pmatrix},$$

and the global matrix obtained was

$$G = \begin{pmatrix} -2.40495 & -0.228481 & 0.0664484 \\ 0.405719 & -1.38551 & 0.0576037 \\ 0.0603367 & 0.0520547 & 1.79789 \end{pmatrix}.$$

5. Conclusion

Here, a new and simple algorithm for blind separation of sources based on geometrical concepts is presented. The algorithm has been presented and tested in the case of two and three sources. The experimental results show that the performances of the algorithms are very satisfactory even for the separation of real nonstationary signals such as the speech signals. Even if the algorithm requires thousands of samples (less than 7000 samples for two signals and over 9000 for three sources), the convergence time is very competitive. The separation can be carried out in a very short time (a few seconds, using Matematica as the programming language and an ultra 30 Sun station). The number of samples can be reduced effectively to a few hundred samples in the case of two stationary signals with a uniform or close-to-uniform PDF (as well as the signals with a histogram similar to a rectangle such as sinusoidal signals). Finally, the experimental studies show that the algorithm is robust even when the signal-to-noise ratio is approximately 10 dB (see Appendix H).

Acknowledgements

We would like to thank the unknown referees and the editor and the co-editor for their suggestions and comments.

Appendix A. Algorithm performances

In this section, we assume that the two sources are zero-mean-bounded signals ($-1 < x_i < 1$, $i \in \{1,2\}$) with uniform PDF.³ The geometrical approach consists of two steps: whitening (using the SOS of the mixing signals $Y(n)$) and rotation (using the coordinate of the farthest point from the origin in the scatter plot of the orthogonal signals $Z(n)$). The estimation errors of the first step basically come from the estimation of SOS and from numerical algorithms (such as Cholesky factorization). In the literature, there are many reports on the estimation and the minimization of such errors [2,15,16,18,20]. Therefore, we focus on the second step or the estimation of the geometrical error assuming that the orthogonal signals are perfect. In this case one can assume that

$$Z(n) = R(\alpha)X(n), \tag{A.1}$$

where $R(\alpha)$ is the rotation matrix of an angle α . The geometrical error occurred when we estimated one vertex of the square by a point in a small square (with a side of ε) near that vertex; see Fig. 10. Based on Eq. (A.1) and the fact that $\det R(\alpha) = 1$, one can conclude that the joint PDF of Z_1 and Z_2 is equal to the joint PDF of the sources. On the other hand, the relationship between the orthogonal signals $Z(n)$ and the sources is bijective. Therefore, one can prove that the probability of finding a point in small squares near the vertices in the scatter plot of $Z(n)$ is the same as the probability of finding a point in small squares (of the same size)

³ The generalization for three sources or signals with different PDFs can be easily developed using approaches similar to those described in this section.

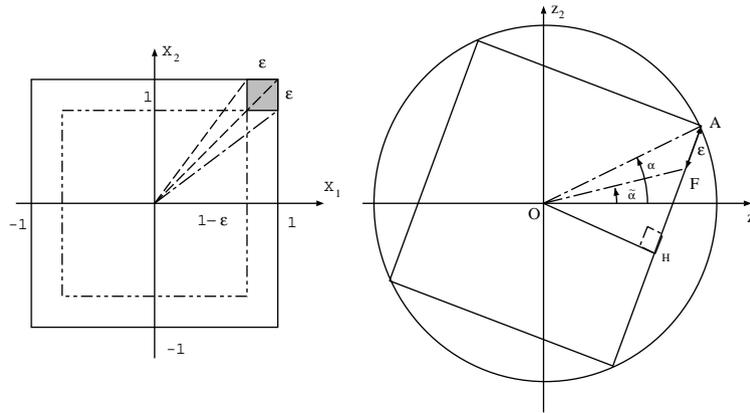


Fig. 10. Performances.

