Relay Selection for Full-Duplex FSO Relays Over Turbulent Channels

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***Abstract*—This paper investigates the performance of the best relay selection, based on the max-min signal-to-noise ratio criterion for dual-hop free-space optical (FSO) full-duplex (FD) relays communication system. Decode-and-forward relays over log-normal (LN) channels for weak-to-moderate turbulence and gamma-gamma (G-G) channels for strong turbulence are considered. We assume that the relays have full channel knowledge and the channel is symmetrical. Considering path loss effects and misalignment errors, the outage probability (OP) of the selection is obtained for both half-duplex (HD) and FD relays using the cumulative distribution function (CDF) of the best selection for LN and G-G random variables. Moreover, the average bit error rate (ABER) expressions for FSO communication system over LN channels are derived with the help of Gauss-Laguerre's quadrature rule for HD relays, FD relays and direct link. Our results show that FD relays have lowest ABER and OP compared with the direct link and HD relays. Monte Carlo simulations corroborate the correctness of the obtained analytical results.**

***Index Terms*—Free-space optical communications; atmospheric turbulence; full-duplex; half-duplex; cooperative relay; decode and forward; outage probability; bit error rate; relay selection.**

I. Introduction

The increasing demand for high data rates along with the congested radio frequency (RF) band necessitate the search for alternative solutions. While optical fiber is poised to solve both problems, it remains an expensive alternative that requires special infrastructure. Free-space optical (FSO) communication, on the other hand, has the same advantages of high-speed optical fiber communication in addition to being cost effective, license free wide-spectrum technology and requires no heavy infrastructure. FSO systems have been significantly used as back up for fiber optic, backhaul for wireless cellular networks, as well as high definition video broadcasting applications. However, FSO has many challenges such as atmospheric turbulence-induced fading, sensitivity to weather conditions, background noise, geometric losses and misalignment problem [1], [2].

Relay-assisted techniques mitigate turbulence effect and path-loss attenuation for FSO communication systems through shortening hops yielding significant performance improvements [3]. Amplify-and-forward (AF) or decode-and-forward (DF) relays can be considered for FSO communication systems [3]. The spectral efficiency degrades when all relays participate in forwarding their received signals to the destination since orthogonal time slots are assigned to each relay [4]. Alternatively, best relay selection techniques can be considered to enhance the performance without degrading the spectral efficiency.

A study in [4] proposes and analyzes the outage probability (OP) of an asynchronous low-complexity cooperative relaying (CR) with best relay selection based on the max-min signal-to-noise ratio (SNR) criterion. In [5], an upper bound on the average bit error rate (ABER) for the best relay selection according to the max-min criterion of SNR for log-normal (LN) and Rayleigh channels is derived. The ABER for the best relay selection scheme according to source-relay SNR is derived using the power series expansion for gamma-gamma (G-G) channels [6]. The ABER performance of best relay selection according to the max-min criterion of SNR for dual-hop parallel DF FSO over G-G channels employing adaptive subcarrier quadrature amplitude modulation is obtained using power series expansion method [7]. In [8], the ABER performance of best relay selection according to the max-min criterion of SNR for multi-hop parallel FSO using DF over exponentiated Weibull (EW) fading channels is derived.

Furthermore, the ABER and the diversity order of relay selection according to the max-min criterion of SNR of parallel DF relays for FSO over G-G channels with misalignment errors are derived in [9]. Moreover, the ABER of relaying selection according to the max-min criterion of SNR of parallel DF relays for FSO over G-G channels without taking into consideration misalignment errors are derived using Gauss-Laguerre quadrature rule [10].

In [11], a DF FSO system is employed with two-way relay (TWR) over LN channel and the OP, the ABER and the ergodic capacity are reported. In [12], TWR DF FSO is employed over $\mathcal{M}$-distribution FSO channels and the ABER and the OP are derived taking into consideration misalignment error effects. Partial relay selection for TWR coherent FSO using AF is employed over G-G channels while considering path losses and pointing errors [13]. The performance of frame error rate of improved adaptive decode-and-forward scheme outperforms adaptive decode-and-forward scheme provided source-relay link and relay-destination link are shorter than source-destination link [14].

