Procedings of the IASTED International Conference
MODELLING, IDENTIFICATION, AND CONTROL
February 18-21, 2002, Innsbruck, Austria

A Survey of The Performance Indexes of ICA Algorithms

Ali MANSOUR[1], Mitsuru KAWAMOTO[2] & Noboru OHNISHI[3]
1- E^3I^2, ENSIETA, 29806 Brest cedex 09, (FRANCE).
2- Department of Electronic and Control Systems, Shimane University, Shimane 690-8504, (JAPAN)
3- Bio-Mimetic Control Research Center (RIKEN), Nagoya 463-003 (JAPAN)
email: mansour@ieee.org, kawa@ecs.shimane-u.ac.jp & ohnishi@nagoya.riken.go.jp
http://ali.mansour.free.fr, www.ecs.shimane-u.ac.jp & www.bmc.riken.go.jp

ABSTRACT
This paper deals with the problem of blind separation of sources (BSS). In the literature, one can find many Independent Component Algorithms (ICA) to solve the BSS. To demonstrate the performances of their algorithms, researchers often use different methods or performance indexes depending on their source signals and their applications. Many methods and performance indexes can not be used to compare two different algorithms applied to different signals. Most of the widely used performance indexes or methods are mentioned and discussed hereafter. We also give many examples to show limitations or drawbacks of some performance indexes or methods.

KEY WORDS
Blind Separation of Sources, BSS, ICA, Crosstalk, SNR, SINR, Gap or Distance to Diagonal Matrix, Performance Indexes, Crosstalk Error, Rejection Level, Global Index, Symbol Error Rate, Scatter Plot, Error Signals, Real World Applications

1. Introduction

Since 1984, when it was firstly introduced by Hérault *et al.* [1, 2], the problem of blind separation of sources (BSS) has been considered as an important problem in the signal processing fields. The blind separation of sources problem involves in the separation of unknown and statistically independent signals (sources), issued by an unknown channel, by only using the observation of their mixture signals given as the sensor outputs. The BSS problem is widely known as the Independent Component Analysis or ICA.

Actually the BSS problem can be found in many applications (such as radio-communication (in mobile-phone as Spatial Division Multiple Access (SDMA)), free-hand phone, speech enhancement, separation of seismic signals, sources separation method applied to nuclear reactor monitoring, airport surveillance, noise removal from biomedical signals) [3]. Hundreds of different publications and papers concerning the BSS can be found in the literature [3].

Since the last decade, dozens of ICA algorithms have been developed and presented. Many researchers from over a dozen different countries around the globe are actually working on this subject.

On the other hand, we should mention that normally the authors use different methods or performance indexes to evaluate their algorithms. This fact makes the comparison between two algorithms, applied in different circumstances to separate different types of sources, can be very difficult (even impossible in some cases). Up to now, there is no "Bench Mark" for the BSS or ICA algorithms. Here we will discuss and present the advantages as well the drawbacks of the different performance methods or indexes.

2. Mixture Model

Let us denote by $S(t) = (s_i(t))$ the $m \times 1$ source vector and by $Y(t) = (y_i(t))$ the $n \times 1$ vector of the observed signals, see Fig. 1. In the case of instantaneous mixture, the observation (mixture) vector Y can be written as:

$$Y(t) = \mathbf{A}S(t) + N(t). \quad (1)$$

Where $\mathbf{A}$ denote the full-rank $n \times m$ mixture matrix and $N(t)$ is the noise. Using the $n \times m$ estimated separating matrix $\mathbf{B}$, the $m \times 1$ estimated (or separated) signal vector $X(t) = (x_i(t))$ can be obtained as:

$$X(t) = \mathbf{B}Y(t). \quad (2)$$

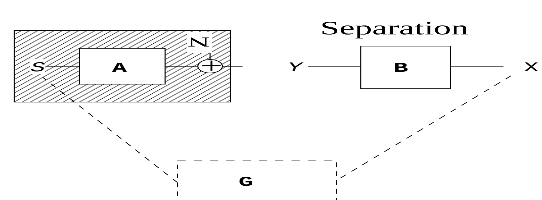

Figure 1. Channel Model

It is well known that the separation of sources can be done up to a scalar factor and a permutation [4, 5]. In this

350-115

case the global matrix $\mathbf{G}$ is given by the following equation:

$$\mathbf{G} = \mathbf{B}\mathbf{A} = \mathbf{P}\mathbf{\Delta} \tag{3}$$

here $\mathbf{P}$ is any permutation matrix and $\mathbf{\Delta}$ is any full rank diagonal matrix. When $\mathbf{G} = \mathbf{P}\mathbf{\Delta}$, it said that the global matrix becomes a general permutation matrix and that the blind separation of sources is done. Equation (3) is equivalent to:

$$\mathbf{B} = \mathbf{P}\mathbf{\Delta}\mathbf{A}^{\#}, \tag{4}$$

here $\mathbf{A}^{\#}$ denotes the pseudo-inverse (or the inverse, when $m = n$) of the mixing matrix $\mathbf{A}$. Generally, researchers assume that $m = n$. In some cases, $m < n$ and the mixture is called undercomplete (underdetermined). Otherwise when $m > n$ the mixture is called overcomplete (overdetermined).

