Blind Detection of Cyclostationary Features in the Context of Cognitive Radio

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***Abstract*—The methods of dynamic access to spectrum developed in Cognitive Radio require efficient and robust spectrum detectors. Most of these detectors suffer from four main limits: the computational cost required for the detection procedure; the need of prior knowledge of Primary User's (PU) signal features; the poor performances obtained in low SNR (Signal to Noise Ratio) environment; finding an optimal detection threshold is a crucial issue. In this paper, we propose a blind detection method based on the cyclostationary features of communication signals to overcome the four limits of spectrum sensors. In order to reduce the computational cost, the FFT Accumulation Method has been adjusted to estimate the cyclic spectrum of the intercepted signal. Then, the spectrum coherence principle is used to catch the periodicity hidden in the cyclic autocorrelation function of this signal. The hidden periodicity is revealed by the crest factor of the cyclic domain profile. The detection of PU's signal is achieved by comparing the embedded periodicity level with a predetermined threshold related to the crest factor. This threshold varies randomly dependent on the SNR. Then, we have modelized the distribution law of the threshold in order to select the optimal value. Using the crest factor of the cyclic domain profile as a detection criterion has permitted to develop a spectrum sensor which is able to work in a blind context. Simulation results corroborate the efficiency and robustness of the proposed detector compared with the classical Energy Detector.**

***Keywords*—Spectrum Sensing, Cyclostaionarity, FFT Acumulation Method, Cyclic Spectrum, Spectrum Coherence, Cognitive Radio.**

I. Introduction

The increasing demand for very high data-rate wireless communication systems and the scarcity of spectrum bandwidths require the development of new technologies to manage wisely the spectrum sharing. Cognitive radio introduced by Mitola [1] addresses the problem of the spectrum lack. It allows two categories of users to share the same bandwidth. The first one, is the Primary User "PU" who holds the license of a bandwidth; and the second one is denoted by Secondary Users "SU" who are all other opportunist transmitters. The main difficulty for cognitive radio is the detection of unoccupied spectral bands. Many methods of spectrum sensing such as Energy Detection (ED), Waveform Detection (WFD), Cyclostationary Features Detection (CFD) have been recently developed [2]. The ED method determines the unoccupied spectrum by comparing the estimated power of the received signal with a predetermined threshold. This method does not perform well under low signal-to-noise ratio (SNR) conditions [3]. The most reliable detection method, WFD performs a correlation between the waveforms of received and reference signals. WFD is very efficient [4] but it requires an accurate knowledge on PU's signal. In real scenario, SU has no prior information on PU's signal. The CFD initially proposed by Gardner [5,6] uses the fact that communication signals can be modelized as cyclostationary signals [7]. The cyclostationary processes are random processes which have periodical statistics [6]. Cyclostationarity can be caused by the modulation or the coding stages, but it may intentionally be introduced in order to aid in the channel estimation or the synchronization process [3]. The CFD can be used in a blind context [8,9]. As no prior information about the PU is needed, the main strategy is based on developing methods to extract cyclostationary features [10,11]. In this paper, we propose a blind detection method based on the cyclostationary features of communication signals to detect the presence of PU's signal in a bandwidth. Our approach can be divided into two major parts. At first, we have adjusted the FFT Accumulation Method (FAM) to estimate the cyclic spectrum of intercepted signal. Then, we have performed spectrum coherence principle to catch the periodicity hidden in the autocorrelation function of this signal. We have achieved the detection of PU's signal by comparing the periodicity level embedded to a predetermined threshold. The value of this threshold varies randomly dependent on the SNR. Therefore, we modelized the threshold variation as a random variable and the optimal value of the threshold is chosen according to a fixed value of Probabilty of False Alarm. Simulation results through Receiver Operating Characteristic (ROC) curves corroborate the efficiency and robustness of the proposed algorithm compared with the classical blind detection method. The rest of this paper is organized in five sections. In section II, the concept of cyclostationarity is briefly reviewed. Section III presents FAM algorithm used to estimate cyclic spectrum. The principle of the detector is presented in section IV. The section V deals with the evaluation of the proposed detector performance. The last section concerns the conclusion and perspectives for future work.