With reference to existing literature, the main contributions of this paper are: The OP and the ABER for best relay selection FSO communication system based on the max-min SNR criterion are obtained for both full-duplex (FD) and half-duplex (HD) DF relays. LN and G-G channels are considered with the effects of weather attenuation, geometric losses and misalignment errors. Detailed performance analysis of FD and HD relays along with direct link are reported.

The remainder of this paper is organized as follows: the system and channel models are discussed in Section II. In Section III, outage performance analysis of the proposed scheme is presented. The ABER performance of the proposed scheme is derived in Section IV. Numerical results and discussions are given in Section V and the paper is concluded in Section VI.

II. System And Channel Models

A. System Model

The considered system is depicted in Fig. 1, where two users communicate through N relay nodes. Only a single relay among existing N relays participates in the communication process. The selected relay should have the highest receive end-to-end SNR and is known to both users. The best relay is selected before data transmission and is defined as [4]

$$j = \max_{i\in\{1:N\}} \left(\min\left(h^2_{\mathrm{SR}_i}, h^2_{\mathrm{R}_i\mathrm{D}}\right)\right) \tag{1}$$

where j is the index of the best relay, i is a relay index, h_{SR_i} and $h_{\mathrm{R}_i\mathrm{D}}$ are the channel fading coefficients between the source and ith relay, R_i, and between R_i and the destination, respectively. In the first time slot, t_1, each user transmits his data to the preselected relay. The user's transceiver steer the signal in the direction of the selected relay. The relay has two transceivers, each of which is directed towards one user. The relay decodes both received signals and in the second time slot, t_2, forwards the received signal to both users. The channel

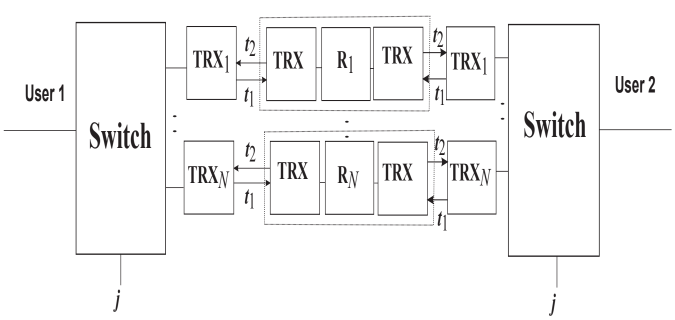

Fig. 1. Block diagram of the proposed FD FSO communication system.

state information (CSI) can be estimated by a simple signaling process due to the advantage of the slowly varying nature of the fading channel in the FSO environment [4], [3], [2]. Hence, it is assumed that all relays have full channel knowledge and the channel is symmetrical [15]. The received signals are affected by an additive white Gaussian noise (AWGN) with zero mean and variance $\sigma_n^2 = N_o/2$ resulting mainly from background noise [15]. The normalized channel coefficient of the considered system can be formulated as follows [15]:

$$h = h_a\ h_p\ h_\beta, \tag{2}$$

where h_a and h_p are the channel fading coefficients due to atmospheric turbulence and misalignment error, respectively. The path losses, h_β, can be calculated by combining weather attenuation with geometric losses as [15]

$$h_\beta = 10^{-\alpha\ell/10} \times \frac{D_R^2}{(D_T + \theta_T \ell)^2}, \tag{3}$$

with ℓ being the link distance for a hop (in km), α is the weather-dependent attenuation coefficient (in dB/km), D_R and D_T are the receiver and transmitter aperture diameters (in m), and θ_T is the optical beam divergence angle (in mrad).