near the vertices in the scatter plot of the sources. Using the independence of the sources and the uniform PDF, one can calculate this probability (see Fig. 10)

$$\begin{aligned}
 & \Pr(\text{Point at the corner}) \\
 &= 4\Pr(\text{Point at one corner}) = 4\Pr(1 - \epsilon < x_1 < 1, 1 - \epsilon < x_2 < 1) \\
 &= 4\Pr(1 - \epsilon < x_1 < 1)\Pr(1 - \epsilon < x_2 < 1) = 4(1 - F_X(x))^2, \tag{A.2}
 \end{aligned}$$

where $F_X(x)$ is the cumulative density function (CDF) of the uniform random variable. In our case, $F_X(x) = (1 + x)/2$ and Eq. (A.2) can be rewritten as

$$\Pr(\text{Point at the corner}) = \epsilon^2. \tag{A.3}$$

Let α be the correct angle of the “farthest point from the origin” (or vertex A ; see Fig. 10) and let $\tilde{\alpha}$ be the estimate of α (i.e., $\overline{AF} = \epsilon$ is the maximum error over the estimate of point A). If we denote $\delta\alpha = \alpha - \tilde{\alpha}$, from Fig. 10 we find that

$$\cos(\delta\alpha) = \frac{2 - \epsilon}{\sqrt{2}\sqrt{2 - 2\epsilon + \epsilon^2}}, \tag{A.4}$$

$$\sin(\delta\alpha) = \frac{\epsilon}{\sqrt{2}\sqrt{2 - 2\epsilon + \epsilon^2}}. \tag{A.5}$$

Assuming that $\epsilon \ll 1$, one can rewrite Eqs. (A.4) and (A.5) as:

$$\cos(\delta\alpha) = 1 - \frac{\epsilon^2}{8} + O(\epsilon^3), \tag{A.6}$$

$$\sin(\delta\alpha) = \frac{\epsilon}{2} + \frac{\epsilon^2}{4} + O(\epsilon^3), \tag{A.7}$$

where $O(\epsilon^3)$ is a residual error of the ϵ^3 order.

The orthogonal signals $Z(n)$ are equal to the sources up to a rotation matrix. An estimation error on α has an immediate effect over the global matrix \mathbf{G} such that:

$$\mathbf{G} = \mathbf{P}\Delta\mathbf{R}(\delta\alpha), \tag{A.8}$$

where \mathbf{P} is any permutation matrix and Δ is any full-rank matrix. It can be easily proved that Eq. (A.8) indicates that the sources will be estimated with a crosstalk of:

$$\text{crosstalk} = \tan^2(\delta\alpha) \simeq \frac{\epsilon^2}{4}. \tag{A.9}$$

If we want to separate the sources with a crosstalk of -20 dB, then:

$$\text{crosstalk} = -20 \text{ dB} \Rightarrow \varepsilon = 0.2. \tag{A.10}$$

and the probability of finding a point in a small square near the vertex to estimate α is $\Pr = \varepsilon^2 = 0.04 = 4\%$. Therefore, using approximately 1000 samples, we consider that one can find approximately 40 good points near the vertex for the estimation of α .

Appendix B. Different PDFs

In this section, we evaluate some transformation functions that can generate signals with specific PDF from a uniform PDF signal (see the previous section). Let u be a uniform PDF $P_U(u)$ signal with $0 \leq u \leq 1$ and y be a signal with (for example) monolateral exponential PDF, i.e.,

$$P_Y(y) = \begin{cases} a \exp(-ay) & \text{where } a > 0 \text{ and } y \geq 0, \\ 0 & y < 0. \end{cases} \tag{B.1}$$

Using the previous definition, the mean $E\{y\}$ and the variance σ_y^2 of y can be easily evaluated as $E\{y\} = 1/a$ and $\sigma_y^2 = 1/a^2$. The CDF of y can also be easily evaluated using Eq. (A.9):

$$F_Y(y) = \int_{-\infty}^y P_Y(y) dy = \int_0^y a \exp(-ay) dy = 1 - \exp(-ay). \tag{B.2}$$

Finally, y can be obtained from samples of u using the inverse of its CDF, i.e., $y = g(u) = F_Y^{-1}(y) = -(\log(1 - u))/a$. We can generate a zero mean and unity variance variable with the monolateral exponential PDF by $y_c = ay - 1$.

In the same manner, one can find Table 1.

Appendix C. Rotate a 3D vector to specific location

In this section, we explain how can we determine a rotation matrix that rotates vector U to make it colinear with vector V :

$$U = |U| \begin{pmatrix} \sin(b) \cos(a) \\ \sin(b) \sin(a) \\ \cos(b) \end{pmatrix} \quad \text{and} \quad V = |V| \begin{pmatrix} \sin(\varphi) \cos(\alpha) \\ \sin(\varphi) \sin(\alpha) \\ \cos(\varphi) \end{pmatrix}. \tag{C.1}$$

Here, $|U|$ is the norm of vector U , and the angles a and b (resp. α and φ) are the polar coordinate of the U (resp. V); see Fig. 11.