II. Cyclostationary spectrum analysis

The hidden cyclic frequencies in cyclostationary signal can be revealed by the Cyclic Autocorrelation Function (CAF). The equivalent of CAF in frequency domain is the cyclic spectrum (CS) [12]. The CAF and the CS are generally used for detecting the presence of a cyclostationary signal [13].

A. Mathematical Background

A zero-mean $x(t)$ is called a second order cyclostationary signal if its time varying autocorrelation function $R_x(t, \tau)$,

$$R_x(t,\tau) = E\left\{x\left(t+\frac{\tau}{2}\right)x^*\left(t-\frac{\tau}{2}\right)\right\} \quad (1)$$

is periodic in time t for any parameter τ; Therefore, it can be decomposed in Fourier series:

$$R_x(t,\tau) = \sum_{k=1}^{M} R_x^{k\alpha_0}(\tau)e^{j2\pi k\alpha_0 t} \quad (2)$$

where the fundamental cyclic frequency is denoted by $\alpha_0 = 1/T_0$ and M represents the rank of the last harmonic. T_0 is the hidden period [14]. The Fourier coefficients $R_x^{k.\alpha_0}(\tau)$ are called the cyclic autocorrelation function (CAF) [15]:

$$R_x^{k\alpha_0}(\tau) = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} R_x(t,\tau)e^{-j2\pi k\alpha_0 t}dt \quad (3)$$

The above coefficients can be as well estimated by the following equation [14]:

$$R_x^{k\alpha_0}(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} R_x(t,\tau)e^{-j2\pi k\alpha_0 t}dt \quad (4)$$

where T is the time duration used to evaluate the CAF. The relation (3) is used with a prior knowledge on the hidden periodicity [14]. Let $\alpha = k.\alpha_0$ be the cyclic frequency, the Fourier Transform of CAF, i.e. the CS [15], becomes:

$$S_x^{\alpha}(\nu) = \int_{-\infty}^{+\infty} R_x^{\alpha}(\tau)e^{-j2\pi\nu\tau}d\tau \quad (5)$$

.

According to [16] and [17], $R_x^{\alpha}(\tau)$ can be approximated as follows:

$$R_x^{\alpha}(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{-\frac{T}{2}} x\left(t+\frac{\tau}{2}\right)x^*\left(t-\frac{\tau}{2}\right)e^{-j2\pi\alpha t}dt \quad (6)$$

Using $e^{-j\pi\alpha(\frac{\tau}{2}-\frac{\tau}{2})} = 1$, equation (6) can be rewritten as follows:

$$R_x^{\alpha}(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{-\frac{T}{2}} x\left(t+\frac{\tau}{2}\right)e^{-j2\pi\frac{\alpha}{2}(t+\frac{\tau}{2})}\times x^*\left(t-\frac{\tau}{2}\right)e^{-j\pi\frac{\alpha}{2}(t-\frac{\tau}{2})}dt \quad (7)$$

Let us denote $y(\tau) = x(t+\frac{\tau}{2})e^{-j2\pi\frac{\alpha}{2}(t+\frac{\tau}{2})}$; by applying Fourier Transform (FT) on (7), we obtain CS, which can be viewed as a spectral correlation function [18]:

$$\begin{aligned} S_x^{\alpha}(\nu) &= TF\left\{R_x^{\alpha}(\tau)\right\} && (8)\\ &= \lim_{T\to\infty}\frac{1}{T}TF\left\{y(\tau)*y^*(-\tau)\right\} && (9)\\ &= \lim_{T\to\infty}\left\{\frac{1}{T}X_T\left(\nu+\frac{\alpha}{2}\right)X_T^*\left(\nu-\frac{\alpha}{2}\right)\right\} && (10)\end{aligned}$$

where $X_T(\nu)$ is the Fourier transform of the product between our signal $x(t)$ and rectangular window of width T:

$$X_T(\nu) = \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)e^{-j2\pi\nu t}dt \quad (11)$$

By definition, $\frac{1}{T}X_T\left(\nu+\frac{\alpha}{2}\right)X_T^*\left(\nu-\frac{\alpha}{2}\right)$ is the cyclic periodogram [19]–[21].