The general model of misalignment fading proposed in [16] is considered hereinafter

$$h_p \approx A_o \exp\left(\frac{-2r^2}{w^2_{\mathrm{zeq}}}\right), \tag{4}$$

where r is the radial displacement which is modeled by Rayleigh distribution, the equivalent beam width (in m), w_{zeq}, is calculated as $w^2_{\mathrm{zeq}} = (w_z^2\sqrt{\pi}\mathrm{erf}(v))/(2v\exp(-v^2))$, $v = (\sqrt{\pi}D_R)/(2\sqrt{2}w_z)$, $A_o = [\mathrm{erf}(v)]^2$, $\mathrm{erf}(\cdot)$ is the error function in [17, Eq. (8.250/1)] and w_z is the beam waist (in m) (radius calculated at e^{-2}). The radial displacement can be modeled as

$$f_r(r) = \frac{r}{\sigma_s^2}\exp\left(-\frac{r}{2\sigma_s^2}\right), \qquad r > 0 \tag{5}$$

where σ_s^2 is the jitter variance at the receiver.

B. LN Channels

The probability density function (PDF) of LN channels is given by [16]

$$\begin{aligned} f_h(h) = & \frac{\xi^2 h^{(\xi^2-1)}}{2(A_o\beta)^{\xi^2}} \\ & \times \mathrm{erfc}\left(\frac{\ln\left(\frac{h}{A_o\beta}\right) + q}{\sqrt{8}\sigma_x}\right)\exp\left(2\sigma_x^2\xi^2(1+\xi^2)\right), \end{aligned} \tag{6}$$

where $\mathrm{erfc}(\cdot)$ denoting the error function in [17, Eq. (8.250/4)], β denotes the normalized path loss coefficient, $\beta = h_\beta/\beta_h$ with β_h being the path loss for the first-hop, $q = 2\sigma_x^2(1+2\xi^2)$ and $\xi = \frac{w^2_{\mathrm{zeq}}}{2\sigma_s^2}$ the ratio between the equivalent beam width at the receiver and the pointing error displacement standard deviation at the receiver.

The channel fading coefficient $h_a = \exp(2x)$, with x being an independent and identically distributed (i.i.d.) Gaussian random variable (RV) with a mean μ_x and a variance σ_x^2. To ensure that the fading channel does not attenuate or amplify the average power, the fading coefficients are normalized [18]. Hence, a plane wave propagation is assumed and the log-amplitude variance is calculated based on the Rytov theory as a function of the distance [16]

$$\sigma_x^2 = 0.30545\ k^{7/6} C_n^2\ \ell^{11/6}, \tag{7}$$

with $k = 2\pi/\lambda$ is the wave number, λ is the wavelength and C_n^2 is the refractive index constant ($\mathrm{m}^{-2/3}$).

C. G-G Channels

The PDF of G-G channels is given by [19]

$$f_h(h) = \frac{ab\xi^2}{A_o\beta\Gamma(a)\Gamma(b)} \times G_{1,3}^{3,0}\left[\frac{abh}{\beta A_o}\middle| \begin{matrix} \xi^2 \\ \xi^2-1, a-1, b-1 \end{matrix}\right], \tag{8}$$

where $G_{p,q}^{m,n}[.]$ is the Meijers G-function in [17, Eq. (9.301)], $\Gamma(.)$ is the Gamma function in [17, Eq. (8.310)], a and b are the effective number of large-scale and small-scale eddies of scattering environment, respectively. Their values for plane wave are given as [20]

$$a = \left[\exp\left(\frac{0.49\sigma_R^2}{(1+1.11\sigma_R^{\frac{12}{5}})^{\frac{7}{6}}}\right) - 1\right]^{-1}, \tag{9}$$

$$b = \left[\exp\left(\frac{0.51\sigma_R^2}{(1+0.69\sigma_R^{\frac{12}{5}})^{\frac{7}{6}}}\right) - 1\right]^{-1}, \tag{10}$$

and the Rytov variance, σ_R^2, is given as [20]

$$\sigma_R^2 = 1.23\ k^{7/6} C_n^2\ \ell^{11/6}. \tag{11}$$

III. Outage Performance Analysis

The OP at each node can be obtained directly from the cumulative distribution function (CDF) of SNR, $F_\gamma(\gamma)$, [4]

$$P_{\mathrm{out_H}} = \Pr(\gamma < \gamma_{th}) = F_\gamma(\gamma_{th}), \tag{12}$$

where $\gamma = h^2\bar{\gamma}$ is the SNR and $\bar{\gamma}$ is the average SNR. If SNR exceeds γ_{th}, no outage occurs and the signal can be decoded with an arbitrarily low error probability at the receiver.