Finally, let N denote the total number of the observed samples of the vector Y. In this case, one can denote by $\mathbf{Y}(N) = (Y(1), \cdots, Y(N))^T$ the $n \times N$ matrix of the mixing signals, $\mathbf{X}(N) = (X(1), \cdots, X(N)^T$ the $m \times N$ matrix of the estimated signals and by $\mathbf{S}(N) = (S(1), \cdots, S(N))^T$ the $m \times N$ matrix of the sources.

3. Algorithms' Performances

To evaluate their algorithms, researchers are using different methods and performance indexes. In this section, we focus on the performance indexes or evaluation methods that have been used by different researchers and that can be generally applied to any algorithm. We should notice here that in some papers, authors proposed some evaluation methods or performance indexes which are linked to their own researches or algorithms and they can not be used easily to evaluate different algorithms. Such performance indexes will be omitted in this paper.

3.1 Crosstalk, SNR & SINR

To indicate the performance of their algorithms, researchers are using the crosstalk index [6, 7, 8, 9, 10, 11]. Let us suppose that the jth output signal x_j is the estimation of the ith source s_i, due to the permutation problem. By definition the crosstalk index of the jth estimated signal, is given by:

$$\text{CrossTalk}_j \stackrel{\text{def}}{=} 10 \log_{10} \frac{\text{E}(x_j - s_i)^2}{\text{E}s_i^2} \tag{5}$$

here E stands for the expectation. It is clear from the above definition (5) of the crosstalk that:

- We can obtain m different crosstalk values. The significant value of the crosstalk should be considered as the minimum (or the average) of these m different values.
- To calculate the crosstalk from its definition (5), one should have the original sources which means that the crosstalk can be estimated in our simulations but it can not directly be applied to real-world experiments.

On the other hand the crosstalk can be considered as the inverse of the Signal to Noise Ratio (SNR) which has been used by many other researchers [12, 13, 14, 15]. Similar index has been used by other researchers as the Signal to Interference Noise Ratio (SINR) [16, 17]:

$$\text{SINR}_j \stackrel{\text{def}}{=} \frac{|B_j A_i|^2 E s_i^2}{B_j \mathbf{R} B_j^T}, \tag{6}$$

where B_j is the jth column of $\mathbf{B}$, A_i is the ith row of $\mathbf{A}$ and $\mathbf{R}$ is the true autocovariance matrix of the interference consisting of the other sources and the background additive noise (if the channel has an additive noise). Same as for the crosstalk, the estimation of SNR and the SNIR needs the knowledge of the sources (at least the power with other features of the signals) and the mixture matrix. On the other hand, these indexes can be easily used to compare different algorithms or to evaluate an algorithm in different cases.

3.2 Gap or Distance to Diagonal Matrix

Let $\hat{\mathbf{A}} = \mathbf{B}^{\#}$ denote the estimated mixture matrix. Let $\mathbf{A}$ and $\hat{\mathbf{A}}$ be two invertible matrices, and define the matrices with unit-norm columns:

$$\begin{aligned} \underline{\mathbf{A}} &= \mathbf{A}\mathbf{\Delta}^{\#} \\ \underline{\hat{\mathbf{A}}} &= \hat{\mathbf{A}}\hat{\mathbf{\Delta}}^{\#} \end{aligned} \tag{7}$$

here $\mathbf{\Delta} = (\delta_{ij})$ (resp. $\hat{\mathbf{\Delta}} = (\varepsilon_{ij})$) is a diagonal matrix such that $\delta_{ii} = \|A_i\|$ (resp. $\varepsilon_{ii} = \|\widehat{A}_i\|$) is the norm of the ith column of $\mathbf{A}$ (resp. $\hat{\mathbf{A}}$). Comon in [18] gives a definition of a gap or a distance measure from the matrix $\mathbf{D} = \underline{\hat{\mathbf{A}}}^{\#}\underline{\mathbf{A}}$ to a diagonal matrix by:

$$\begin{aligned} \Upsilon(\mathbf{D}) \stackrel{\text{def}}{=} & \sum_i \left(\sum_j |d_{ij}| - 1 \right)^2 + \sum_j \left(\sum_i |d_{ij}| - 1 \right)^2 \\ & + \sum_i \left| \sum_j |d_{ij}|^2 - 1 \right| + \sum_j \left| \sum_i |d_{ij}|^2 - 1 \right| \end{aligned} \tag{8}$$