B. Cyclic Spectrum estimation

The CS can be estimated by time or frequency smoothing algorithms [22,23]. According to [18,19,22], the time smoothing algorithms are more efficient and reliable than frequency smoothing. Let Δt be the observation time, an estimation of the CS can be obtained by the time-smoothed cyclic periodogram given by:

$$S_x^{\alpha}(\nu) \approx S_{x_{T_w}}^{\alpha}(t,\nu)_{\Delta t} = \frac{1}{\Delta t}\int_{t-\frac{\Delta t}{2}}^{t+\frac{\Delta t}{2}} S_{x_{T_w}}(u,\nu)du \quad (12)$$

where

$$S_{x_{T_w}}(u,\nu) = \frac{1}{T_w}X_{T_w}\left(u,\nu+\frac{\alpha}{2}\right).X_{T_w}^*\left(u,\nu-\frac{\alpha}{2}\right) \quad (13)$$

with T_w is the window width of short-time FFT, and

$$X_{T_w}(t,\nu) = \int_{t-\frac{T_w}{2}}^{t+\frac{T_w}{2}} x(u)e^{-j2\pi\nu u}du \quad (14)$$

is the short-time Fourier transform (STFT). The spectral components generated by STFT have a resolution $\Delta f = \frac{1}{T_w}$. For a reliable estimation of CS, the Grenander's uncertainty condition should be respected [18]:

$$\Delta t.\Delta f \gg 1 \quad (15)$$

According to [19], the most used time smoothing algorithms are the FFT Accumulation Method (**FAM**) and the Strip Spectral Correlation Algorithm (**SSCA**). As FFT Accumulation Method is more computationally efficient than SSCA [18,22], we have used it in our work.

III. FAM Algorithm

Let $x[n]$ be the discrete time version of a signal $x(t)$, the estimation of CS becomes:

$$S_x^{\alpha}(n,\nu) = \frac{1}{N}\sum_{n=0}^{N-1}\frac{1}{N_p}X_{T_w}\left(n,\nu+\frac{\alpha}{2}\right)X_{T_w}^*\left(n,\nu-\frac{\alpha}{2}\right) \quad (16)$$

where N is the total number of discrete samples within the observation time Δt, and N_P the number of points within the discrete short-time FFT. X_{T_w} denotes the Discrete Fourier Transform of $x[n]$:

$$X_{T_w}(n,\nu) = \sum_{k=-\frac{N_P}{2}}^{\frac{N_P}{2}-1} w(k)x(n-k)e^{-j2\pi\nu(n-k)T_s} \quad (17)$$

with $w(k)$, the data tapering window of width $T_w = N_P T_s$ seconds. T_s is the sampling period of $x(t)$. $X_{T_w}(n,\nu+\frac{\alpha}{2})$ is called the complex demodulate of $x(n)$. FAM is derived from equation (16) and it divides the entire bifrequency plane (ν,α) into small areas to calculate the CS of each area. As showed in [3] and [20], the FAM algorithm works as follows:

1) The input sample sequence $x[n]$ of length N is divided into P blocks, where each block contains N_P samples. L data samples are skipped between two successive blocks of N_P samples. The value of L is fixed to be equal to $N_P/4$, which is a good trade-off among the computational efficiency, minimizing the cycle leakage and the cycle aliasing. The value of N_P and P are determined respectively according to the desired resolution in frequency $\Delta\nu$ and resolution in cyclic frequency $\Delta\alpha = \frac{1}{\Delta t}$ and sampling frequency F_s by:

$$N_P = 2^{\left\lfloor \log_2\left(\frac{F_s}{\Delta f}-1\right)+1 \right\rfloor} \quad (18)$$

$$P = 2^{\left\lfloor \log_2\left(\frac{F_s}{L.\Delta\alpha}-1\right)+1 \right\rfloor} \quad (19)$$

where $\lfloor a \rfloor$ denotes the integer part of a.

2) A Hamming window $w(n)$ is applied across each block. We have chosen a Hamming window $w(n)$ because of its low skirts and low sidelobes that allows to reduce the cycle leakage [19];
3) $N_P - points\ FFT$ of each block is computed to obtain the complex envelope $X_{T_w}(n,\nu)$. The result is multiplyed by $e^{j2\pi(\frac{\alpha}{2})nT_s}$, to downshift $X_{T_w}(n,\nu)$ in frequency: $X_{T_w}(n,\nu+\frac{\alpha}{2})$;
4) Complex conjugates of downshifted $X_{T_w}(n,\nu+\frac{\alpha}{2})$ are computed;
5) CS is estimated by multiplying $X_{T_w}(n,\nu+\frac{\alpha}{2})$ and its conjugate;
6) The smoothing operation of the product of sequences is executed by means of $P{-}points\ FFT$.

