Spectrum Sensing Enhancement Using Principal Component Analysis

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***Abstract*—In this paper, Principal Component Analysis (PCA) techniques are introduced in the context of Cognitive Radio to enhance the Spectrum Sensing performance. PCA step increases the SNR of the Primary User's signal and, consequently, enhances the Spectrum Sensing performance. We applied PCA as a combination scheme of a multi-antenna Cognitive Radio system. Analytic results will be presented to show the effectiveness of this technique by deriving the new SNR obtained after applying PCA, which can be considered a pre-processing step for a classical Spectrum Sensing algorithm. The effect of PCA is examined with well known detectors in Spectrum Sensing, where the proposed technique shows its efficiency. The performance of the proposed technique is corroborated through many simulations.**

***Keywords*—Principal Component Analysis, Multi-antennas system, Spectrum Sensing.**

I. INTRODUCTION

The Cognitive Radio (CR) was proposed to address the scarcity in the available frequency bandwidths [1]by sharing spectrum among users, Primary User (PU) and Secondary User (PU). PU has the spectrum license. When PU is idle, a SU can access the channel. However, if PU becomes again active, then SU should immediately vacate the channel to avoid any interference to PU.

The monitoring of the PU activities becomes a challenge for CR. To determine the PU status (active or idle), CR should perform a Spectrum Sensing algorithm to get this information.

In the literature, many spectrum sensing techniques can be identified [5]: Energy Detection (ED), Autocorrelation Detection (ACD), Cyclo-Stationary Detection (CSD) $etc.$
ED method is very simple method and it is still the most widely used [5], [3]. ED measures the energy of the received signal compares it to a predefined threshold depending on the noise variance.
ACD exploits the oversampling aspect of the PU signal received at the SU receiving antenna [4]. The autocorrelation of the PU signal for some non-zero lag leads to non-zero value whereas this autocorrelation vanishes for a white noise. Based on the Cyclic-Autocorrelation Function (CAF), CSD tests the cyclic statistics of the received signal at a given cyclic frequency [5]. Since telecommunication signals are cyclostationary, CAF detect the presence of a cyclostationary signal in a noisy channel.

To enhance the performance of Spectrum Sensing, systems of multi-antennas with hard/soft combining schemes have been proposed [6], [5]. In Hard Combining Scheme (HCS), a decision about the PU presence is made on each antenna. Later on, a fusion center combines all issued decisions using logic rules such as Or, And or a Majority rule [6], [5]. In Soft Combining Scheme (SCS), the fusion center combines linearly the test statistics calculated at the receiving antennas to obtain a global test statistic which is compared to a predefined threshold to make the decision on the PU status.

The Principal Component Analysis (PCA) has been recently used in Spectrum Sensing. PCA techniques were used to enhance the autocorrelation detector [8], [9]. In such situation, Robust PCA [7] technique is used to split the covariance matrix into a diagonal matrix (corresponds to the white noise), and a low-rank matrix (corresponds to the PU oversampled signal).

Our work emphasizes the use of PCA to enhance the SNR of the PU signal. In this manuscript, we consider a SU equipped with m receiving antennas. Using the covariance matrix of the m observation, PCA can be applied for generating m Principal Components (PCs). When PU exists, only one PC contains the noisy PU signal with enhanced SNR, while the other PCs are linear combinations of the noise. For that, The SU should be able to select the appropriate PC to perform the Spectrum Sensing. In this paper, we derive the output signals of the PCA system, and we derive the new SNR obtained after applying PCA. Furthermore, we set a criterion based on which the SU should find the appropriate PCA output that is capable to examine the channel status. Note that our proposed technique is not a Spectrum Sensing algorithm, but it is an efficient pre-processing step.

II. SYSTEM MODEL

The problem formulation on the presence/absence of the PU can be presented in a classic Bayesian detection problem as follows:

$$H_\eta : \mathbf{x}_i = \eta h_i \mathbf{s} + \mathbf{w}_i \tag{1}$$

Where $\eta \in \{0;1\}$. H_0 stands for the case where PU is absent, whereas under H_1 PU is transmitting. $\mathbf{x}_i$ is $1 \times N$ vector representing the observation at the ith SU receiving antenna, N stands for the total number of received samples, $\mathbf{s}$ is $1 \times N$ vector containing the PU user signal. The $1 \times N$ vector $\mathbf{w}_i$ represents zero mean Additive White Gaussian Noise (AWGN) with a variance $\sigma^2_{w_i}$ and a covariance matrix $E[\mathbf{w}_i \mathbf{w}_j] = \sigma^2_w \delta_{ij}$, where δ_{ij} the the Kronecker function, and h_i is the channel gain between the PU base station and the ith SU receiving antenna.

