World Multiconference on Systemics, Cybernetics and Informatics SCI 2001
Orlando, Florida, USA, July 22-25, 2001

A New Geometrical Blind Separation of Sources Algorithm.

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Abstract

In this paper we present a new blind separation of sources (BSS) algorithm based on second order statistics (SOS) and geometrical approaches. The new algorithm can separate multisources from their instantaneous mixtures obtained by multisensors. In the case of p sources and p sensors, the algorithm can be decomposed into p steps: First, one should transform the mixing signals to orthogonal signals using mainly the SOS of the mixing signals. After that, one can separate the sources by using $p-1$ rotations and projections. The experimental studies show that the separation of two or three speech or music signals can be obtained in relatively competitive time and that the obtained results are very satisfactory.

keywords: Decorrelation, Second-Order Statistics, Whiteness, Blind Separation of Sources, Geometrical Algorithms, Independent Component Analysis, Probability Density Function.

1 Introduction

The blind separation of sources (BSS) has been an important problem in signal processing field since it can be found in many different applications [1]: speech enhancement [2], separation of seismic signals [3], sources separation method applied to nuclear reactor monitoring [4], airport surveillance [5], noise removal from biomedical signals [6], multi-tag radio-frequency identification systems [7], robotic and artificial life [8].

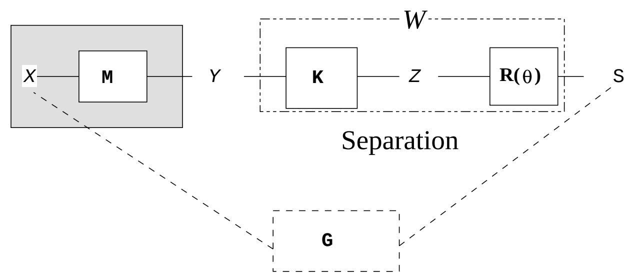


Figure 1: Separation Model

According to that problem [9], one should retrieve the p unknown and independent sources $X(t)$ from the observation of their mixture $Y(t)$ using p sensors, see Fig. 1. In many cases, the channel effect can be considered as memoryless channel (i.e., the mixture of the sources can be considered as instantaneous mixture), or:

$$Y(t) = \mathbf{M}X(t), \tag{1}$$

where $\mathbf{M}$ is a $p \times p$ full rank matrix represents the channel effect. For instantaneous mixtures, many algorithms and criteria were proposed [10, 11, 12, 13, 14, 15, 16, 17]. Most of the proposed algorithms are based on or used High Order Statistics (HOS). Meanwhile, some researchers proposed a simple algorithm for blind separation of binary or n-valued signals using geometrical concepts [18, 19]. Their algorithm is very simple and it can converge in very competitive time. On the other hand, that version of the algorithm cannot be easily generalized for more than 2 sources and to separate real signals such as speech or music signals as well as signals with Gamma or similar PDF. To separate speech signals, another geometrical algorithm was proposed [20]. Unfortunately, the new algorithm can not separate signals with uniform or similar PDF (as sinusoidal signals). The geometrical algorithms generally are based on the fact that the scatter plot of two independent signals (i.e. $x_2(n)$ against $x_1(n)$ for every n) is a rectangular which its edges are parallel to the reference axes.

In this paper, we propose a new geometrical algorithm to separate two or more zero-mean sources which can be either uniform PDF or speech signals. unfortunately, the actual version can not separate the mixing of signals which belong to the two categories (the algorithm can be modified to deal with such cases).

2 New Algorithm

From geometrical point of view, equation (1) can be considered as geometrical transformation that change the rectangular (in the case of two signals) scatter plot of the sources into a parallelogram (i.e., the scatter plot of the observation of the mixing signals). Here, we show how one can separate p sources from their instantaneous mixture by using an algorithm of p steps: First of all, one can transform the scatter plot of the observation signals from an hyper-parallelepiped into an hyper-cube scatter plot of some orthogonal signals in the p dimensional space. The orthogonal signals $Z(t)$ can be easily obtained from $Y(t)$ as:

$$Z(t) = \mathbf{K}Y(t), \tag{2}$$

here, $\mathbf{K} = \mathbf{L}^{-1}$ and $\mathbf{L}$ is the Cholesky factorization of the observation covariance matrix, i.e. $\mathbf{R}_Y = \mathrm{E}YY^T = LL^T$ [21] and $\mathrm{E}(X)$ denotes the statistical mean of X.