Fig. 1 illustrates the different stages of FAM algorithm. The reliable CS estimation is a great and difficult stage in cyclostationary signal detection.

IV. Detection model

Let $x(t)$ be the received signal, $s(t)$ stands for the communication signal and $n(t)$ is the Additive White Gaussian Noise (AWGN).

$$x(t) = \theta.s(t) + n(t) \quad (20)$$

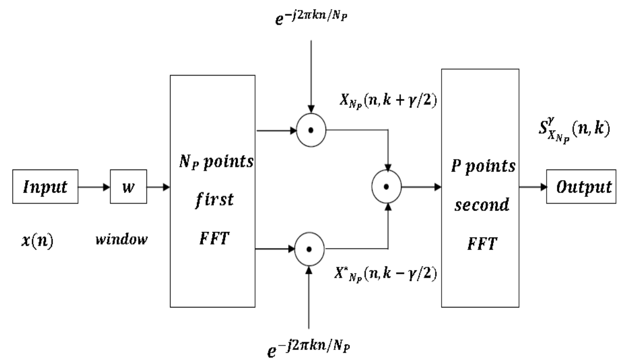

Fig. 1: Block diagram of FAM algorithm [18]

Let us consider the following hypotheses H_0 and H_1:

- H_0 : absence of communication signal; $\theta = 0$
- H_1 : presence of communication signal; $\theta = 1$

The main idea of signal detection consists in measuring the periodicity contained in the CAF of $x(t)$. Indeed, communication signals are second order cyclostationary processes [24]. However, AWGN is assumed to be a stationary signal. In this paper, we establish a detection threshold based on the spectral autocoherence function $C_x^{\alpha}(\nu)$:

$$C_x^{\alpha}(\nu) = \frac{S_x^{\alpha}(\nu)}{\sqrt{S_x^0(\nu+\frac{\alpha}{2}).S_x^0(\nu-\frac{\alpha}{2})}} \quad (21)$$

We should note that $|C_x^{\alpha}(\nu)| \leq 1$ [14,24,25]. From $C_x^{\alpha}(\nu)$, we define the Cyclic Domain Profile $I(\alpha)$ which contains only the peak values of $|C_x^{\alpha}(\nu)|$, as follows [13,26]:

$$I(\alpha) = \max_{\nu} |C_x^{\alpha}(\nu)| \quad (22)$$

Using the definition of $I(\alpha)$, the crest factor F_c, is defined as follows [27]:

$$F_c = \frac{\max_{\alpha}\{I(\alpha)\}}{\sqrt{\frac{1}{k}\sum_{\alpha=\alpha_1}^{\alpha_k}\left[I(\alpha)\right]^2}} \quad (23)$$

Where k is the length of vector $I(\alpha)$. F_c characterizes the level of periodicity contained in the CAF of a signal. Hereinafter, the crest factor F_c is used as the detection criterion. At first, F_c is evaluated under H_0 (there is just noise in the channel). This value is denoted C_{TH}. During spectrum sensing period, the detector evaluates the factor F_c and applies the following criterion:

$$F_c \underset{H_0}{\overset{H_1}{\gtrless}} C_{TH} \quad (24)$$

By performing several Monte Carlo simulations, we noticed that C_{TH} can be modelized by a random variable depending on SNR. The main difficulty at this stage is to determine an optimal value of C_{TH} in order to minimize detection errors according to Neymann-Pearson detection theory [28]. The Probability Density Function (PDF) of C_{TH} can be approximated using the following steps:

1) Calculate different values of C_{TH} when the PU is absent.
2) Estimate the PDF of C_{TH} variation by the empirical histogram.
3) Select analytical PDF for this C_{TH} variation and validate the selection by using Kolmogorov-Smirnov's test (see Fig. 3).

Fig. 2 shows the evolution of crest factor F_c with SNR variation from $-30\ dB$ to $30\ dB$; from a context where there is no signal to a context where there is no noise.