A. LN Channels

For LN channels, the OP is derived as [16]

$$\begin{aligned} P_{\mathrm{out_H}} &= \Pr\left(h < \frac{1}{\rho P_M}\right) \\ &= \frac{1}{2}\exp\left(\xi^2\Psi - 2\sigma_x^2\xi^4\right)\mathrm{erfc}\left(\frac{\Psi}{\sqrt{8}\sigma_x}\right) \\ &+ \frac{1}{2}\ \mathrm{erfc}\left(\frac{4\sigma_x^2\xi^2 - \Psi}{\sqrt{8}\sigma_x}\right) = F_\gamma(\gamma_{th}), \end{aligned} \tag{13}$$

where $P_M = \sqrt{\frac{\bar{\gamma}}{\gamma_{th}}}$ is the power margin and $\Psi = \ln\left(\frac{1}{A_o\beta\rho P_M}\right) + 2\sigma_x^2(1+2\xi^2)$. The power of HD systems, such as direct link and HD relays, are assumed to be half the power of FD systems in order to achieve a fair comparison using the same equipment of Fig. 1 [21]. Hence, the power margin is multiplied by a constant, ρ. The value of ρ is unity for FD systems and one-half for HD systems. The OP for a dual-hop system is given by [4]

$$P_{\mathrm{out}} = 1 - \left[\left(1 - P_{\mathrm{out}_1}\right)\left(1 - P_{\mathrm{out}_2}\right)\right]. \tag{14}$$

where P_{out_1} and P_{out_2} are the outage probability for the first node and the second node, respectively. Due to the assumption that the relays have full channel knowledge and the network is symmetrical [15], under this assumption, (14) can be used for both FD and HD relays taking into consideration the power constraint. The values of β and σ_x^2 for each node in dual-hop scheme decrease in directly proportion to the distance, which enhances the performance of multi-hop systems. If multiple branches of dual-hop exist, as in Fig. 1, the OP for the best relay selection as in (1) can be calculated as [4]

$$P_{\mathrm{out}} = \left[1 - \left[\left(1 - P_{\mathrm{out}_1}\right)\left(1 - P_{\mathrm{out}_2}\right)\right]\right]^N = F_\gamma(\gamma_{th}). \tag{15}$$

It is evident that a diversity gain of N is achieved for a dual-hop system with the best relay selection.

B. G-G Channels

For G-G channels, the OP at each node is derived as [22]

$$\begin{aligned} P_{\mathrm{out_H}} &= \frac{\xi^2}{\Gamma(a)\Gamma(b)} \times G_{2,4}^{3,1}\left[\frac{ab}{\beta\rho P_M}\middle| \begin{matrix} 1, \xi^2+1 \\ \xi^2, a, b, 0 \end{matrix}\right] \\ &= F_\gamma(\gamma_{th}), \end{aligned} \tag{16}$$

The OP for the best relay selection can be calculated by substituting (16) into (15). For dual-hop system, the values of a, b and β should be changed as the link distance is decreased.

IV. ABER Performance Analysis

The ABER can be calculated directly using the CDF approach as [23]

$$\begin{aligned} \mathrm{ABER} = d\ \mathbb{E}\left[Q(\sqrt{c\gamma}\ )\right] &= \frac{d}{\sqrt{2\pi}} \\ \times \int_0^\infty F_\gamma\left(\frac{y^2}{c}\right) &\exp\left(-\frac{y^2}{2}\right) dy, \end{aligned} \tag{17}$$

where $Q(\cdot)$ is the Gaussian Q-function, $\mathbb{E}[\cdot]$ denotes the average over channel fading distributions, and c and d are constants determined by the modulation format. In this study, multiple pulse amplitude modulation ($M-$PAM) using intensity modulation with direct detection (IM/DD) is considered as in [24]. The spectral efficiency of $M-$PAM is equal to $\log_2(M)$ bits/s/Hz [24]. Hence, 2-PAM is considered for direct link (1 bit per slot) and FD relays (2 bits from two users per 2 slots), while 4-PAM is considered for HD relays (2 bits per 2 slots) to maintain similar spectral efficiency [24]. The conditional bit error probability (BEP) of M-PAM is given by [25]

$$\Pr(e|\gamma) \approx \frac{2(M-1)}{M\log_2(M)}\left[Q\left(\sqrt{\frac{\gamma\log_2(M)}{2(M-1)^2}}\right)\right]. \tag{18}$$

