Multicoset Sampling Based on Frequency Shifting and Filtering

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***Abstract*— In this manuscript, the problem of compressive sampling of intercepted sparse signals is addressed. Assuming that the intercepted signals are sparse in frequency domain, we propose a new algorithm based on shifting and filtering concepts to perform a compressive sampling. Schematic circuits to carry out the undersampling step and the recovery of original signals are also proposed. Simulation results are discussed. Finally, conclusions and future works are presented.**

Keywords-component; Wireless Communication, Compressive sampling, Compressed Sensing, Cognitive Radio, Software Defined Radio, Intercepted Signals, Electronic Warfare.

I. INTRODUCTION

A major challenge of modern societies is the limitation of natural resources. From wireless telecommunication engineering point of view, the limitation of available spectrum is a strong restriction for developing new systems and applications [1]. To deal with an increasing demand on frequency bandwidths, new concepts of radio communication have recently been proposed. In fact, for developing new civil and military applications, many researchers and engineers are involved in cognitive radio techniques and concepts. A main idea of cognitive radio consists on sensing the spectrum to use available parts of it. In this case, the signal of the primary user, i.e. the one who is holding the license of that part of spectrum, should be intercepted and analyzed. In order to wisely exploit the spectrum, one should monitor a large radio frequency bandwidth. To reach such goal, researchers and engineers are developing two major research fields: Compressive Sensing and Compressive Sampling [2].

In this manuscript, the compressive sampling problem is considered [3]. To explain the importance of such problem, let us suppose that our wideband antenna can intercept signals with frequency between few hundred kilo Hz till almost 2GHz, in this case the intercepted signal should sampled at a 4GHz sampling rate which create huge amount of data to be stored or processed. In many situations, the spectrum is not well used and the intercepted signals are sparse in frequency domain. In this case, compressive sampling techniques consist on using a much lower sampling rate but keeping all necessary information [4].

In 1967, the pioneer works of Landau [5] proved the under sampling concept. However, the first implemented algorithms are mostly proposed at the beginning of this century [6-7]. Venkataramani and Bresler in [7] proposed an optimal sub-Nyquist algorithm to deal with multi-band signals. In [8], the authors show that under sampling techniques can be deployed to deal with correlated signals. Recently, Chen *et al.* conducting sub-Nyquist non-uniform sampling using a filter-bank [9-10].

Finally, many wireless algorithms deal with baseband signals. In this case, transmission channels should be modeled by complex random variables; it is worth mentioning that complex random variables could be considered as a vector of two real random variables.

II. MATHEMATICAL MODEL & BACKGROUNDS

Let $x(t)$ be a continuous complex sparse signal in the frequency domain, i.e. frequency band-limited to a subset F which is a union of m bounded intervals [11]:

$$F = \cup_{i=1}^{m} [a_i, b_i) \tag{1}$$

Let us denote by $f_{max} = b_m$ the maximum frequency of $x(t)$. The Fourier transform $X(f)$ of $x(t)$ is defined by:

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi f t} dt \tag{2}$$

In the following, small letter are used to represent signals in time, while capital letter to represent them in frequency domain. In addition, $x(t)$ is representing a continuous time signal and $x[n]$ is the corresponding sampled signal. The spectral occupancy Ω of $x(t)$ over the spectral support F is defined as follows [12]:

$$0 \le \Omega = \frac{\lambda(F)}{f_{max}} \le 1 \tag{3}$$

where $\lambda(F) = \sum_i (b_i - a_i)$ is the Lebesgue's measure for the spectral support F. It is clear that the Nyquist sampling frequency f_{Ny} should respect the following constraint:

$$f_{Ny} \geq 2 f_{max} \tag{4}$$

Hereinafter, we derive a compressed sampling algorithm to reduce the sampling frequency to its minimum limit $\lambda(F)$ which is the Nyquist - Landau minimal sampling rate [5].