III. PCA USING m ANTENNAS

In this section, we present the PCA technique on a system of m antennas, $m > 1$. PCs are found using the covariance matrix of the observed signal at m antennas [10], [11].
Let X be the matrix collecting the observations on m antennas:

$$X = \left[\mathbf{x}_1^T \ \mathbf{x}_2^T \ ... \ \mathbf{x}_m^T\right]^T \tag{2}$$

In this case X becomes $m \times N$ matrix and the covariance matrix C becomes $m \times m$ matrix as follows:

$$C = \begin{bmatrix} \eta|h_1|^2\sigma_s^2 + \sigma_w^2 & \eta h_1 h_2^* \sigma_s^2 & \cdots & \eta h_1 h_m^* \sigma_s^2 \\ \eta h_2 h_1^* \sigma_s^2 & \eta|h_2|^2\sigma_s^2 + \sigma_w^2 & \cdots & \eta h_2 h_m^* \sigma_s^2 \\ \vdots & \vdots & \ddots & \vdots \\ \eta h_m h_1^* \sigma_s^2 & \eta h_n h_2^* \sigma_s^2 & \cdots & \eta|h_m|^2\sigma_s^2 + \sigma_w^2 \end{bmatrix} \tag{3}$$

η^2 is replaced in (3) by η for simplicity, since $\eta^2 = \eta$. The covariance matrix C can be estimated as follows:

$$\hat{C} = \frac{1}{N}\sum_{n=1}^{N} X X^H \tag{4}$$

By using the independence assumption between the PU signal and the noise, the matrix C can be written as the sum of two covariance matrices, C_s and C_w.

$$C = C_s + C_w \tag{5}$$

Where C_s is the covariance matrix of the PU signal received on m antennas. C_s becomes null under H_0. Under H_1, C_s is a matrix of rank one. C_w is the covariance of the noise components, which is diagonal: $C_w = \sigma^2_w I_n$. Since C_w is diagonal, the eigenvalues of C are the sum of those of C_s and C_w:

$$\lambda(C) = \lambda(C_s) + \lambda(C_w) \tag{6}$$

Being diagonal, the eigenvalues of C_w, $\lambda_i^w, \ 1 \leq i \leq m$ are equal to σ^2_w, while the eigenvalues of C_s, λ_i^s, $1 \leq i \leq m$, should be zeros except one is equal to the trace of C_s, $tr(C_s)$.

$$tr(C_s) = \sum_{i=1}^{m} |h_i|^2 \sigma_s^2 \tag{7}$$

This is because C_s is of rank one. Consequently, the eigenvalues of C, $\lambda_i, \ 1 \leq i \leq m$, become:

$$\lambda_1 = \lambda_2 = ... = \lambda_{m-1} = \sigma^2_w \tag{8}$$

$$\lambda_m = \sum_{i=1}^{m} |h_i|^2 \sigma_s^2 + \sigma_w^2 \tag{9}$$

The eigenvectors can be found based on the eigenvalues by solving the following equations:

$$(C - \lambda_i I_2) v_i = 0 \tag{10}$$

where v_1 is the ith eigenvector corresponding to the ith eigenvalue λ_i and I_2 is the identity matrix. Once the eigenvectors are found, the PCs can be obtained as follows:

$$p_i = v_i^H X \tag{11}$$

A. Finding the Principal Components under H_1 and H_0

Under H_1, PCA yields m PCs, among them, only one component contains a filtered PU signal. This component, $\mathbf{p}_m$, corresponds to the highest eigenvalue $\lambda_m = \sum_{i=1}^{m} |h_i|^2 \sigma_s^2 + \sigma_w^2$ where $\sum_{i=1}^{m} |h_i|^2 \sigma_s^2$ and σ_w^2 stand for the power of the PU and the power of the noise component signal existing in $\mathbf{p}_m$ respectively [10], [11]. The other $m-1$ components are a mixture of the noises observed at the m antennas. The last discussion shows the impact of PCA on the SNR. The new SNR, γ_{pca}, which is obtained after applying the PCA technique is presented as follows:

$$\gamma_{pca} = \frac{\sum_{i=1}^{m} |h_i|^2 \sigma_s^2}{\sigma_w^2} \tag{12}$$

Asssuming that $|h_i|^2 = |h_j|^2, \ \forall \ 1 \leq i, j \leq m$, the new SNR becomes linearly proportional to the number of used antennas in PCA.

Under H_0 ($i.e.$ $\eta = 0$), (3) yields a diagonal matrix:

$$C_0 = C_w \tag{13}$$

Since $C_0 = \sigma_w^2 I_m$, the eignevalues of C_0 are given as follows:

$$\lambda_1 = \lambda_2 = ... = \lambda_m = \sigma^2_w \tag{14}$$

Since C_0 is a diagonal matrix, then the $m \times m$ identity matrix, I_m, can be the matrix collecting the eigenvectors, $v_i, \ 1 \leq i \leq m$.

$$[v_1 \ v_2 \ \ldots \ v_m] = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \end{bmatrix} \tag{15}$$

According to (11), the PCs under H_0 are nothing but the noise components. However, any rotation of the set of eigenvectors do not affect the PCs' statistical properties under H_0, since the m noise components at the m SU receiving antennas are white Gaussian and independent. Consequently, $\mathbf{p}_i \ \forall \ 1 \leq i \leq m$, becomes a linear combination of $\mathbf{w}_i, \ 1 \leq i \leq m$, and then $\mathbf{p}_i$ remains white Gaussian noise.