He proved that the above gap is invariant by post-multiplication of the form $\mathbf{P}\mathbf{\Delta}$ (i.e. by any general permutation) and that $\Upsilon(\mathbf{D}) \iff \mathbf{B} = \mathbf{P}\mathbf{\Delta}\mathbf{A}^{\#}$. Similar to the case of the SNR, the gap of Comon can not be estimated without the knowledge of the mixture matrix.

3.3 Performance Index or Crosstalk Error

Many researchers have used the "Crosstalk Error" or the "Performance Index" to demonstrate the performances of their algorithms [19, 20, 21, 22, 23, 24, 25, 26, 27]. They defined the Crosstalk Error as:

$$Ce(\mathbf{G}) \stackrel{\text{def}}{=} \sum_{i=1}^{n}\left(\sum_{j=1}^{n}\frac{|g_{ik}|}{\max_k |g_{ik}|}-1\right) + \sum_{j=1}^{n}\left(\sum_{i=1}^{n}\frac{|g_{ik}|}{\max_k |g_{ik}|}-1\right) \quad (9)$$

It is clear that the "crosstalk error" is invariant by a multiplication of a permutation matrix. At the same time, one can easily verify that the crosstalk has a positive value and it is equal to zero when the global matrix $\mathbf{G}$ satisfies equation (3). Normally researchers gives the value of Ce in dB.

Theoretically, this index is a good performance index. On the other hand, it is very sensitive to numerical error and meaningless in many practical cases. In fact, let us consider the following two matrices:

$$\mathbf{G}_1 = \begin{pmatrix} 100000 & 10 & 1 & 2 & 4 \\ 1 & 0 & 0 & 0 & 0 \\ 10 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (10)$$

$$\mathbf{G}_2 = \begin{pmatrix} 100000 & 10 & 1 & 2 & 4 \\ 1 & 0 & 0 & 0 & 0 \\ 10 & 0 & 0.01 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (11)$$

One can verify that $Ce(\mathbf{G}_1) = 0.00037 = -34.32$ dB and that $Ce(\mathbf{G}_2) = 0.01137 = -19.44$ dB. Neither $\mathbf{G}_1$ nor $\mathbf{G}_2$ verify equation (3).

3.4 Rejection Level

The Mean Rejection Level or Rate (MRL or MRR) has been defined in many papers [28, 29, 30, 31, 32] as the mean power of the interference of the jth source into the ith estimated sources, i.e.:

$$MRL_{ij} \stackrel{\text{def}}{=} \mathrm{E}\, g_{ij}^2. \quad (12)$$

Considering the fact that the estimation of the global matrix depends on the total number of samples N, other researchers prefer to use the Asymptotic Rejection Level (ARL) [33, 34] defined as:

$$ARL_{ij} \stackrel{\text{def}}{=} \lim_{N\to\infty} \mathrm{E}\, g_{ij}^2. \quad (13)$$

Based on the definition of the MRL, one can defined the Global Rejection Level (GRL) as:

$$GRL \stackrel{\text{def}}{=} \sum_{i\neq j} MRL_{ij}. \quad (14)$$

Unfortunately, the GRL also is sensitive to numerical values and in some cases it can give a wrong impression: Let us consider the following three matrices

$$\mathbf{G}_4 = \begin{pmatrix} 1 & 10 & 10 & 10 & 10 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (15)$$

$$\mathbf{G}_5 = \begin{pmatrix} 1000000 & 10 & 10 & 10 & 10 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 10 \end{pmatrix} \quad (16)$$

$$\mathbf{G}_6 = \begin{pmatrix} 10 & 10 & 0 & 0 & 0 \\ 0 & 10 & 10 & 0 & 0 \\ 0 & 0 & 10 & 10 & 0 \\ 0 & 0 & 0 & 10 & 10 \\ 0 & 0 & 0 & 0 & 10 \end{pmatrix} \quad (17)$$

One can verify that the above three matrices have the same value of GRL. On the other hand it is clear that:

- $\mathbf{G}_5$ corresponds to an acceptable solution of the blind separation problem.
- In the case of $\mathbf{G}_4$ all the sources have been estimated correctly except the first one.
- $\mathbf{G}_6$ means that the estimated signals are not independent signals except the last one.