III. Compressive Sampling

Here, we introduce the concept of conducting a compressed multicoset sampling by using frequency-shifting and filtering techniques in order to satisfy the Nyquist - Landau minimal sampling rate. The idea of frequency shifting and filtering is well known in the digital signal processing community and it has been used in various applications especially in wireless telecommunication, such as the removal of co-channel interference [13]. With any loss of generality, intercepted signal $x(t)$ can be considered as analytical signal, i.e. its spectrum only contents positive frequency ($\forall f < 0 \Rightarrow |X(f)| = 0$). In fact, in the case of real signals, the following steps of the proposed algorithm should be adjusted to take into consideration two main modifications:

- The symmetrical part of the spectrum (for real signals, the amplitude of the Fourier transform $|X(f)|$ should be an even function, i.e. ($\forall f \in \mathbb{R} \Rightarrow |X(f)| = |X(-f)|$).
- The frequency shifting should be realized by using a cosine function $s_{sh1}(t) = \cos(2\pi f_0 t)$ instead of a complex exponential function $s_{sh2}(t) = \exp(j2\pi f_0 t)$. The main difference is that the Fourier transform of the cosine is the sum of two impulse functions $S_{sh1}(f) = \frac{\delta(f+f_0)+\delta(f-f_0)}{2}$, where $\delta(f)$ is Dirac's impulse function. In this case, one should pay attention to spurious frequency shifting, i.e. any frequency shift is a kind of a dual of increasing and decreasing frequency shifting.

In real scenarios, the spectrum of intercepted signals is unknown. However, the frequency bandlimits of signals of interest could be perfectly or roughly known by guessing or recognizing the wireless transmission activities of the primary user (for example, the primary user is using a modem or a cell phone operating in GSM- GPRS, General Packet Radio Service, 3G mode or in WiMAX, Worldwide Interoperability for Microwave Access as defined in the IEEE 802.16, etc.) It is worth pointing out that the frequency bandlimits of signals of interest could be also estimated [12-14]. In [15-16], the authors proposed a blind sequential forward selection (SFS) algorithm to estimate the frequency bandlimits. Hereinafter, we consider that the frequency bandlimits are well known or well estimated. The effect of the estimation error is beyond the scope of this manuscript.

To simplify our discussion, we will discuss the case of analytical complex signal $x(t)$ who is the sum of two other signals $x(t) = x_1(t) + x_2(t)$ where $x_1(t)$ is a low pass analytical signal and $x_2(t)$ is a band-pass one, see Fig. 1.

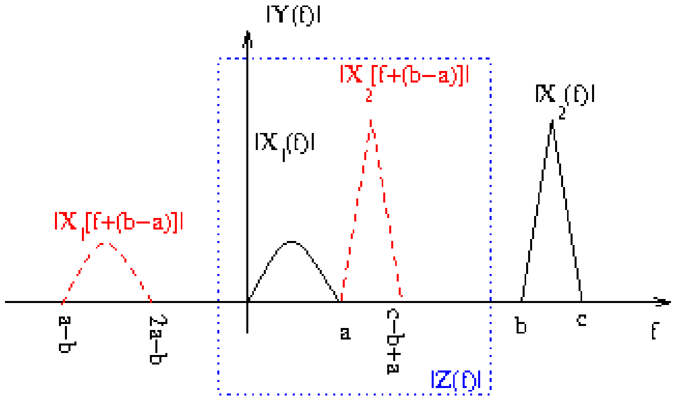


Figure 1: Signal Spectrums

As the highest frequency of $X_2(f)$ is c, the Nyquist frequency should be $f_{Ny} = 2c$. To justify the computation efforts of compressive sampling techniques, signals should be narrow passband ones, i.e. in our case $c - b \ll b$ as well as $a \ll b$. However, the Nyquist-Landau lower limit of $x(t)$ is given by:

$$f_{NL} = 2(c + a - b) \ll f_{Ny} = 2f_{max} = 2c \tag{5}$$

The main idea of our algorithm consists on creating a new lowpass analytical signal $z(t)$ by shifting $x_2(t)$ to lower frequency band, see the schematic presented in Fig. 2.

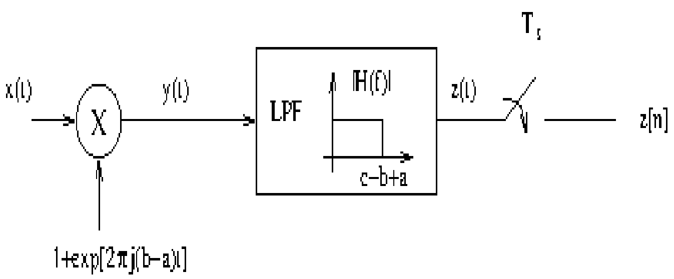


Figure 2: The modified low-pass signal $z(t)$.