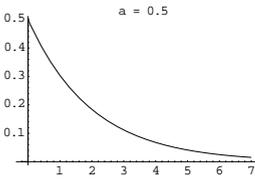
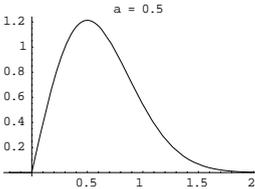
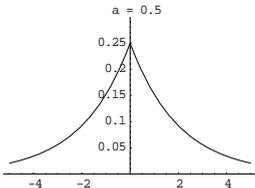
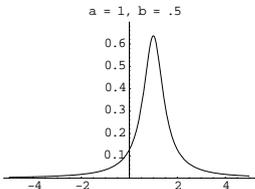
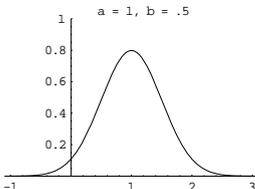
After some algebraic and trigonometric operations, one can determine the global rotation matrix that rotates U to V as

$$\mathbf{R} = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\varphi - b) & 0 & \sin(\varphi - b) \\ 0 & 1 & 0 \\ -\sin(\varphi - b) & 0 & \cos(\varphi - b) \end{pmatrix} \begin{pmatrix} \cos(a) & \sin(a) & 0 \\ -\sin(a) & \cos(a) & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{C.2}$$

The rotation matrix \mathbf{R} can be seen as a rotation of an angle $-a$ around oz plus a rotation of an angle $\varphi - b$ around oy , then a rotation of an angle α around oz .

Finally, the angles a and b (resp. α and φ) can be determined very easily from the coordinate of the vector U (resp. V). Let $U = (U_x, U_y, U_z)^T$; in this case one can find that $a = \arctan(U_y/U_x)$ if $U_x > 0$, and that

Table 1
Different PDFs

PDF	Definitions	Shapes	Transformation functions
Monolateral exponential	$\begin{cases} a \exp(-ay) & a > 0 \text{ and } y \geq 0 \\ 0 & y < 0 \end{cases}$	 <p>Monolateral exponential PDF</p>	$-\log(1 - u)/a$
Rayleigh	$\begin{cases} \frac{y}{a^2} \exp(-\frac{y^2}{2a^2}) & y \geq 0 \\ 0 & y < 0 \end{cases}$	 <p>Rayleigh's PDF</p>	$\sqrt{-2a^2 \log(1 - u)}$
Laplace	$\frac{ a }{2} \exp(- ay)$	 <p>Laplacian PDF</p>	$\begin{cases} \log(2u)/a & \text{if } 0 \leq u \leq 0.5 \\ -\log(2 - 2u)/a & \text{if } 0.5 \leq u \leq 1 \end{cases}$
Cauchy	$\frac{ b }{\pi(b^2 + (x-a)^2)}$	 <p>Cauchy's PDF</p>	$ b \tan(\pi u - \pi/2) + a$
Gaussian	$\frac{\exp(-\frac{(y-a)^2}{2b^2})}{b\sqrt{2\pi}}$	 <p>Gaussian PDF</p>	$a + b\sqrt{-2 \log(u)} \cos(2\pi v)$ v is similar to u

$a = \pi + \arctan(U_y/U_x)$ if $U_x < 0$. Therefore, one can write:

$$a = \frac{1 - \text{Sign}(U_x)}{2} \pi + \arctan\left(\frac{U_y}{U_x}\right), \tag{C.3}$$

$$b = \frac{1 - \text{Sign}(U_z)}{2} \pi + \arctan\left(\frac{\sqrt{U_x^2 + U_y^2}}{U_z}\right). \tag{C.4}$$

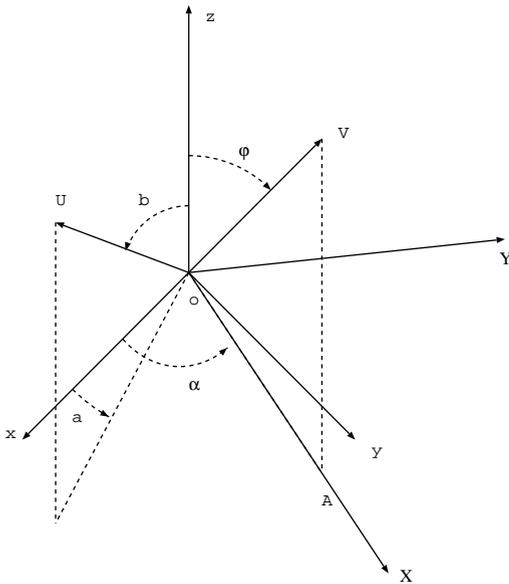


Fig. 11. Rotation of 3D vector.