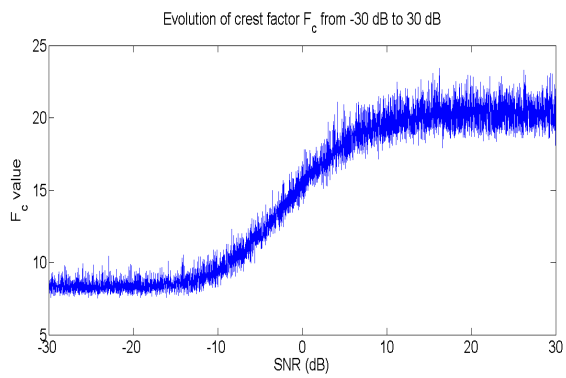

Fig. 2: Evolution of crest factor F_c from SNR= $-30\ dB$ to SNR= $30\ dB$.

By determining the threshold C_{TH} at the absence of communication signal, the corresponding histogram illustrated in Fig. 3 can be approximated reliably by a Generalized Extreme Value (GEV) distribution defined as follows [29]:

$$f(x;\mu,\sigma,\xi) = \frac{1}{\sigma}\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}-1} \times \exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\} \quad (25)$$

where ξ is the shape parameter, σ the scale parameter and μ the location parameter. To estimate these parameters, we have used the minimizing quadratic error principle. The following values have been obtained:

$\xi = -0.027$, $\sigma =\ 0.295$ and $\mu = 8.20$.

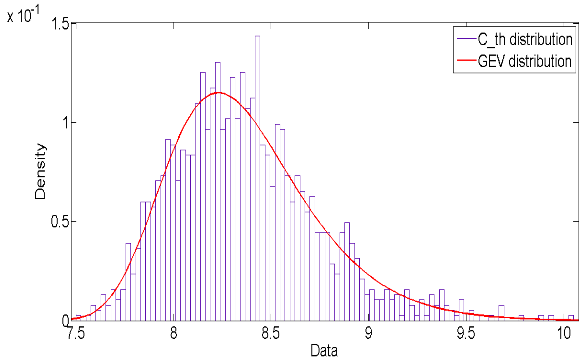

Fig. 3: Approximation of the PDF of C_{TH} by GEV distribution (equation (25))

Let us define the conditional PDF of the crest factor F_c:

- $f_{F_c|H_0}(f_c)$ is the conditional PDF of F_c under H_0.
- $f_{F_c|H_1}(f_c)$ is the conditional PDF of F_c under H_1.

Simulations show that the conditional PDF $f_{F_c|H_0}(f_c)$ and $f_{F_c|H_1}(f_c)$ are overlapped as shown in Fig. 4.

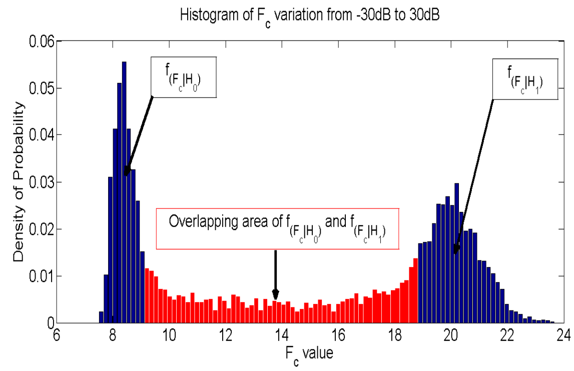

Fig. 4: Histogram of the crest factor for a SNR from -30 dB to 30 dB. Detection errors are from the overlapping region between $f_{F_c|H_0}$ and $f_{F_c|H_1}$

From Neymann-Pearson detection theory [28], the detection probability P_d can be defined by:

$$P_d = P(F_c > C_{TH}|H_1) = \int_{C_{TH}}^{+\infty} f_{F_c|H_1}(u)du \quad (26)$$

and the Probability of false alarm P_{fa} becomes:

$$P_{fa} = P(F_c > C_{TH}|H_0) = \int_{C_{TH}}^{+\infty} f_{F_c|H_0}(u)du \quad (27)$$

To evaluate the performance of the proposed detector, we have generate the Receiver Operational Characteristic (ROC) curves using several Monte-Carlo simulations [28].