It is clear that $z(t)$ can be obtained as the output of a lowpass filter excited by $y(t)$, see appendix A:

$$y(t) = x(t)\big(1 + exp(j2\pi(b-a)t)\big) \tag{6}$$

The spectrum $Y(f)$ of $y(t)$ is given in Fig. 1.

$$Y(f) = X(f) * \left(\delta(f) + \delta\big(f + (b-a)\big)\right)$$
$$= X(f) + X\big(f + (b-a)\big)$$

In figure (1), the Fourier transform of $z(t) = y(t) *h(t)$ becomes the highlighted dashed rectangular part. Fig. 2 shows that the recovery of the original signal $x(t)$ requires the following constraint:

$$b > c + a - b \Longrightarrow b > \frac{c+a}{2} \qquad (7)$$

The above constraint could be easily satisfied when the bandpass parts of the original signal are relatively narrow bands. It becomes obvious that $z(t)$ is an analytical lowpass signal witch its highest frequency is below $f_{max} = c + b - a$. Therefore a sampling circuit with a sampling frequency of f_{NL}, see equation (5), can correctly generate a digital copy of $z(t)$ and allow us to easily recover $z(t)$ using $z[n] = z_s(nT_s)$ (T_s is the sampling period and the sampled signal $z_s(n) = z(t)\sum_n \delta(t - nTs)$) according to a lowpass recovering techniques, see appendix B.

In order to complete our study, two major problems should be addressed:

- Once, $z[n]$ is obtained, any digital signal processing algorithm can be used to process this signal. However, how can we recover the original signal $x(t)$ from $z[n]$? In order to recover the original signal, we propose the following circuit based on a frequency shifting and using two different low-pass filters (where the cut off frequency (COF) of the first one should be $f_{max} = c + b - a$ and the COF of the second filter is a) in addition with a bandpass filter that its lower COF is b and its upper COF should be c, see Fig. 3.

In some identification or information extraction applications, one should have the exact frequency band of the original signal. In our case, we obtained $z[n]$ which is a modified copy of $x[n]$ using compressive sampling. The second problem to be addressed is the follows: Is it possible to have $x[n]$ by only using $z[n]$? if $x[n]$ is needed, then one can use the schematic shown in Fig. 4.

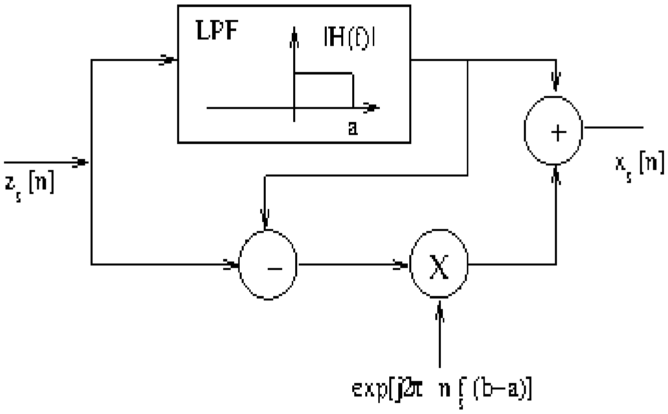


Figure 4: The recovered signal $\hat{x}[n]$.

IV. Simulation Results

Various simulation results have been conducted to evaluate the overall performance of our algorithm. In this section, the outcomes results of a typical and generic study is shown and discussed.

The original intercepted signal $x(t)$, as shown in Fig. 5, is set as the sum of two chrip signals: The instantaneous frequency of the first one is linearly increasing between [0, a = 400Hz], while the second chrip instantaneous frequency is also linearly increasing in [b = 2Khz, c = 2100Hz]. The original sampling frequency (oversampling) was fixed at 10kHz. It is clear that the used signal satisfies the constraint (1).