3.5 Global Index

In [35, 36], the Global index has been used and defined by the following equation:

$$\rho\left(\mathbf{G}(k)\right) \stackrel{\text{def}}{=} 100 \sum_j \left(\max_i \left\{\frac{|g_{ij}|}{\sum_i |g_{ij}|}\right\} - \frac{1}{2}\right), \quad (18)$$

The authors describe that index in the case of $j = 1, 2$ (i.e. we have two sensors and two sources[1]). The idea of having a percentage performance index is a nice idea and one can verify, in the case of $m = n = 2$, that this index will be equal to hundred (which indicate the best solution) when the equation (3) holds. Unfortunately, the opposite isn't true, i.e. that index can attain the 100 without meaning that the global matrix $\mathbf{G}$ satisfies the equation (3), that can be easily verified by using the following matrices $\mathbf{G}_7 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\mathbf{G}_8 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\mathbf{G}_9 = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$, here $\rho(\mathbf{G}_7) = \rho(\mathbf{G}_8) = \rho(\mathbf{G}_9) = 100$.

[1] This index can be generalized as: $\rho\left(\mathbf{G}(k)\right) \stackrel{\text{def}}{=} 100 \sum_{j=1}^{n} \left(\max_i \left\{\frac{|g_{ij}|}{\sum_i |g_{ij}|}\right\} - \frac{1}{n}\right)$.

3.6 Error Norm

Using a matrix norm, the authors of [37] define the Error Norm as:

$$EN(\mathbf{A}) \stackrel{\text{def}}{=} \|\mathbf{A} - \widehat{\mathbf{A}}\| \tag{19}$$

where $\widehat{\mathbf{A}}$ is the estimated mixing matrix (i.e. $\widehat{\mathbf{A}} = \mathbf{B}^{\#}$) and $\|\mathbf{A}\|$ is the matrix norm of $\mathbf{A}$ (one can use by example the Frobenius matrix norm [38], i.e. $\|\mathbf{A}\| = \sqrt{\sum_{i,j} a_{ij}^2}$). To use the EN index, one should know the mixing matrix.

Unfortunately, this index is variant by matrix multiplication (i.e., $\widehat{\mathbf{A}}$ should be exactly the matrix $\mathbf{A}$ and it can not be equal to $\mathbf{AP\Delta}$, for any $\mathbf{\Delta}$ full-rank diagonal matrix and a $\mathbf{P}$ permutation matrix). In real world applications, the EN is not a good index, as one can conclude from the following example:

$$\mathbf{A}_1 = \begin{pmatrix} 10000 & -10000 \\ 5000 & 600 \end{pmatrix} \qquad \widehat{\mathbf{A}}_1 = \begin{pmatrix} 9999 & -10001 \\ 5001 & 599 \end{pmatrix}$$

$$\mathbf{A}_2 = \begin{pmatrix} 0.5 & 0.6 \\ 0.4 & 0.8 \end{pmatrix} \qquad \widehat{\mathbf{A}}_2 = \begin{pmatrix} -0.5 & 1.6 \\ 1.4 & -0.2 \end{pmatrix}$$

One can verify that the $EN(\mathbf{A}_1) = EN(\mathbf{A}_2)$ but it is clear that the estimation in the case of $\mathbf{A}_1$ is much better than the one of $\mathbf{A}_2$.

3.7 Symbol Error Rate

In some applications, the sources can be special signals (as for example Nbinary signals) and one can find some performance indexes related to the type of the signals [37] as the ”Symbol Error Rate” (SER):

$$SER \stackrel{\text{def}}{=} \frac{\text{Number of erroneous estimated source bits}}{\text{Number of total source bits}} \tag{20}$$

3.8 Mixture & Global Matrices

By writing down the global and mixing matrix, one can get an idea about the performances of the algorithms presented on [39, 40, 41, 42].

By plotting the diagonal elements and the off-diagonal elements of the global matrix, the algorithm performances were shown in [43]

Other researchers prefer to give the mixture matrix and the root-mean-square error and estimation mean estimated in 100 Monte Carlo runs [44].

3.9 Scatter Plot

Using the fact that two independent signals have a rectangular shape in their own (or phase) plan which is called the scatter plot of the two signals, in [45, 46, 47, 48] the authors plot the scatter plots of the sources, the mixing signals and the estimated signals to show that the separation was done.

3.10 Plotting Of The Estimated Sources

Some researchers plot all the signals involved in their problem as the sources, the mixing signals and the estimated sources and let the conclusion to the readers as in [49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63]. The problems of the plotting strategy, as in this subsection and the previous one, that it can not be used easily to compare two different algorithms applied to different signals. At the same time, one should mention that the sources here should be known.