The values of d and c are (d, c) = (1, 0.5) for 2-PAM and (d, c) = (0.75, 0.1111) for 4-PAM. It is worth mentioning that $M-$PAM has different intensity levels, I_i^{PAM}, according to the symbol sequence as: $I_i^{\mathrm{PAM}} = \frac{\overline{I}}{M-1} \quad (i-1)$, where $i = (1, 2, \cdots, M)$, and $\overline{I}$ denotes the average light intensity. For the sake of fair comparison, the average light intensity levels for arbitrary M-PAM modulation scheme is fixed to $\frac{1}{M}\sum_{i=1}^{M} \frac{\overline{I}}{M-1} \quad (i-1) = \frac{\overline{I}}{2}$. Hence, guaranteeing the same average optical power for the family of M-PAM modulation schemes. CSI is assumed available at the receiver side and a maximum likelihood (ML) decoder is used to decode the received signals as [24]

$$\hat{i} = \arg\min_{i} \|\mathbf{r} - \eta \mathrm{h}_i\|_{\mathrm{F}}^2 , \tag{19}$$

where $\boldsymbol{r}$ denotes the received signal, η is the optical to electrical conversion coefficient and $||.||_F$ is the Frobenius norm [17].

A. LN Channels

The ABER at the receiving node over LN channels can be derived by substituting (13) into (17) as

$$\begin{aligned} &\mathrm{ABER_H} = \qquad (20)\\ &\frac{d}{\sqrt{2\pi}} \int_0^\infty \frac{1}{2} \exp\left(\xi^2 \grave{\Psi} - 2\sigma_x^2 \xi^4 - \frac{y^2}{2}\right) \mathrm{erfc}\left(\frac{\grave{\Psi}}{\sqrt{8}\sigma_x}\right) dy \\ &+ \frac{d}{\sqrt{2\pi}} \int_0^\infty \frac{1}{2}\ \mathrm{erfc}\left(\frac{4\sigma_x^2\xi^2 - \grave{\Psi}}{\sqrt{8}\sigma_x}\right) \exp\left(-\frac{y^2}{2}\right) dy, \end{aligned}$$

where $\grave{\Psi} = \ln\left(\frac{y}{A_o \beta\rho\sqrt{\overline{\gamma} c}}\right) + 2\sigma_x^2(1+2\xi^2)$. After performing a simple transformation of $x = \frac{y^2}{2}$, (20) is easily obtained as

$$\begin{aligned} &\mathrm{ABER_H} = \qquad (21)\\ &\frac{d}{2\sqrt{\pi}} \int_0^\infty \frac{1}{2\sqrt{x}} \exp\left(\xi^2 \tilde{\Psi} - 2\sigma_x^2 \xi^4 - x\right) \mathrm{erfc}\left(\frac{\tilde{\Psi}}{\sqrt{8}\sigma_x}\right) dx \\ &+ \frac{d}{2\sqrt{\pi}} \int_0^\infty \frac{1}{2\sqrt{x}}\ \mathrm{erfc}\left(\frac{4\sigma_x^2\xi^2 - \tilde{\Psi}}{\sqrt{8}\sigma_x}\right) \exp\left(-x\right) dx, \end{aligned}$$

where $\tilde{\Psi} = \ln\left(\frac{\sqrt{2x}}{A_o \beta\rho\sqrt{\overline{\gamma} c}}\right) + 2\sigma_x^2(1+2\xi^2)$.