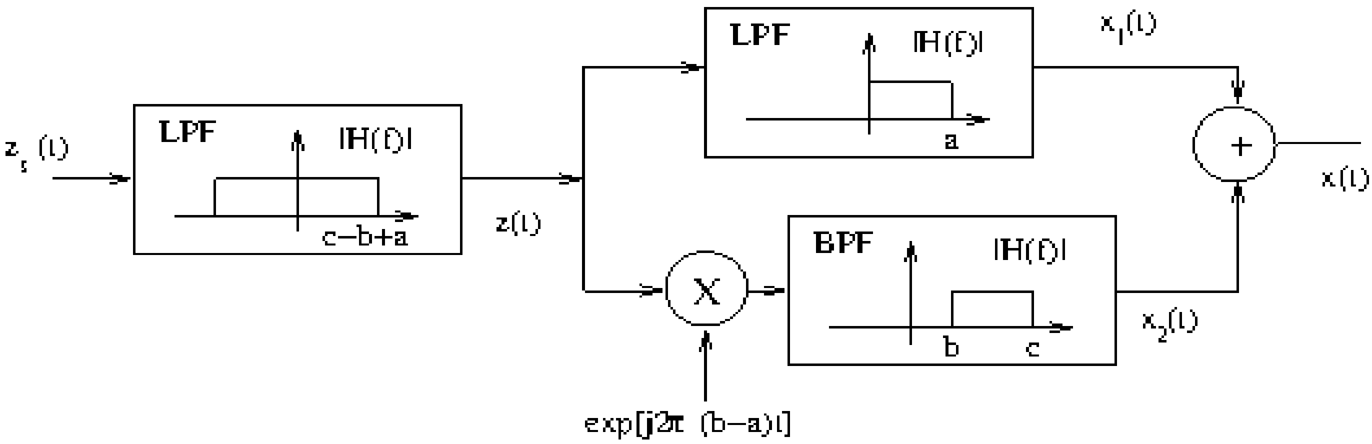


Figure 3: The recovered signal $\hat{x}(t)$

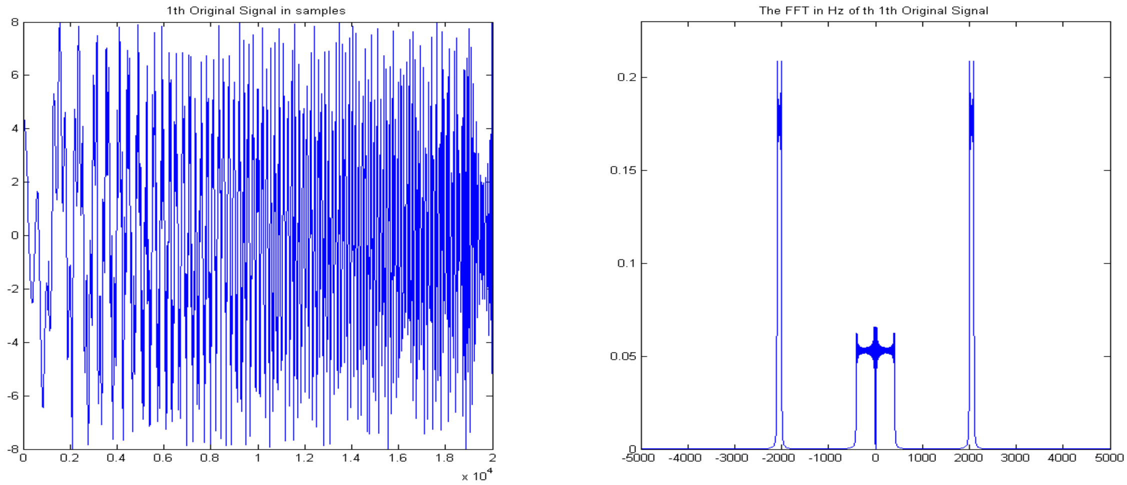


Figure 5: The intercepted signal $x(t)$ and its Fourier transform $X(f)$.

The analytical signal $x_a(t)$ has been obtained using a Hilbert transform (TH):

$$x_a(t) = x(t)+j\ \mathrm{TH}(x(t))$$

where j stands for the complex number, see Fig. 6. The latter signal was used as the input of the circuit proposed in Fig. 2. In this case the Nyquist-Landau sampling rate of $x_a(t)$ should be equal to $2(c + b - a)$ = 2*(2100-2000+400)=1KHz. However to avoid using high order passband filter (in our experimental studies, 4th order lowpass and passband butterworth filters are implemented) and some instability problems due to the simulation environment of Matlab, we decided to shift the $x_2(t)$ by b - 1.1 a = 1560Hz. Fig. 7 shows the shifted signal $y(t)$.The compressive sampling rate is selected to be 2 KHz instead of the original 10Khz. Fig. 8 shows that the obtained $z[n]$ sampled at 10 Khz or 2 KHz have exactly the same information. Fig. 9 shows the recovered original signal $\hat{x}[n]$

Finally, it is worth be mentioning that the digital lowpass and bandpass analytical filters could be implemented using the method proposed in [17] and the references cited therein.