3.11 Plotting of the Error Signals

In some cases, the plot of the signals does not have any special meaning and it can be difficult to show the performance by comparing the sources with the estimated sources. In [64] the authors propose the performance of their algorithm by plotting the error signals (the difference between the original signal and the estimated one). As the previous two methods, one should have the original signals. In addition here, we should know the mixing matrix (an estimated signal should be the exact estimation of the corresponding source signal, no permutation neither a scalar factor are allowed).

3.12 Real World Signals

In all of the previous methods, one should assume that the mixing parameters or the sources are known. In this section we will briefly mention three different methods can be used to deal with the real world applications (i.e. neither the sources nor the mixing matrix are known).

In [65], the authors measure the performance of their algorithm by rate of the speech recognition results: They test their algorithm using an automatic speech recognition system trained on the Wall-Street Journal task to test its performances on the recorded and separated signals.

In a mobil phone application, the source signal can be considered as the signal recorded with the mobil unite mounted on a mouth simulator and the measurement done without any disturbance in anechoic room [66].

To show the performance of their algorithms, the authors in [67] used some recorded speech signals as inputs to two speaker devices. The sources signals were the speaker output signals. The observed signals were detected by two microphones (nondirectional microphones). In their experiments, the authors compare the separated signals with the original recorded signals.

4. Conclusion

In this paper, we emphasize and discuss the methods to compare and demonstrate the performances of blind separation of source algorithms. The most used performance indexes or performance plotting methods have been discussed here. We show that some methods can be used to compare different algorithms applied to different signals.

Up to now, most of the mentioned methods can be used uniquely in the case of simulated experiments (i.e. the sources or the mixing parameters are known). However some methods for real world signals have been presented, but these methods depend on the applications and they can not be considered as standard indexes or methods. Finally, we should mention just till nowadays, no "Bench Mark" has been made or used for the blind separation of sources algorithms.

References

[1] J. Hérault and B. Ans, "Réseaux de neurones à synapses modifiables: Décodage de messages sensoriels composites par un apprentissage non supervisé et permanent," *C. R. Acad. Sci. Paris*, vol. série III, pp. 525–528, 1984.

[2] J. Hérault, C. Jutten, and B. Ans, "Détection de grandeurs primitives dans un message composite par une architecture de calcul neuromimétique en apprentissage non supervisé," in *Actes du Xème colloque GRETSI*, Nice, France, 20-24, May 1985, pp. 1017–1022.

[3] A. Mansour, A. Kardec Barros, and N. Ohnishi, "Blind separation of sources: Methods, assumptions and applications.," *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, vol. E83-A, no. 8, pp. 1498–1512, August 2000.

[4] C. Jutten and J. Hérault, "Une solution neuromimétique du problème de séparation de sources," *Traitement du signal*, vol. 5, no. 6, pp. 389–403, 1988.

[5] P. Comon, "Analyse en composantes indépendantes et identification aveugle," *Traitement du signal*, vol. 7, no. 5, pp. 435–450, December 1990.

[6] C. Jutten, L. Nguyen Thi, E. Dijkstra, E. Vittoz, and Caelen J., "Blind separation of sources: An algorithm for separation of convolutive mixtures," in *International Signal Processing Workshop on Higher Order Statistics*, Chamrousse, France, July 1991, pp. 273–276.

[7] C. Jutten, A. Guerin, and Nguyen Thi L., "Adaptive optimization of neural algorithms," in *IWANN 91, Artificial neural networks*, Granada, Spain, September 1991, pp. 54–61.

[8] C. Jutten, L. Nguyen Thi, and J. Hérault, "Nouveaux algorithmes de séparation de sources," in *Con. satellite du cong. Europée de mathématiques: Aspects théoriques des reseaux de neurones*, Paris, France, July 1992.

[9] S. Van Gerven, D. Van Compernolle, L. Nguyen Thi, and C. Jutten, "Blind separation of sources: A comparative study of a 2nd and a 4th order solution," in *Signal Processing VII, Theories and Applications*, M.J.J. Holt, C.F.N. Cowan, P.M. Grant, and W.A. Sandham, Eds., Edinburgh, Scotland, September 1994, pp. 1153–1156, Elsevier.

[10] L. Nguyen Thi and C. Jutten, "Blind sources separation for convolutive mixtures," *Signal Processing*, vol. 45, no. 2, pp. 209–229, 1995.

[11] A. Mansour and C. Jutten, "A direct solution for blind separation of sources," *IEEE Trans. on Signal Processing*, vol. 44, no. 3, pp. 746–748, March 1996.