The integration in (21) can be computed using the generalized Gauss-Laguerre quadrature function as [26]

$$\int_0^\infty x^f \exp(-x) g(x) dx \approx \sum_{i=1}^{S} w_i g(x_i), \tag{22}$$

where S is the order of the approximation and x_i and w_i are the roots and the weights of the generalized Laguerre polynomial, respectively. The first hundred values of x_i and w_i are well tabulated in [27]. Thus (22) can be expressed by truncated series as

$$\begin{aligned} &\mathrm{ABER_H} \approx \qquad (23)\\ &\frac{d}{2\sqrt{\pi}} \sum_{i=1}^{S} \frac{w_i}{2} \exp\left(\xi^2 \bar{\Psi} - 2\sigma_x^2 \xi^4\right) \mathrm{erfc}\left(\frac{\bar{\Psi}}{\sqrt{8}\sigma_x}\right) \\ &+ \frac{d}{2\sqrt{\pi}} \sum_{i=1}^{S} \frac{w_i}{2}\ \mathrm{erfc}\left(\frac{4\sigma_x^2\xi^2 - \bar{\Psi}}{\sqrt{8}\sigma_x}\right) \approx \frac{d}{2\sqrt{\pi}} \sum_{i=1}^{S} w_i I_{\mathrm{H}}, \end{aligned}$$

where $\bar{\Psi} = \ln\left(\frac{\sqrt{2x_i}}{A_o \beta\rho\sqrt{\overline{\gamma} c}}\right) + 2\sigma_x^2(1+2\xi^2)$, $\mathrm{H} \in \{1, 2\}$ and $I_{\mathrm{H}} = \frac{1}{2}\left[\exp\left(\xi^2\bar{\Psi} - 2\sigma_x^2\xi^4\right)\mathrm{erfc}\left(\frac{\bar{\Psi}}{\sqrt{8}\sigma_x}\right) + \mathrm{erfc}\left(\frac{4\sigma_x^2\xi^2 - \bar{\Psi}}{\sqrt{8}\sigma_x}\right)\right]$.
For a dual-hop system, an approximated ABER can be calculated by [28]

$$\mathrm{ABER} \approx \frac{1}{2}\left[1 - \left(1 - 2\ \mathrm{ABER_1}\right)\left(1 - 2\ \mathrm{ABER_2}\right)\right]. \tag{24}$$

where $\mathrm{ABER_1}$ and $\mathrm{ABER_2}$ are the ABER for the first node and the second node, respectively. Following the same steps of (23), an approximated ABER expression of the best relay selection for LN channels can be calculated by substituting the CDF, $F_\gamma(\gamma_{th})$, of (15) into (17) and using (22) as

$$\mathrm{ABER} \approx \frac{d}{2\sqrt{\pi}} \sum_{i=1}^{S} w_i \left[1 - (1 - I_1)(1 - I_2)\right]^N. \tag{25}$$

V. Numerical Results and Discussions

In the presented results, a target ABER of 10^{-9}, OP for 10^{-15} are assumed and the first-hop link distance and the second-hop link distance are 600 m and 400 m, respectively. Derived analytical results are corroborated via Monte Carlo simulations. In the obtained simulation results, 10^7 bits are transmitted for each depicted SNR value and the Gauss-Laguerre quadrature approximation order is $S \leq 50$. Table I shows the system parameters under investigation which are used in various FSO communication systems [4], [16], [29]. Using Table I, atmospheric turbulence conditions are calculated by (7), (9), (10) and (11), and are presented in Table II as follows σ_{x_1}, a_1, b_1 and $\sigma_{R_1}^2$ turbulence parameters for the first-hop and σ_{x_2}, a_2, b_2 and $\sigma_{R_2}^2$ turbulence parameters for the second-hop.

The OP for HD relays, FD relays and direct link for FSO links over weak, moderate and strong turbulence channels are depicted in Figs. 2, 3 and 4 respectively. Fig. 2 shows that FD relays, with a single relay or multiple relays, outperforms their counterparts HD relays and direct link systems. Furthermore, it can be noticed that the performance of FD relays with the best relay selection is enhanced by increasing the number of relays. Performance gains of about 3 dB can be noticed as compared to HD relays.