V. Conclusions & Future Works

In this manuscript, a simple concept of compressive sampling is proposed and discussed. In the cases of noise-free signals, while the proposed algorithm can reach the Nyquist-Landau lower sampling rate, it is still using a sample regular classic sampling circuit. Simulation results corroborate the performance of our algorithm.

The following two problems, which are beyond the scope of the actual manuscript, should be addressed in our future works:

- Can the proposed circuit be simplified to cope with a variable and relatively big number of frequency segments, i.e. a big number m in equation (1)?
- Can the frequency segment estimation algorithms proposed in the literature, such as [15-16], be tuned and optimized to fit our schematic circuit and enhance the overall performances?

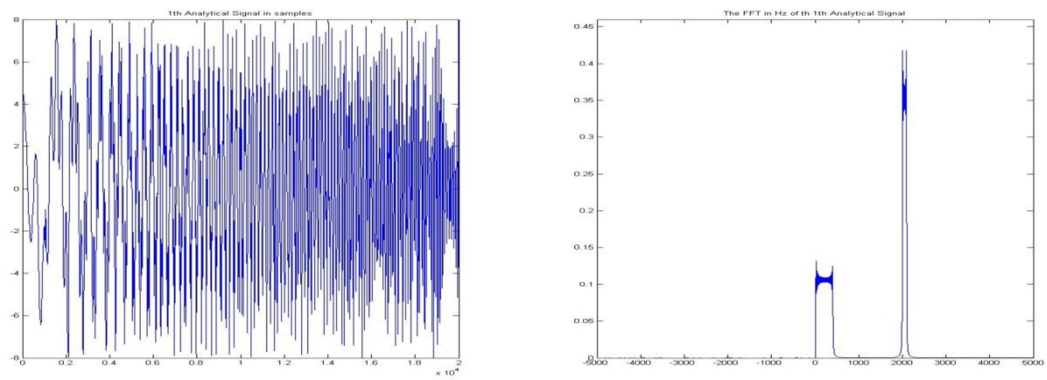


Figure 6: The intercepted analytical signal $x_a(t)$ and its Fourier transform.

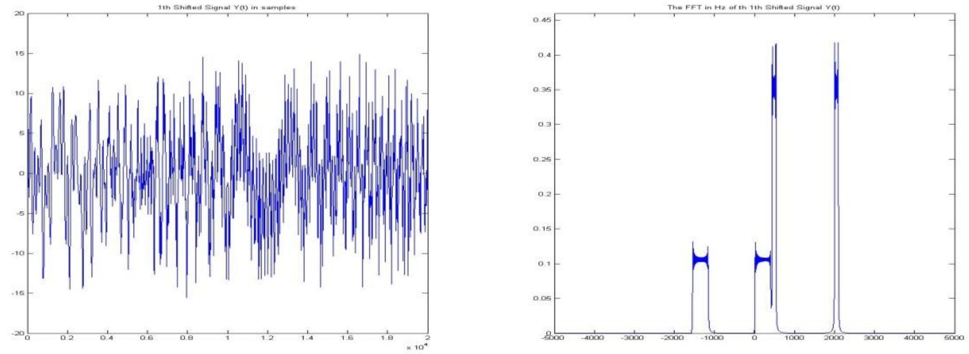
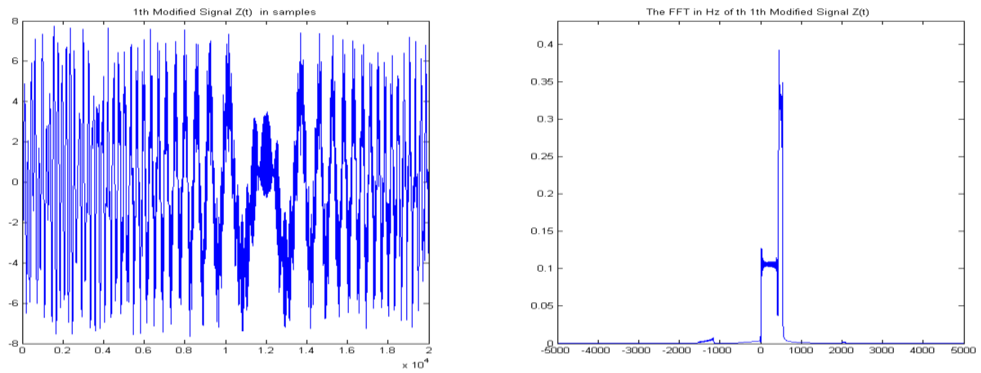
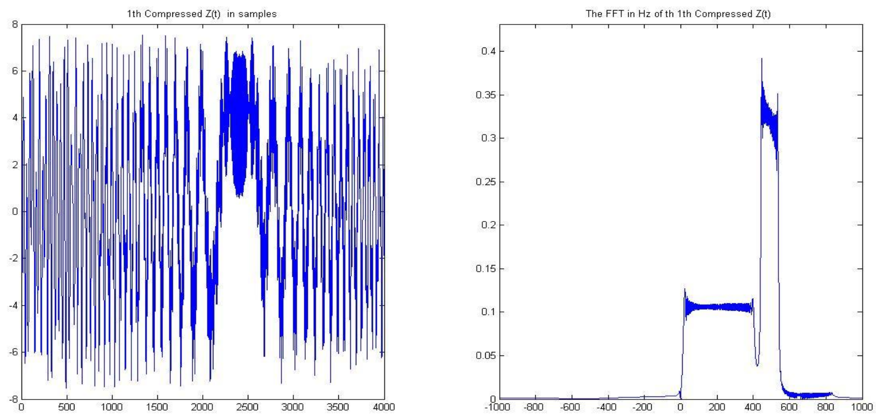