V. Simulations results

To analyse the performance of the proposed algorithm, we applied the equation (24) to different signals: 4-QAM, 16-QAM, BPSK, QPSK and 4-ASK. In this paper, we give the results about 16-QAM and BPSK signals. Let us consider the intercepted signal characteristics as follows: data frequency $F_d = 1024\ Hz$, carrier frequency $F_c = 2048\ Hz$, sampling frequency $F_s = 8192\ Hz$. The parameters of the detector are the observation duration of intercepted signal $\Delta t = 125\ ms$ and the duration of sliding window $T_w = 19.53\ ms$. Consequently, the detector has a frequency resolution $\Delta\nu = \frac{1}{T_w} = 512\ Hz$ and a cyclic frequency resolution $\Delta\alpha = \frac{1}{\Delta t} = 8\ Hz$. The Grenander's uncertainty condition $\Delta t.\Delta\nu \gg 1$ is respected for a reliable CS estimation. The choice of $\Delta\alpha = 8\ Hz$ implies that the number of samples contained in the intercepted signal is $N = \frac{F_s}{\Delta\alpha} = 1024$ samples. Equations (26) and (27) show that when the value of C_{TH} increases, P_{fa} and P_d decrease. By using the inverse of complementary cumulative distribution function, we determine the corresponding C_{TH} value for a fixed value of P_{fa}. The Table I gives some C_{TH} values based on the P_{fa}.

P_{fa}	0.01	0.02	0.05	0.1	0.2	0.3
C_{TH}	9.48	9.29	9.04	8.88	8.65	8.50

TABLE I: Correspondance values of C_{TH} and P_{fa}

The P_d curves on Fig.5 based on SNR values show that the proposed method is able to detect the presence of communication signal buried in noise with good performances. For example, concerning 16-QAM signal, our algorithm is able to detect the signal with $P_d \geq 0.9$ in a Gaussian channel where $SNR = -10$ dB. The fixed value of P_{fa} in this case is $P_{fa} = 0.1$.
When P_{fa} is fixed to 0.01, the proposed algorithm is able to detect the PU's signal with $P_d \geq 0.93$ in a Gaussian channel with $SNR = -8$ dB. In addition, from the Fig.5, we notice that for $SNR \geq -7$ dB, the PU's signal can be detected easily with $P_{fa} \leq 0.1$

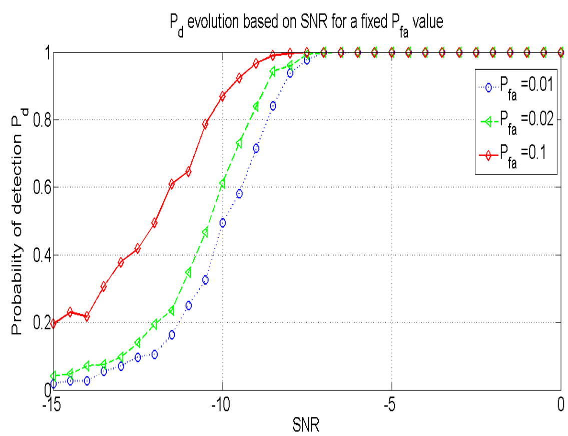

Fig. 5: Evolution of probability of detection following the SNR values for a fixed P_{fa}.The signal used is a 16-QAM signal.

Afterwards, we have generated the ROC curves for different SNR values to evaluate the robustness of our detector (see Fig. 6).These ROC curves corroborate the previous results. They allow us to understand the relation between the P_d and the P_{fa} evolution for different values of the SNR.

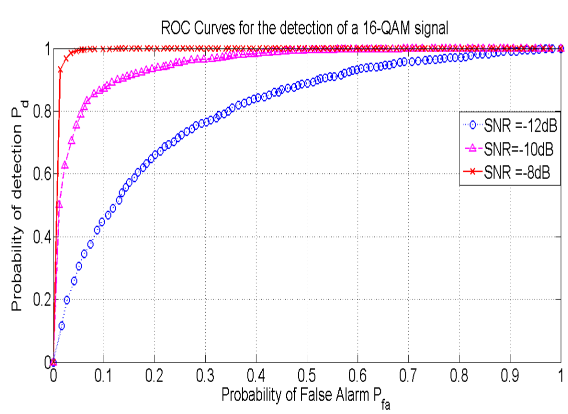

Fig. 6: Evaluation of detector robustness on 16-QAM signal by ROC Curves of P_d versus P_{fa} for fixed SNR.