Similar behaviors can be noticed as well in Figs. 3 and 4, where FD relays are shown to outperform HD relays and direct link systems. It is worth mentioning that the performance of FD relays degrades by more than 2.5 dB for moderate turbulence and by more than 8.5 dB for strong turbulence. HD relays performance is shown to degrade as well. FD relays outperform HD relays by about 3 dB for moderate and strong turbulence. Moreover, Figs. 2-4 show that misalignment error effect is more sensitive to weak turbulence than moderate and strong turbulence. For the negligible misalignment error case when ($\xi \to \infty$), (13) is consistent with the formula obtained in [3, Eq. 21] and (16) is consistent with the formula obtained in [4, Eq. 54].

The ABER of FD relays, HD relays and direct link for FSO links over weak and moderate turbulent channels with respect

TABLE I
SYSTEM CONFIGURATION [4], [16], [29]

Parameter	Symbol	Value
Wavelength	λ	1550 nm
Receiver diameter	D_R	0.2 m
Transmitter diameter	D_T	0.2 m
Divergence angle	θ_T	2 mrad
Distance between the source and the destination	L	1 km
Attenuation coefficient	α	0.43 dB/km
Jitter standard deviation	σ_s	0.3 m
Beam waist	w_z	2 m
Pointing error parameter	ξ	3.3377
Refractive index constant (weak-to-strong turbulence)	C_n^2	$0.5\times\ 10^{-14}\ \mathrm{m}^{-2/3}$, $2\times\ 10^{-14}\ \mathrm{m}^{-2/3}$, $5\times\ 10^{-14}\ \mathrm{m}^{-2/3}$

TABLE II
ATMOSPHERIC CONDITIONS

Atmospheric turbulence	Link type	σ_{x_1}	σ_{x_2}	a_1	a_2	b_1	b_2	$\sigma^2_{R_1}$	$\sigma^2_{R_2}$
Weak turbulence	Direct	0.16							
	Dual-hop	0.1	0.07						
Moderate turbulence	Direct	0.31							
	Dual-hop	0.2	0.14						
Strong turbulence	Direct			4.4		3.15		1	
	Dual-hop			6.99	12.41	5.87	11.21	0.39	0.19

to the average SNR are shown in Figs. 5 and 6, respectively. To guarantee consistent spectral efficiency for all compared systems, 2-PAM is considered for direct and FD relays while 4-PAM is used for HD relays. The significant enhancement of FD relays as compared to HD relays and direct link are clearly shown in Figs. 5-6. Performance gains of more than 13 dB can be clearly noticed in Figs. 5-6. Additionally, for negligible misalignment error case when ($\xi \to \infty$), (23) is consistent with the formula obtained in [30, Eq. 12]. The ABER expression of the best of relay selection shown in (25) is obtained as approximated and series based analytical expressions. Thus, the truncation errors of (25) leads to a narrow gap between (25) and Monte Carlo simulations at high ABER.

VI. CONCLUSIONS

In conclusion, we investigated the selection of a single relay based on the max-min SNR criterion for FSO communication systems. Dual-hop DF relaying system over different atmospheric turbulence channels affected with path losses and misalignment errors are employed. OP are obtained for LN and G-G channels under the considered challenges. Moreover, approximated ABER expressions for FD relays, HD relays and direct link are derived using Gauss-Laguerre quadrature rule assuming full CSI. Our simulation results show the superiority of FD relays systems as compared to their counterparts in terms of ABER and OP especially for strong turbulence.

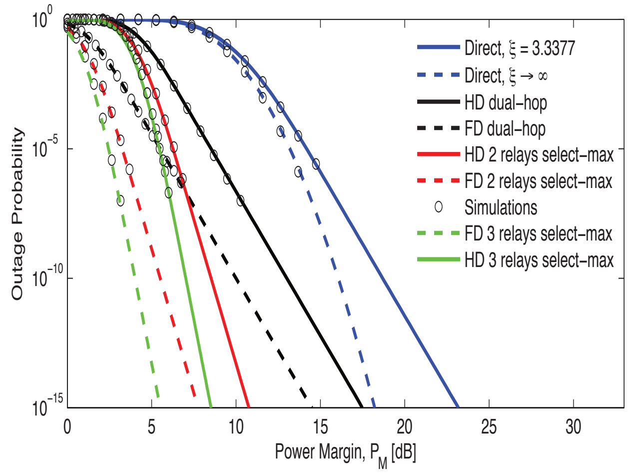

Fig. 2. OP for FSO over weak fading channel.