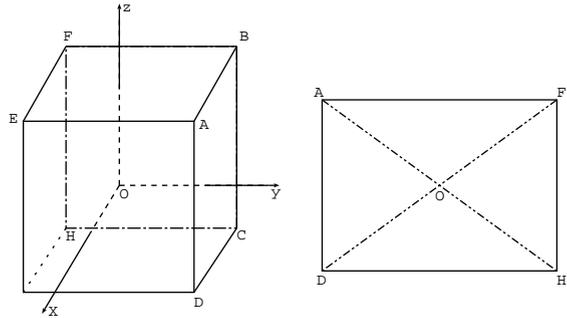


Fig. 12. The cube vertices.

Appendix D. Finding another vertex

Let us assume that one vertex denoted by point A (see Fig. 12) of a given cube is known. How can we find another vertex (let us say B , E or D). These three points are similar and can be used to solve the problem of blind separation of sources up to a permutation. These are the point of the cube such that $|\vec{OB}| = |\vec{OA}| = \frac{\sqrt{3}}{2}|\vec{AB}|$. To find one point among the above three vertices, one could select a set of points Σ_1 such that:

$$B \in \Sigma_1 \Leftrightarrow \begin{cases} \left| |\vec{AB}| - \frac{2}{\sqrt{3}}|\vec{OA}| \right| < e_1, \\ \text{and} \\ \left| |\vec{OB}| - |\vec{OA}| \right| < e_2, \end{cases} \quad (\text{D.1})$$

where e_1 and e_2 are small numbers that describe the precision of the estimation⁴. Finally, we select a point “ B ” in the previous set Σ_1 such that:

$$\min_{B \in \Sigma_1} \left(\left(\left| |\vec{AB}| - \frac{2}{\sqrt{3}}|\vec{OA}| \right| \right)^2 + (|\vec{OB}| - |\vec{OA}|)^2 \right). \quad (\text{D.2})$$

⁴ In our experiments, we chose $\frac{|\vec{OA}|}{100} < e_1 = e_2 < \frac{|\vec{OA}|}{10}$. The experimental results show that the values of e_1 and e_2 do not have a major effect on the performance of the algorithm.

Appendix E. Rotation matrix around a fixed straight line

In this section, we determine the rotation matrix of an angle ψ around a straight line Δ defined by the parametric equation

$$\Delta \begin{cases} x = at, \\ y = bt, \\ z = ct. \end{cases} \quad (\text{E.1})$$

Using the previous Eq. (E.1), one can determine a plan P perpendicular to the line Δ ; $P: ax + by + cz = 0$. Any straight line in P is perpendicular to Δ . Let us consider straight line D_1 given by

$$D_1 \begin{cases} x = -ct, \\ y = 0, \\ z = at. \end{cases} \quad (\text{E.2})$$

Let P_2 be a plan perpendicular to D_1 given by $P_2: cx - az = 0$. Now let $D_2 = P \cap P_2$ such that

$$D_2 \begin{cases} x = at, \\ y = -\frac{a^2+c^2}{b}t, \\ z = ct. \end{cases} \quad (\text{E.3})$$

Let \vec{u} , \vec{v} and \vec{w} be unite norm vectors colinear to D_1 , D_2 and Δ , respectively. It is clear that $(o, \vec{u}, \vec{v}, \vec{w})$ can be considered orthonormal basis axes.