Figure 7: The shifted signal $y(t)$ and its Fourier transform.



(a) The modified signal $z[n]$ sampled with 10KHz.



(b) The modified signal $z[n]$ sampled with 2KHz.

Figure 8: The Lowpass Modified Signal $z[n]$

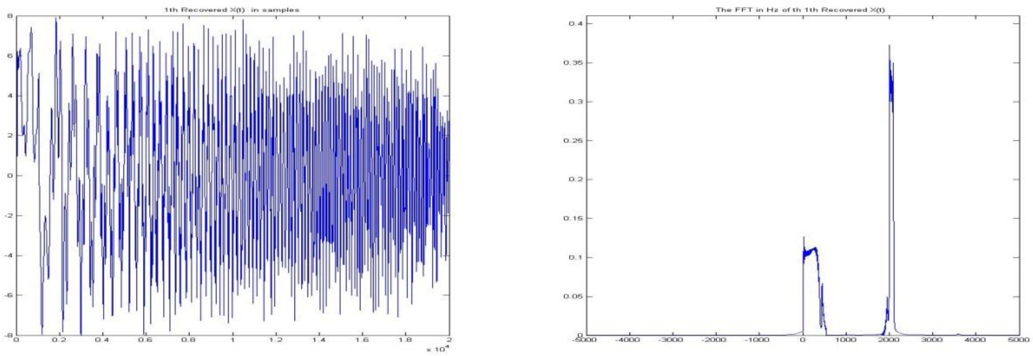


Figure 9: The recovered signal $\hat{x}[n]$

Appendix A: Filtering by rectangular window

Let h(t) denotes the impulse time response of a rectangular filtering window H(f) defined by the following equation:

$$|H(f)| = \begin{cases} 1 & \forall f \in [a,b] \\ 0 & elsewhere \end{cases} \qquad (8)$$

In this case, one can evaluate $h(t)$ using the following equations:

$$\begin{aligned} h(t) &= \int_{-\infty}^{+\infty} H(f) e^{j2\pi ft} df = \int_a^b e^{j2\pi ft} df \\ &= \frac{e^{j2\pi bt} - e^{j2\pi at}}{j2\pi t} \\ &= e^{j2\pi(b+a)t}\left(\frac{e^{j2\pi(b-a)t} - e^{-j2\pi(b-a)t}}{j2\pi t}\right) \\ &= (b-a)\sin c(\pi(b-a))e^{j\pi(b+a)t} \end{aligned} \qquad (9)$$

where $sinc(x) = \frac{sin(x)}{x}$. One should notice two special cases:

- Real Low pass Filter: In this case, $b = -a = f_m$ then

  $h(t) = 2f_m\, sinc(2\pi f_m t)$ (10)