The second kind of signal used in our simulations is a BPSK signal. Here, the detector is able to detect this signal with $P_d = 0.96$ and $P_{fa} = 0.02$ for $SNR = -8$ dB. With $SNR = -10$ dB, we obtain $P_d = 0.90$ versus $P_{fa} = 0.1$. When SNR is less than -10 dB, it is difficult for the CSD to have good performances in PU's signal detection (see Fig. 7).

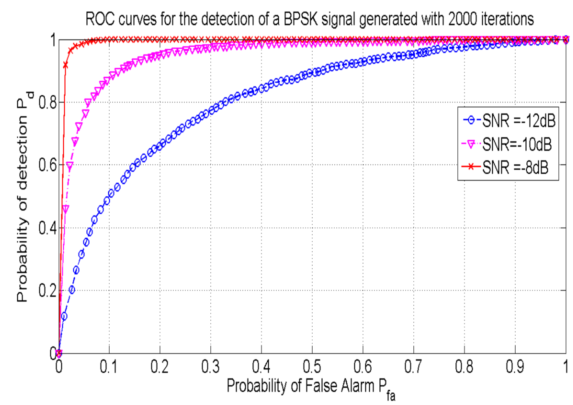

Fig. 7: Evaluation of detector robustness on BPSK signal by ROC Curves of P_d versus P_{fa} for fixed SNR.

One can notice that our proposed CSD acts in blind context. We have confronted his robustness to the classical energy detection which acts in blind context too. We used the same simulation parameters. The number of samples contained in the intercepted signal is $N = 1024$. The signal used is a 16-QAM signal. For different SNR values (from -16dB to 3 dB), we evaluate the probability of detection of CSD and ED for $P_{fa} = 0.1$. The simulation results are presented on Fig. 8. We notice that for $P_{fa} = 0.1$, the CSD is able to detection the presence of PU's signal with $P_d = 0.9$ in a channel where $SNR = -10$ dB. Whereas, the ED can detect the same signal with a $P_d = 0.9$ only when $SNR = -3$ dB. The ROC curves of P_d versus SNR on Fig.8 prove that the CSD is able to detect the PU's signal in a very low SNR environment contrary to classical ED.

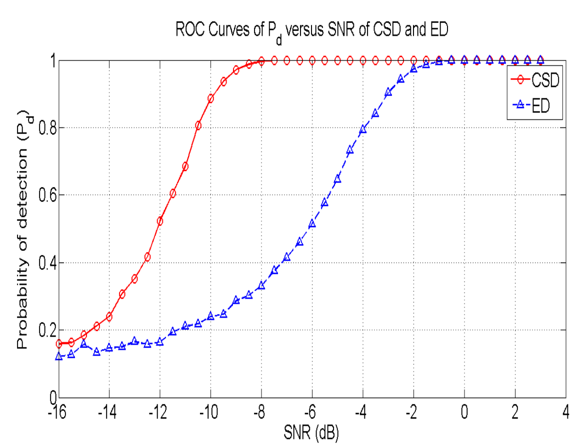

Fig. 8: ROC curves of P_d versus P_{fa} for $P_{fa} = 0.1$: Comparaison of CSD to classical ED.

VI. CONCLUSION

PU's signal detection in a low SNR is crucial to cognitive radio. In this paper, we propose an efficient blind detection method based on the cyclostationary features of communication signals to detect the presence of PU's signal in a bandwidth. This proposed detector overcomes four limits of most of spectrum sensors. After estimating cyclic spectrum by FFT Accumulation Method, the crest factor of cyclic domain profile is calculated and compared with a predetermined threshold to make a decision about the presence or not of PU's signal. The crest factor is the feature which shows the periodicity level embedded in the cyclic autocorrelation function of intercepted signal. The efficiency of our cyclostationary features detector for a very low SNR in Gaussian channel is emphasized through ROC curves over the classical energy detector. During our simulations, we have noticed that cyclic spectrum is a sparse matrix. In future works, we will apply the sparsity methods to extract only essential features of cyclic spectrum. It will allow us to further reduce the computational cost and volume of data used in cyclostationary features detection procedure. In addition, we will study the behaviour of our proposed cyclostationary features detector in various transmission channels such as Rayleigh channel.

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