To rotate a vector or a point around Δ by an angle ψ , one can first transform the coordinate of that point from the original basis axes $(o, \vec{i}, \vec{j}, \vec{k})$ to $(o, \vec{u}, \vec{v}, \vec{w})$ by simple multiplication of the vector (or point) coordinates by the matrix $\mathbf{M} = (\vec{u} \vec{v} \vec{w})$. In the new coordinate system, the rotation can be carried out using a Givens rotation matrix. Later on, we transform the obtained coordinates in the old reference basis axes $(o, \vec{i}, \vec{j}, \vec{k})$. One can assume that using a rotation matrix:

$$\vec{\mathbf{R}}(\psi) = \mathbf{M} \begin{pmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{M}^T, \quad (\text{E.4})$$

\mathbf{M} is an orthogonal matrix.

In the case where Δ is given by $x = y = z = t/\sqrt{3}$, one can find that

$$D_1 \begin{cases} x = -\frac{t}{\sqrt{2}} \\ y = 0 \\ z = \frac{t}{\sqrt{2}} \end{cases} \quad \text{and} \quad D_2 \begin{cases} x = \frac{t}{\sqrt{6}} \\ y = -\frac{2t}{\sqrt{6}} \\ z = \frac{t}{\sqrt{6}} \end{cases}.$$

Finally, the necessary rotation matrix around Δ is given by

$$\vec{\mathbf{R}}(\psi) = \begin{pmatrix} \frac{1+2\cos(\psi)}{3} & \frac{1-\cos(\psi)+\sqrt{3}\sin(\psi)}{3} & \frac{1-\cos(\psi)-\sqrt{3}\sin(\psi)}{3} \\ \frac{1-\cos(\psi)-\sqrt{3}\sin(\psi)}{3} & \frac{1+2\cos(\psi)}{3} & \frac{1-\cos(\psi)+\sqrt{3}\sin(\psi)}{3} \\ \frac{1-\cos(\psi)+\sqrt{3}\sin(\psi)}{3} & \frac{1-\cos(\psi)-\sqrt{3}\sin(\psi)}{3} & \frac{1+2\cos(\psi)}{3} \end{pmatrix}. \quad (\text{E.5})$$

Appendix F. The necessary angle ψ

Here we assume that one vertex “A” of a cube is known and that one of its principal diagonals is fixed. Using the approach described in Appendix D, we can determine another vertex “B”. In this section, we answer

the following question: what is the angle ψ that can be used to rotate the cube around the fixed diagonal such that \vec{AB} becomes parallel to one of the reference axes ($\vec{o}\vec{x}, \vec{o}\vec{y}, \vec{o}\vec{z}$)?

Let B_2 be the desired position of B and let \vec{AB}_2 be parallel to $\vec{o}\vec{x}$ (see Fig. 12):

$$\vec{OA} = \begin{pmatrix} x_A \\ y_A \\ z_A \end{pmatrix} \tag{F.1}$$

and

$$\vec{AB}_2 // \vec{o}\vec{x} \Rightarrow \vec{OB}_2 = \begin{pmatrix} -x_A \\ y_A \\ z_A \end{pmatrix}.$$

Let $\vec{OB} = (x_B, y_B, z_B)^T$. Using the results in Appendix E, one can find that the angle ψ needed to rotate the cube such that B becomes B_2 , should satisfy the following equation:

$$\begin{pmatrix} -x_A \\ y_A \\ z_A \end{pmatrix} = \begin{pmatrix} \frac{1+2\cos(\psi)}{3} & \frac{1-\cos(\psi)+\sqrt{3}\sin(\psi)}{3} & \frac{1-\cos(\psi)-\sqrt{3}\sin(\psi)}{3} \\ \frac{1-\cos(\psi)-\sqrt{3}\sin(\psi)}{3} & \frac{1+2\cos(\psi)}{3} & \frac{1-\cos(\psi)+\sqrt{3}\sin(\psi)}{3} \\ \frac{1-\cos(\psi)+\sqrt{3}\sin(\psi)}{3} & \frac{1-\cos(\psi)-\sqrt{3}\sin(\psi)}{3} & \frac{1+2\cos(\psi)}{3} \end{pmatrix} \begin{pmatrix} x_B \\ y_B \\ z_B \end{pmatrix}. \tag{F.2}$$

Using the previous equation, one can write

$$\begin{pmatrix} 2x_B - y_B - z_B & \sqrt{3}(y_B - z_B) \\ -x_B + 2y_B - z_B & \sqrt{3}(z_B - x_B) \\ -x_B - y_B + 2z_B & \sqrt{3}(x_B - y_B) \end{pmatrix} \begin{pmatrix} \cos(\psi) \\ \sin(\psi) \end{pmatrix} = \begin{pmatrix} -3x_A - x_B - y_B - z_B \\ 3y_A - x_B - y_B - z_B \\ 3z_A - x_B - y_B - z_B \end{pmatrix}. \tag{F.3}$$