- Analytic Filter: Where $a = 0$ and $b = f_m$ then

  $h(t) = f_m\, e^{j\pi f_m t} sinc(\pi f_m t)$ (11)

Appendix B: Recovering a sampled signa

Let $s(t)$ be a low pass signal with a maximum frequency f_m and let $s_s(t)$ the sampled signal, i.e:

$$s_s(t) = s(t) \sum_{n\in\mathbb{Z}} \delta(t - nT_s) \qquad (12)$$

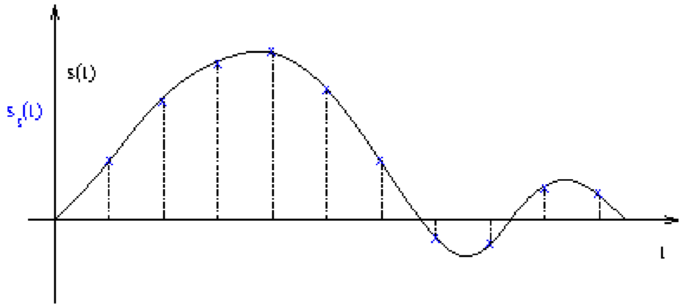
where T_s is the sampling period and $\delta(t)$ is the dirac impulse. Applying Fourier transform and Poisson's summation theorem, the previous equation becomes:

$$S_s(f) = S(f) * \frac{1}{T_s}\sum_{m\in\mathbb{Z}} \delta(f - mf_s) = \frac{1}{T_s}\sum_{m\in\mathbb{Z}} S(f - mf_s) \qquad (13)$$

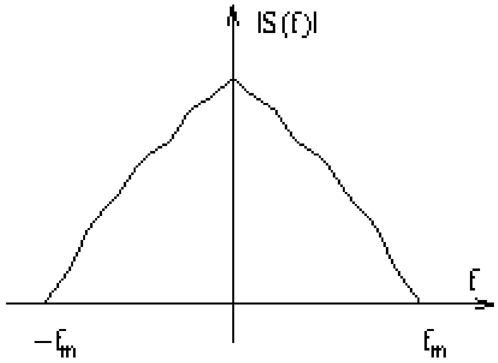
Here * denotes the convolution product and $f_s = \frac{1}{T_s} \geq 2f_m$ is the sampling frequency. Equation (13) shows that the Fourier transform of the sampled signal is the moderated sum of shifted copies of $S(f)$. Therefore, it is clear to prove that $S(f)$ could be easily recovered from $S_s(f)$ using a simple LPF, see Fig. 10:

$$\begin{aligned} s(t) &= [s(t) \textstyle\sum_{n\in\mathbb{Z}} \delta(t - nT_s)] * h(t) \\ &= \textstyle\sum_{n\in\mathbb{Z}} s(nT_s)h(t - nT_s) \end{aligned} \qquad (14)$$

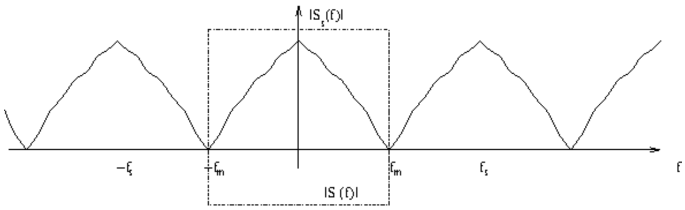
Here $h(t)$ is the impulse response of a real low pass filter, see Appendix A.



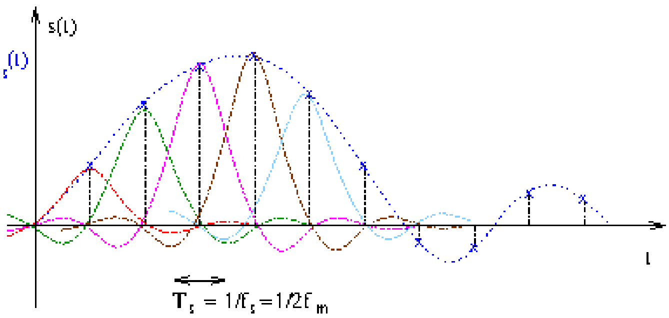
(a) Original signal and sampled signal



(b) The spectrum of the original signal



(c) The spectrum of the sampled signal



(d) The recovered signal

Figure 10: Fundamental steps in sampling and recovering of a lowpass signal

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