After the orthogonalization of the signals, the separated signals $S(t)$ can be obtained as the multiplication of the orthogonal signals $Z(t)$ by a rotation matrix $\mathbf{R}(\theta)$. The latter matrix $\mathbf{R}(\theta)$ should rotate the scatter plot G_Z of the orthogonal signals such that the edges of the rotated scatter plot G_S are parallel to the references axes. In the following, we explain how $\mathbf{R}(\theta)$ can be obtained as the product of $p-1$ simple rotation matrix. But one should distinguish two cases: uniform or similar PDF, and Speech or Gamma PDF signals.

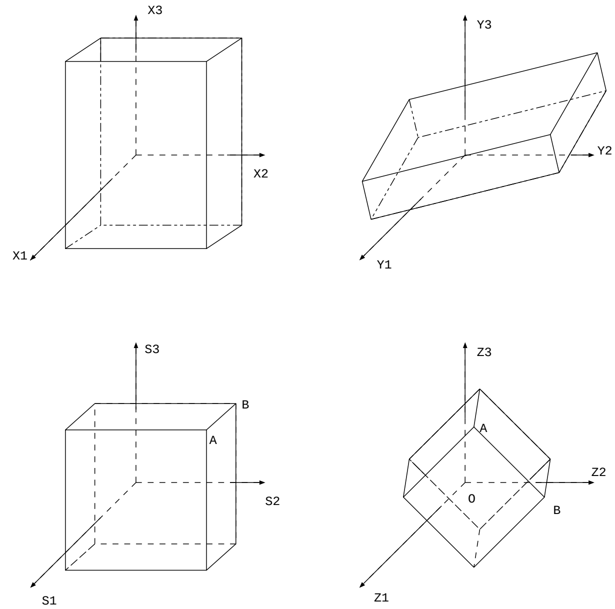


Figure 2: Separation Algorithm for Uniform PDF

2.1 Uniform PDF

In this section we assume that the sources have uniform or similar PDF. The scatter plot of such signals can be represented by Fig. 2.

Let O be the center of the reference axes. It is clear that the farthest point A from the origin is actually one of the vertices of the hyper-cube G_Z (G_Z is the scatter plot of $Z(t)$). It is known that the separation of sources can be achieved [13, 1] up to a permutation and a factor, i.e., the global matrix $\mathbf{G}$ can be considered as:

$$\mathbf{G} = \mathbf{WM} = \mathbf{P\Delta}, \tag{3}$$

where $\mathbf{P}$ is any permutation matrix, $\mathbf{\Delta}$ is any full-rank diagonal matrix, and $\mathbf{W}$ denotes the separating matrix, in our case $\mathbf{W} = \mathbf{KR}(\theta)$. From equation (3), one can consider, without loss of generality, that the estimated sources $S(t)$ are bounded signals with the same boundary. In other words, one vertices of G_Z should belongs to a $p-1$ hyper-surface D_1 defined by its parametric equation $x_1 = x_2 = \cdots = x_p$, x_i is the coordinate of any point the p space. Therefore by using a rotation matrix $\mathbf{R}_1$, one should rotate the hyper-cube G_Z such that the vertex A should belong to D_1, the rotated hyper-cube is denoted by G_1. In the case of three sources, D_1 becomes a straight line which should be colinear to a principal diagonal of the cube G_S. In this case, the necessary rotation matrix $\mathbf{R}_1$ is given by:

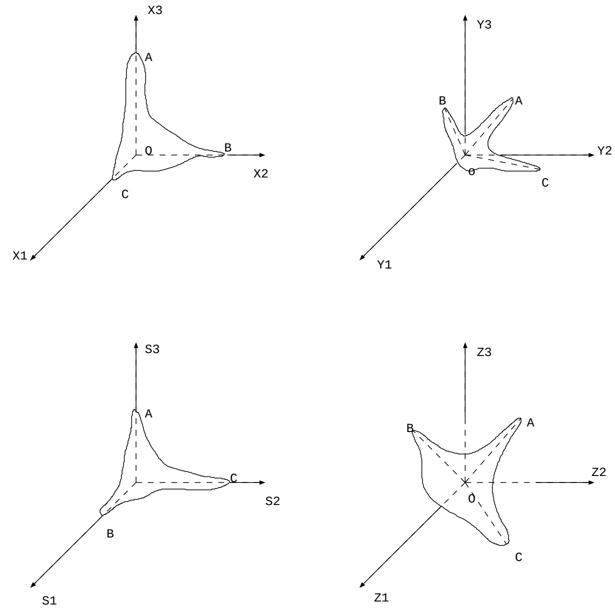


Figure 3: Separation Algorithm for Speech signals

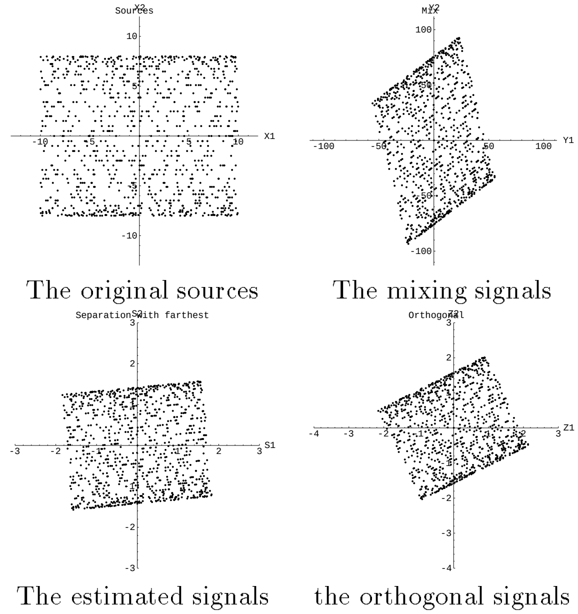


Figure 4: The separation of mixing of a uniform signal with a sinusoidal signal

$$\mathbf{R}_1 = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(\varphi-b) & 0 & \sin(\varphi-b) \\ 0 & 1 & 0 \\ -\sin(\varphi-b) & 0 & \cos(\varphi-b) \end{pmatrix} \cdot \begin{pmatrix} \cos(a) & \sin(a) & 0 \\ -\sin(a) & \cos(a) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4}$$

here, $\alpha = \pi/4$, $\varphi = \arctan(\sqrt{2})$, $a = \frac{1-\mathrm{Sign}(A_x)}{2}\pi + \arctan(\frac{A_y}{A_x})$ and $b = \frac{1-\mathrm{Sign}(A_z)}{2}\pi + \arctan(\frac{\sqrt{A_x^2+A_y^2}}{A_z})$.

using $p-1$ constraints, one can find another vertex B of rotated hyper-cube G_1. now we should rotate G_1 around OA such that hyper-straight line AB becomes parallel to one reference axe, say $\vec{ox}$. We should continue rotating the hyper-cube around different axes and using different vertices to finally obtained an hyper-cube G_s such that its edges are parallel to the references axes. When $p=3$, B is a point of the cube G_1 such that $|\vec{OB}| = |\vec{OA}| = \frac{2}{\sqrt{3}}|\vec{AB}|$ (the indeterminacy over B is coming from the permutation indeterminacy, see equation (3)). The separated signals can be obtained after the rotation of G_1 around OA such that AB becomes parallel to ox. We can prove that such rotation is given by:

$$\mathbf{R}_2 = \begin{pmatrix} \frac{1+2\cos(\psi)}{3} & \frac{1-\cos(\psi)+\sqrt{3}\sin(\psi)}{3} & \frac{1-\cos(\psi)-\sqrt{3}\sin(\psi)}{3} \\ \frac{1-\cos(\psi)-\sqrt{3}\sin(\psi)}{3} & \frac{1+2\cos(\psi)}{3} & \frac{1-\cos(\psi)+\sqrt{3}\sin(\psi)}{3} \\ \frac{1-\cos(\psi)+\sqrt{3}\sin(\psi)}{3} & \frac{1-\cos(\psi)-\sqrt{3}\sin(\psi)}{3} & \frac{1+2\cos(\psi)}{3} \end{pmatrix}$$

here ψ is the rotation angle, which its cosine and sine can be obtained by:

$$\begin{pmatrix} \cos(\psi) \\ \sin(\psi) \end{pmatrix} = (\mathbf{F}^T\mathbf{F})^{-1}\mathbf{F}^T W \tag{5}$$