Let us consider that $U = (2x_B - y_B - z_B, -x_B + 2y_B - z_B, -x_B - y_B + 2z_B)^T$, $V = (\sqrt{3}(y_B - z_B), \sqrt{3}(z_B - x_B), \sqrt{3}(x_B - y_B))^T$, $W = (-3x_A - x_B - y_B - z_B, 3y_A - x_B - y_B - z_B, 3z_A - x_B - y_B - z_B)^T$, and $\mathbf{F} = (U \ V)$. Eq. (F.3) derived from three equations and two variables ($\cos(\psi)$ and $\sin(\psi)$). Therefore, it will have a unique solution if and only if (iff) $\text{Rank}(U \ V \ W) = 2$, no solution iff $\text{Rank}(U \ V \ W) = 3$ and multiple solutions⁵ iff $\text{Rank}(U \ V \ W) < 2$.

In our case, point “A” should be a point of straight line $D_1: x = y = z = t$, thus:

$$x_A = y_A = z_A, \tag{F.4}$$

and point “B” should satisfy the following two constraints (see Fig. 12):

$$|\vec{OB}| = |\vec{OA}|, \tag{F.5}$$

$$|\vec{AB}| = 2|x_A|. \tag{F.6}$$

Using Eqs. (F.4)–(F.6), one can prove that:

$$x_B^2 + y_B^2 + z_B^2 = 3x_A^2, \tag{F.7}$$

$$x_B + y_B + z_B = x_A. \tag{F.8}$$

From Eqs. (F.4), (F.5) and (F.6), one can prove that $\det(U \ V \ W) = 0$ and $\text{Rank}(\mathbf{F}) = \text{Rank}(U \ V) = 2, \forall x_A \neq 0$. The last statement indicates that Eq. (F.3) has a unique solution. To reduce the effect of the estimation error,

⁵ In our case, the two variables ($\cos(\psi)$ and $\sin(\psi)$) are dependents, therefore a solution for the system should satisfy the dependence relation of the two variables.

one can determine the corresponding value⁶ of $\cos(\psi)$ and $\sin(\psi)$ by

$$\begin{pmatrix} \cos(\psi) \\ \sin(\psi) \end{pmatrix} = (\mathbf{F}^T \mathbf{F})^{-1} \mathbf{F}^T \mathbf{W}. \quad (\text{F.9})$$

Appendix G. The algorithm

In this section, we summarize the proposed algorithm. Note that if the number of sources is p then the algorithm consists of p steps, as one transformation and $p - 1$ rotation. For example, in the case of $p = 3$, we can separate the sources using the following three steps:

1. Using SOS, one can transform the scatter plot of mixing signals from a parallelepiped to a cube C_1 .
2. One can find the “farthest point A ” from the origin and use its coordinates to rotate the cube C_1 obtained such that this axis of symmetry \vec{oA} will be colinear with straight line $D_1: x = y = z = t$ if the signals have PDF close to uniform, or the vertical axis $\vec{o\hat{z}}$ if the signals are speech or have PDF similar or close to Laplacian PDF. After rotation, the rotated cube is denoted by C_2 .
3. Finding another axis of symmetry $\vec{o\hat{B}}$ of the cube C_2 . The separation is obtained when the C_2 is rotated around its first axis of symmetry \vec{oA} such that \vec{AB} becomes parallel to the horizontal axis $\vec{o\hat{x}}$.

Appendix H. Instantaneous mixture with additive noise

It is known that the geometrical approach is more sensitive to noise than the statistical approach. On the other hand, it seems that our geometrical algorithm (the second step) is very non-robust and has bad performance.

To improve the robustness of our algorithm (see Appendix G), we propose here a heuristic and simple modified approach. In the following, we will assume that the mixture is an instantaneous one with additive noise. In this case, the orthogonal signals $Z(n)$ are given by

$$Z(n) = \mathbf{U}X(n) + B(n). \quad (\text{H.1})$$

Here $\mathbf{U} = \mathbf{K}\mathbf{M}$ is the orthogonal matrix and $B(n)$ is the zero-mean noise vector. In this case, point A of Fig. 10 cannot be estimated correctly. Instead of point A , one can estimate a point A' within a disk centered at A and its radius depends on the noise power (and amplitude). On the other hand, it has been shown in Appendix A that any point within the trust region (i.e., the small square near the vertex) is sufficient to estimate the diagonal angle of the scatter plot of the orthogonal signals. The last statement proves that the actual version of the algorithm is robust against small noise perturbations.