[12] L. Nguyen Thi, C. Jutten, and J. Caelen, "Speech enhancement: Analysis and comparison of methods in various real situations," in *Signal Processing VI, Theories and Applications*, J. Vandewalle, R. Boite, M. Moonen, and A. Oosterlinck, Eds., Brussels, Belgium, August 1992, pp. 303–306, Elsevier.

[13] B. Laheld and J. F. Cardoso, "Séparation adaptative de sources en aveugle. implantation complexe sans contraintes," in *Actes du XIVème colloque GRETSI*, Juan-Les-Pins, France, 13-16 September 1993, pp. 329–332.

[14] L. Shaolin and T. J. Sejnowski, "Adaptive separation of mixed broad-band sound sources with delays by a beamforming hérault-jutten network," *IEEE Journal of Oceanic Engineering*, vol. 20, no. 1, pp. 73–79, January 1995.

[15] O. Grellier and P. Comon, "Blind separation of discrete sources," *IEEE Signal Processing Letters*, vol. 5, no. 8, pp. 212–214, August 1998.

[16] A. Belouchrani, K. Abed-Meraim, J. F. Cardoso, and E. Moulines, "A blind separation technique using second-order statistics," *IEEE Trans. on Signal Processing*, vol. 45, pp. 434–444, 1997.

[17] L. Castedo, C. J. Escudero, and A. Dapena, "A blind signal separation method for multiuser communications," *IEEE Trans. on Signal Processing*, vol. 45, no. 5, pp. 1343–1348, May 1997.

[18] P. Comon, "Independent component analysis, a new concept?," *Signal Processing*, vol. 36, no. 3, pp. 287–314, April 1994.

[19] O. Macchi and E. Moreau, "Self-adaptive source separation by direct and recursive networks," in *in Proc. International Conference on Digital Signal Processing (DSP'93)*, Limasol, Cyprus, May 1993, pp. 1154–1159.

[20] E. Moreau and O. Macchi, "High-order contrasts for self-adaptive source separation," *International journal of adptive control and signal processing*, pp. 19–46, 1996.

[21] S. I. Amari, "Neural learning in structured parameter spaces: Natural riemannian gradient," in *Neural Information Processing System-Natural and Synthetic*, San Diego, Colorado, USA, 2-7 December 1996.

[22] H. H. Yang and S. I. Amari, "Adaptive on-line learning algorithms for blind separation - maximum entropy and minimum mutual information," *Neural Computation*, vol. 9, pp. 1457–1482, 1997.

[23] E. Moreau and O. Macchi, "Self-adptive source separation part ii: Comparison of the direct, feedback and mixed linear network," *IEEE Trans. on Signal Processing*, vol. 46, no. 1, pp. 39–50, 1998.

[24] O. Macchi and E. Moreau, "Adaptive unsupervised separation of discrete sources," *Signal Processing*, vol. 73, pp. 49–66, 1999.

[25] T. W. Lee, M. Girolami, and T. J. Sejnowski, "Independent component analysis for mixed sub-gaussian and super-gaussian sources," *Neural Computation*, vol. 11, no. 2, pp. 417–442, 1999.

[26] J. F. Yang, "Serial updating rule for blind separation derived from the method of scoring," *IEEE Trans. on Signal Processing*, vol. 47, no. 8, pp. 2279–2285, August 1999.

[27] J. Basak and S. I. Amari, "Blind separation of uniformly distributed signals: a general approach," *IEEE Trans. on Neural Networks*, vol. 10, no. 5, pp. 1173–1185, September 1999.

[28] J. F. Cardoso and B. Laheld, "Adaptive blind source separation for channel spatial equalization," in *Proc. cost229*, Bordeaux, France, 1992.

[29] A. Belouchrani and K. Abed-Meraim, "Séparation aveugle au second ordre de sources corrélées," in *Actes du XIVème colloque GRETSI*, Juan-Les-Pins, France, September 1993, pp. 309–312.

[30] E. Moreau and O. Macchi, "Séparation auto-adaptative de sources sans blanchiment prealable," in *Actes du XIVème colloque GRETSI*, Juan-Les-Pins, France, 13-16 September 1993, pp. 325–328.

[31] A. Belouchrani and J. F. Cardoso, "Maximum likelihood source separation for discrete sources," in *Signal Processing VII, Theories and Applications*, M.J.J. Holt, C.F.N. Cowan, P.M. Grant, and W.A. Sandham, Eds., Edinburgh, Scotland, September 1994, pp. 768–771, Elsevier.