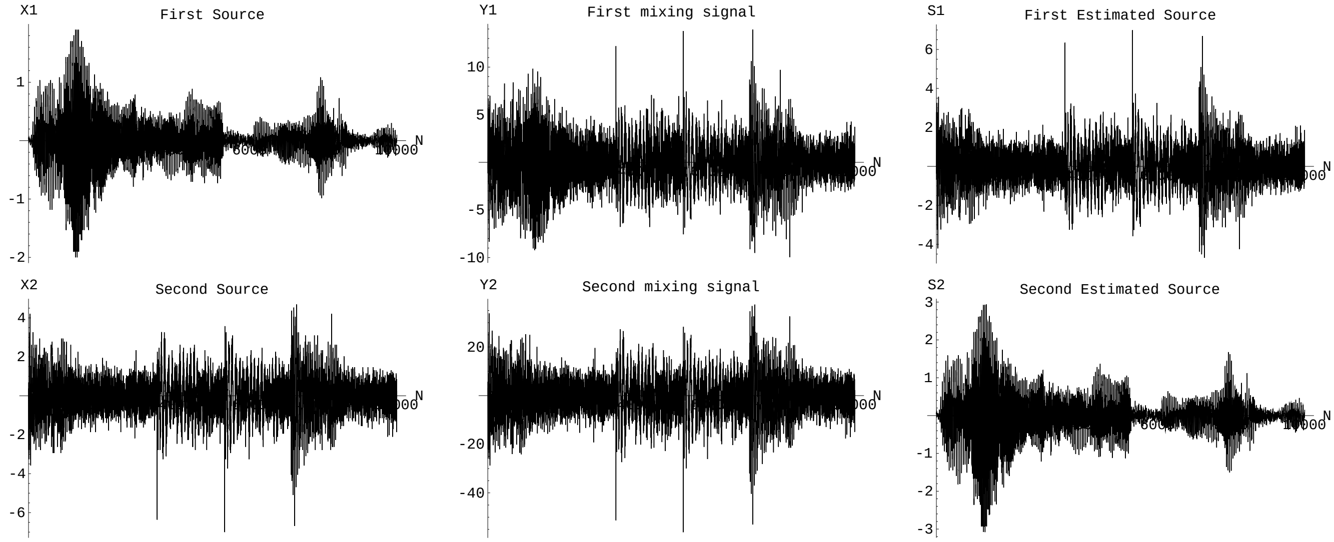
here

$$U = \begin{pmatrix} 2x_B - y_B - z_B \\ -x_B + 2y_B - z_B \\ -x_B - y_B + 2z_B \end{pmatrix}, \ V = \begin{pmatrix} \sqrt{3}(y_B - z_B) \\ \sqrt{3}(z_B - x_B) \\ \sqrt{3}(x_B - y_B) \end{pmatrix},$$

$$W = \begin{pmatrix} -3x_A - x_B - y_B - z_B \\ 3y_A - x_B - y_B - z_B \\ 3z_A - x_B - y_B - z_B \end{pmatrix} \quad \text{and } \mathbf{F} = (U \ V).$$

2.2 Speech Signals

For independent signals with Gamma or similar PDF and real signals such as speech or musical signals, the scatter plots can not be in hyper-cube form, instead of that it will be more like a star (a cross for two signals) which the symmetrical axes are colinear to the reference axes. Instead of the vertices of the hyper-cube, one should consider the radiating points of the star. The algorithm can be applied by considering the particularity of this case, (one should change the rotation angles, the constraints to determined the different radiating points).



The original sources   The mixing signals   The estimated signals

Figure 5: The separation of two speech signals: The voice of man and music signal

For three signals, one can use the rotation matrix $\mathbf{R}_1$ with $\varphi = \alpha = 0$. Now B is the farthest point from the origin in the plan Oxy. Finally the separation is obtained after the rotation of the scatter plot G_1 around the vertical axe $\vec{oZ}$ such that $\vec{oB}$ becomes colinear to the horizontal axe $\vec{ox}$ using the following Givens matrix:

$$R(\alpha) = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 071 & \end{pmatrix} \qquad (6)$$

here $\alpha = (1 - \text{ Sign}(X_B))\frac{\pi}{2} + \arctan(\frac{Y_B}{X_B})$, X_B and Y_B are the coordinate of B.

3 Experimental Results

Many experiments have been conducted and good results have been obtained. The experimental study shows that with less than 6000 samples the algorithm can separate successfully the mixing of two speech, musical or Gamma PDF signals. Just few hundred of samples are enough to separate two signals with uniform or similar PDF. Unfortunately, it needs more than 10000 samples to separate the mixing of three signals with uniform or similar PDF and more than 20000 samples (around 30000) to separate successfully the mixing of three speech, musical or Gamma PDF signals.

Fig. 4 shows the scatter plots of the different signals. Fig. 5 shows the experimental results obtained from the separation of two speech signals. Finally, the separation of three speech or music signals is shown in Fig. 6.