To improve the robustness of our algorithm, one can consider reference point A_0 as the barycenter of some candidate points within set \mathcal{E} (i.e., $A' \in \mathcal{E} \Rightarrow A'$ is near vertex A). This simple idea can be realized by the following algorithm:

1. Let $V_{\text{ref}} = 0$ be the initial value of the barycenter vector (i.e., \vec{OA}_0).
2. Let V be the coordinate of the actual sample point.
3. If $\|V\| > (1 + \varepsilon_1)\|V_{\text{ref}}\|$ then put $V_{\text{ref}} = V$ and $k = 0$.
4. Otherwise, if the scalar product $V \cdot V_{\text{ref}} > (1 - \varepsilon_2)\|V_{\text{ref}}\|^2$, then V is a point near V_{ref} and it is in set \mathcal{E} . Therefore, we should increase K (i.e., $k++$) and put $V_{\text{ref}} = ((k - 1)V_{\text{ref}} + V)/k$.

⁶ In the rotation matrix (E.5), we use the cosine and the sine of ψ . However, the value of ψ can be easily estimated.

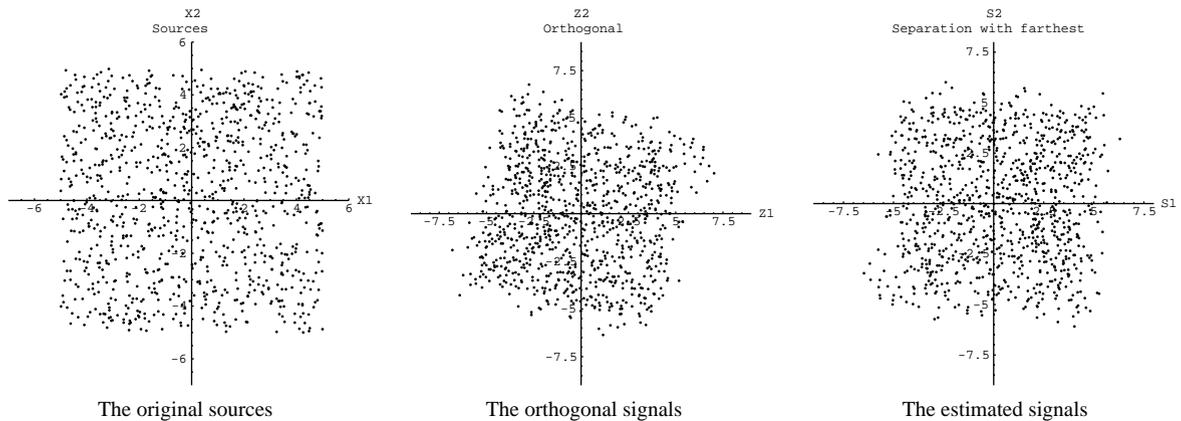


Fig. 13. The separation of instantaneous mixture with additive noise. (a) The original sources, (b) the orthogonal signals, (c) the estimated signals.

- Repeat from step 2 till the stop of the algorithm. The stop can be determined based on the number of samples or any other criterion.

Here, ε_1 and ε_2 are two small numbers. Using the modified algorithm, many experiments have been conducted and the separation has been obtained even with noise mixture. Fig. 13 shows the separation results of model (H.1). In that figure, the signals are two uniform PDF signals, the noise is Gaussian noise with SNR = 13 dB, the orthogonal mixing matrix

$$\mathbf{U} = \begin{pmatrix} 0.25885 & -0.965918 \\ 0.965918 & 0.25885 \end{pmatrix}$$

and global matrix

$$\mathbf{G} = \begin{pmatrix} -0.0650625 & 0.997881 \\ -0.997881 & -0.0650625 \end{pmatrix}.$$

Finally, we should mention that we used 1000 samples in that experiment, $\varepsilon_1 = 0.12$ and $\varepsilon_2 = 0.5$.

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