[32] J. F. Cardoso, A. Belouchrani, and B. Laheld, "A new composite criterion for adaptive and iterative blind source separation," in *Proceedings of International Conference on Speech and Signal Processing 1994, ICASSP 1994*, Australia, 1994.

[33] A. Souloumiac and J. F. Cardoso, "Performances en séparation de sources," in *Actes du XIVème colloque GRETSI*, Juan-Les-Pins, France, Septmber 1993, pp. 321–324.

[34] J. F. Cardoso, "On the performance of orthogonal source separation algorithm," in *Signal Processing VII, Theories and Applications*, M.J.J. Holt, C.F.N. Cowan, P.M. Grant, and W.A. Sandham, Eds., Edinburgh, Scotland, September 1994, pp. 776–779, Elsevier.

[35] A. Kardec Barros, A. Mansour, and N. Ohnishi, "Adaptive blind elimination of artifacts in ecg signals," in *International ICSC Workshop on Independence & Artificial Neural Networks 98*, Tenerife-Spain, 11-13 February 1998.

[36] A. Kardec Barros, A. Mansour, and N. Ohnishi, "Removing artifacts from ecg signals using independent components analysis," *NeuroComputing*, vol. 22, pp. 173–186, 1999.

[37] K. Diamantaras and E. Chassioti, "Blind separation of n binary sources from one observation: A deterministic approach," in *International Workshop on Independent Component Analysis and blind Signal Separation*, Helsinki, Finland, 19-22 June 2000, pp. 93–98.

[38] G. H. Golub and C. F. Van Loan, *Matrix computations*, The johns hopkins press- London, 1984.

[39] A. J. Bell and T. J. Sejnowski, "An information-maximization approach to blind separation and blind deconvolution," *Neural Computation*, vol. 7, no. 6, pp. 1129–1159, November 1995.

[40] C. G. Puntonet, A. Prieto, C. jutten, M. Rodriguez Alvarez, and J. Ortega, "Separation of sources: A geometry-based procedure for reconstruction of n-valued signals," *Signal Processing*, vol. 46, no. 3, pp. 267–284, 1995.

[41] P. Comon, "Separation of stochastic processes whose a linear mixture is observed," in *Workshop on Higher-Order Spectral Analysis*, Vail (CO), USA, June 1989, pp. 174–179.

[42] P. Comon, "Separation of sources using higher-order cumulants," in *SPIE Vol. 1152 Advanced Algorithms and Architectures for Signal Processing IV*, San Diego (CA), USA, August 8-10, 1989.

[43] B. C. Ihm and D. J. Park, "Blind separation of sources using higher-order cumulants," *Signal Processing*, vol. 73, pp. 267–276, 1999.

[44] L. Tong, Y. Inouye, and R. W. Liu, "Waveform-preserving blind estimation of multiple independent sources," *IEEE Trans. on Signal Processing*, vol. 41, no. 7, pp. 2461–2470, July 1993.

[45] C. Jutten and J. Hérault, "Independent components analysis versus principal components analysis," in *Signal Processing IV, Theories and Applications*, J. L. Lacoume, A. Chéhikian, N. Martin, and J. Malbos, Eds., Grenoble, France, September 1988, pp. 643–646, Elsevier.

[46] A. Prieto, B. Prieto, C. G. Puntonet, A. Cañas, and P. Martín-Smith, "Geometric separation of linear mixtures of sources: Application to speech signals," in *International Workshop on Independent Component Analysis and Blind Separation of Signals*, J. F. Cardoso, Ch. Jutten, and Ph. loubaton, Eds., Aussois, France, 11-15 January 1999, pp. 295 – 300.

[47] A. Mansour, C. G. Puntonet, and N. Ohnishi, "A new geometrical blind separation of sources algorithm," in *The 5th World Multi-Conference on Systemics, Cybernetics and Informatics (SCI'2001)*, Florida, USA, 22-25 July 2001, pp. 350–355, This paper has been selected as one of the best papers presented in the session Image and Multidimensional Signal Processing IV.

[48] A. Mansour, C. G. Puntonet, and N. Ohnishi, "A simple ica algorithm based on geometrical approach.," in *Sixth International Symposium on Signal Processing and its Applications (ISSPA 2001)*, M. Deriche, Boashash, and W. W. Boles, Eds., Kuala-Lampur, Malaysia, August 13-16 2001, pp. 9–12.

[49] E. Weinstein, M. Feder, and A. V. Oppenheim, "Multichannel signal separation by decorrelation," *IEEE Trans. on Speech, and Audio Processing*, vol. 1, no. 4, pp. 405–414, October 1993.