4 Conclusion

In this paper, a new geometrical algorithm for blind separation of sources problem is presented.

The new algorithm is based on second order statistics and geometrical concepts (as rotations, projections into hyper-surfaces or volumes). The algorithm can be derived very simply in the case of three or two signals.The experimental results obtained in the case of two or three signals are very encouraging.

Unfortunately, the number of samples increases very fast is the number of sources increases: in the case of speech and musical signals, 6000 samples are enough to separate two signals. However, more than 20000 samples are required to separate three musical signals.

Finally, the new algorithm is not a time consuming algorithm (few second are enough to separate three speech signals using high level of computer language, as Mathematica or Matlab) and it does not require high computational efforts.

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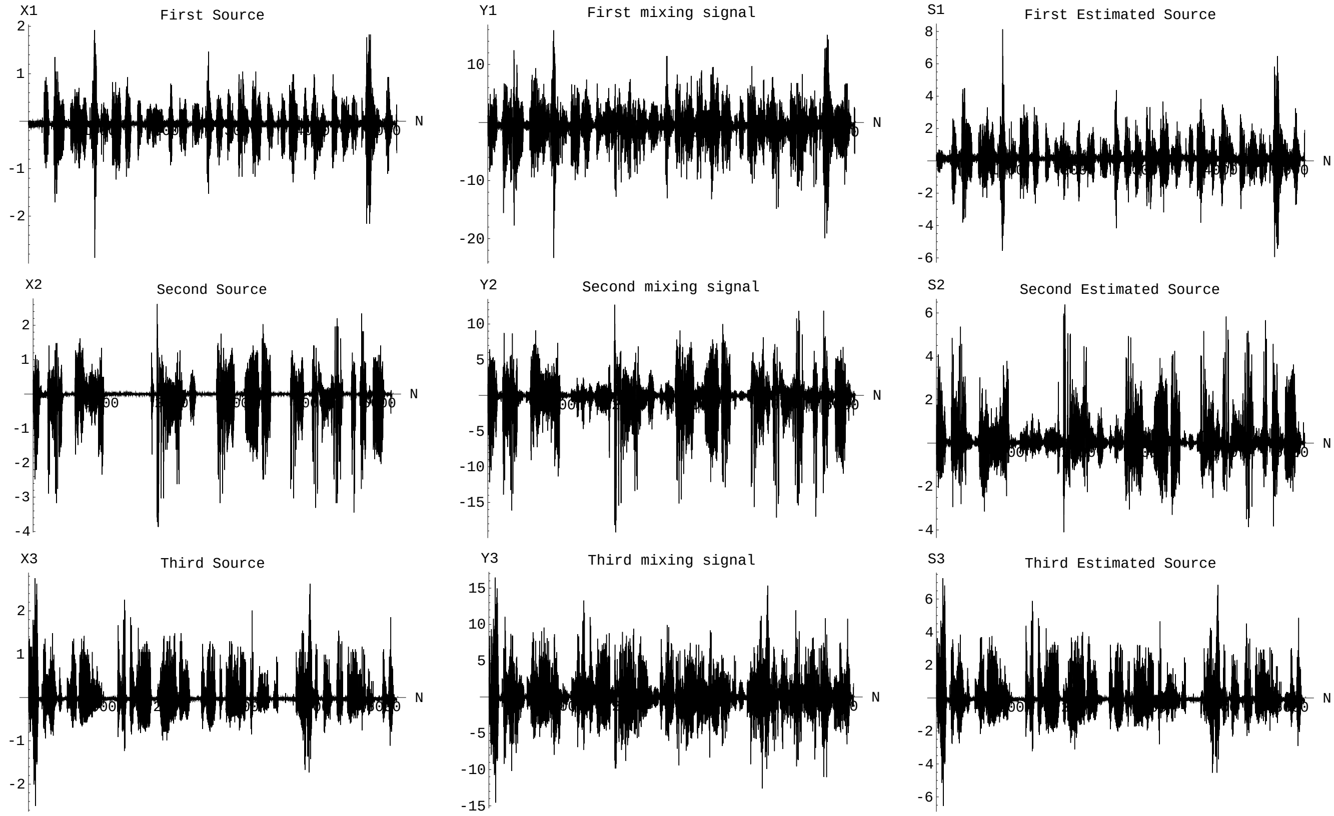

The original sources The mixing signals The estimated signals

Figure 6: The separation of three speech signals: Voices of two females and one male.

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