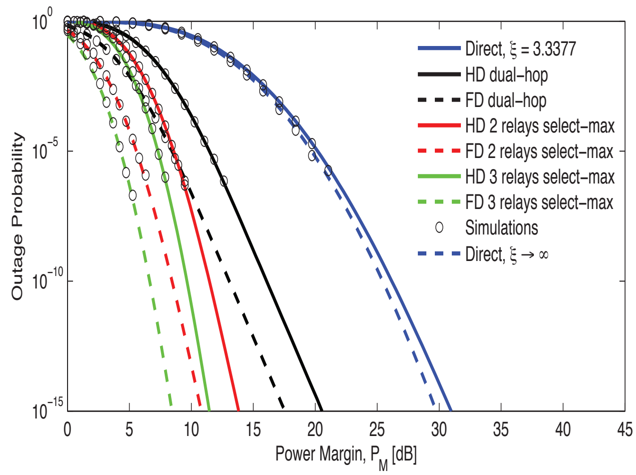

Fig. 3. OP for FSO over moderate fading channel.

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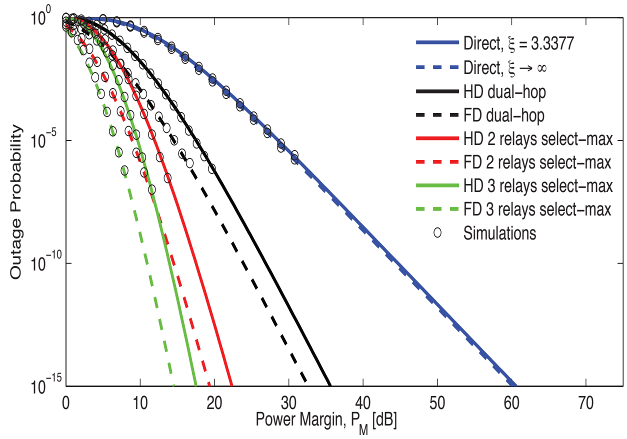

Fig. 4. OP for FSO over strong fading channel.

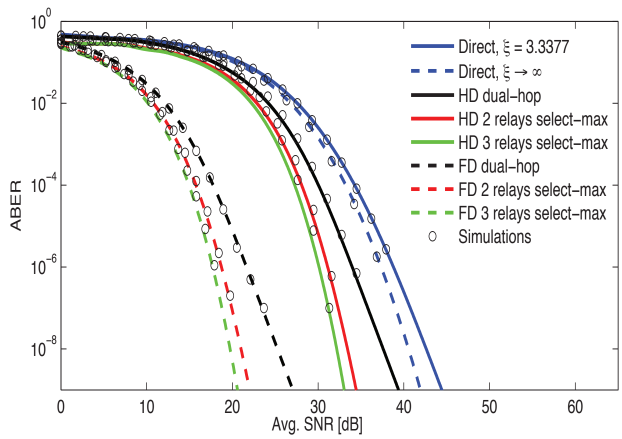

Fig. 5. ABER of FSO over weak fading channel.

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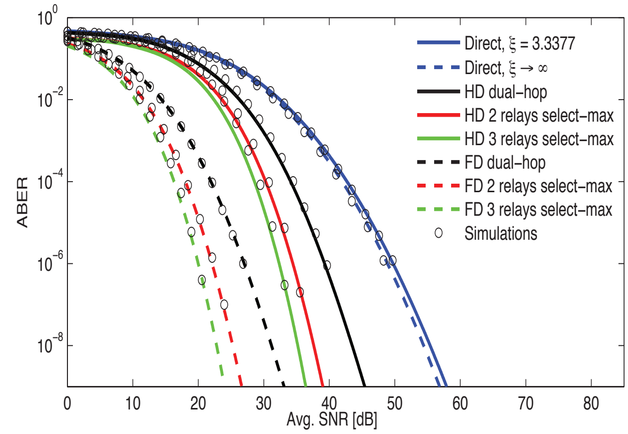

Fig. 6. ABER of FSO over moderate fading channel.

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