[50] A. Cichocki, W. Kasprzak, and S. I. Amari, "Multi-layer neural networks with a local adaptive learning rule for blind separation of source signals," in *Proceedings of the 1995 International Symposium on Nonlinear Theory and Its Applications (NOLTA '95)*, Tokyo, Japan, 1995, pp. 61–65.

[51] K. Matsuoka, M. Oya, and M. Kawamoto, "A neural net for blind separation of nonstationary signals," *Neural Networks*, vol. 8, no. 3, pp. 411–419, 1995.

[52] A. Cichocki and R. Unbehauen, "Robust neural networks with on-line learning for blind identification and bilnd separation of sources," *IEEE Trans. on circuits & systems*, vol. 43, pp. 894–906, 1996.

[53] D. Yellin and E. Weinstein, “Multichannel signal separation: Methods and analysis,” *IEEE Trans. on Signal Processing*, vol. 44, no. 1, pp. 106–118, January 1996.

[54] A. Cichocki, I. Sabala, S. Choi, B. Orsier, and R. Szupiluk, “Self-adaptive independent component analysis for sub-gaussian and super-gaussian mixtures with an unknown number of sources and additive noise,” in *Proceedings of the 1997 International Symposium on Nonlinear Theory and Its Applications (NOLTA ’97)*, Hawaii, U.S.A, December 1997, pp. 731–734.

[55] A. Hyvärinen and E. Oja, “Independent componenet analysis by general nonlinear hebbian-like learning rules,” *Signal Processing*, vol. 64, pp. 301–313, 1998.

[56] A. Cichocki, I. Sabala, and S. I. Amari, “Intelligent neural networks for blind signal separation with unknown number of sources,” in *Proc. of Conf. Engineering of Intelligent Systems, ESI-98*, 1998.

[57] H. H. Yang, S. I. Amari, and A. Cichocki, “Information-theoritic approach to blind separation of sources in non-linear mixture,” *Signal Processing*, vol. 64, pp. 291–300, 1998.

[58] A. Mansour and N. Ohnishi, “Multichannel blind separation of sources algorithm based on cross-cumulant and the levenberg-marquardt method.,” *IEEE Trans. on Signal Processing*, vol. 47, no. 11, pp. 3172–3175, November 1999.

[59] E. Oja, “Nonlinear pca criterion and maximum likelihood in independent component analysis,” in *First International Workshop on Independent Component Analysis and signal Separation (ICA99)*, J. F. Cardoso, Ch. Jutten, and Ph. loubaton, Eds., Aussois, France, 11-15 January 1999, pp. 143–148.

[60] T. W. Lee, M. S. Lewicki, M. Girolami, and T. J. Sejnowski, “Blind source separation of more sources than mixtures using overcomplete representations,” *IEEE Signal Processing Letters*, vol. 6, no. 4, pp. 87–90, April 1999.

[61] A. Mansour and N. Ohnishi, “Discussion of simple algorithms and methods to separate non-stationary signals,” in *Fourth IASTED International Conference On Signal Processing and Communications (SPC 2000)*, M. H. Hamza, Ed., Marbella, Spain, 19 - 22 September 2000, pp. 78–85.

[62] A. Mansour, M. Kawamoto, and N. Ohnishi, “Blind separation for instantaneous mixture of speech signals: Algorithms and performances.,” in *IEEE Conf. of Intelligent Systems and Technologies for the Next Millennium (TENCON 2000)*, Kuala Lumpur, Malaysia, 24-27 September 2000, pp. I–26 – I–32.

[63] Y. Tan, J. Wang, and J. M. Zurada, “Nonlinear blind source separation using a radial basis function network,” *IEEE Trans. on Neural Networks*, vol. 12, no. 1, pp. 124–134, January 2001.

[64] A. Mansour and C. Jutten, “Fourth order criteria for blind separation of sources,” *IEEE Trans. on Signal Processing*, vol. 43, no. 8, pp. 2022–2025, August 1995.

[65] T. W. Lee, A. J. Bell, and R. Orglmeister, “Blind source separation of real world signals,” in *Proceding of International Conference on Neural Networks*, Houston, June 1997.

[66] H. Sahlin and H. Broman, “Separation of real-world signals,” *Signal Processing*, vol. 64, pp. 103–113, 1998.

[67] M. Kawamoto, A. Kardec Barros, A. Mansour, K. Matsuoka, and N. Ohnishi, “Real world blind separation of convolved non-stationary signals.,” in *First International Workshop on Independent Component Analysis and signal Separation (ICA99)*, J. F. Cardoso, Ch. Jutten, and Ph. loubaton, Eds., Aussois, France, 11-15 January 1999, pp. 347–352.