IV. SPECTRUM SENSING USING PCA

PCA generates up to m components (the same number of observations), the detector has to choose the validate one to perform the Spectrum Sensing. As discussed in the section above, under H_0, $p_0^i(n)$ are equivalent since $\mathbf{w}_i$ are AWGN having the same variance. Unlike H_0, H_1 leads to non-equivalent PCs. $\mathbf{p}_1^i, 1 \leq i \leq m-1$ are nothing but a combination of the noise components, whereas $\mathbf{p}_1^m$ is a combination of the PU signal and the noise. Therefore, by applying a test statistic on $\mathbf{p}_1^m$, the SU is able to diagnose the channel status. Consequently, the SU should be able to choose the good PCA output that leads to an efficient decision on the PU status.
Since $\mathbf{p}_1^i$ and $\mathbf{p}_1^m$ corresponds to two different eigenvalues $\forall\ i \neq m$, where $\lambda_i = \sigma_w^2, 1 \leq i \leq m-1$ and $\lambda_m = \sum_{i=1}^{m} |h_i|^2\sigma_s^2 + \sigma_w^2$, the SU can choose the validate output, $\mathbf{p}_{val}$, as the PC that correspond to the maximal eigenvalue.

$$\mathbf{p}_{val} = \mathbf{p}_k \quad \text{subject to } \lambda_k = \max\{\lambda_i\},\ i = 1,..,m. \tag{16}$$

Where $\{\mathbf{p}_k\}$ is the set of the output signal after applying PCA. Note that this test does not affect the performance of the Spectrum Sensing under H_0 since the m PCs are equivalent. Once the SU chooses the appropriate PC, then a test statistic, T, is calculated by applying a Spectrum Sensing method and compared to a threshold, ξ to make a decision on the PU status. Motivated by the discussion above, the new channel hypothesis can be presented as follows:

$$\begin{cases} H_0 : \mathbf{p}_{val} = \mathbf{w} \\ H_1 : \mathbf{p}_{val} = \mathbf{y} + \mathbf{r} \end{cases} \tag{17}$$

Where $\mathbf{w}$ corresponds to the noise component which should be obtained under H_0, $\mathbf{y}$ and $\mathbf{r}$ stands for the PU signal and the noise existing in $\mathbf{p}_{val}$ under H_1 respectively.
The following algorithm summarizes the steps followed to make a decision on the channel using the PCA.

Algorithm 1 Spectrum Sensing using PCA

1. Collect the received samples from m antennas
2. Calculate the covariance matrix C according to (4)
3. Calculate the eigenvalues of C
4. Find the maximum eigenvalue λ_m
5. Calculate the Eigenvector, v_m, corresponding to λ_m
6. Find $\mathbf{p}_{val}$, the PC corresponding to v_m
7. Apply a certain Spectrum Sensing method on $\mathbf{p}_{val}$ to obtain a test statistic
8. Compare the test statistic to a threshold to make a decision on the channel status

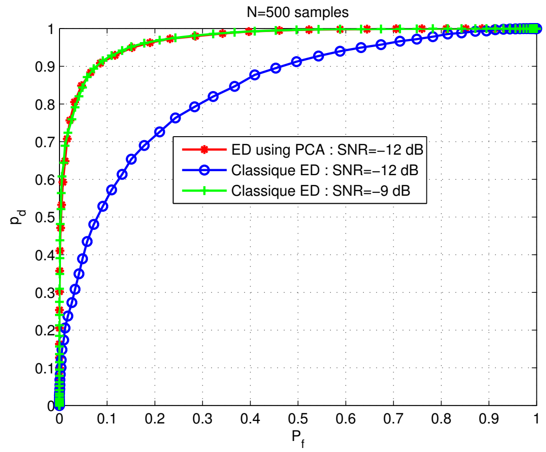


Fig. 1. The PCA technique effect on ROC curve

V. NUMERICAL RESULTS

In this section, we show by simulation the efficiency of the PCA technique. The PU signal is assumed to be 16-QAM baseband modulated signal with a symbol duration of 8μs, and a sampling frequency of 1 MHz.
Two types of simulations will be performed, the first one deals with a perfect knowledge of the covariance matrix, and in the second one the covariance matrix is estimated according to (4).

a) Perfect Knowledge of Covariance matrix : To show the effect of PCA on the SNR of the PU signal and to show the the accurate analytic relation of (12), we assume that the Covariance Matrix, C, is perfectly known. Figure (1) shows the Receiver Operating Characteristic (ROC) curve for a number of receiving antennas $m = 2$. As shown in figure (1), at $\gamma = -12$ dB we obtain the same performance as that when $\gamma = -9$ dB. Therefore a gain of 3 dB is achieved (The SNR is doubled).

b) Estimated Covariance matrix: In this section we consider the covariance matrix estimation effect on the Spectrum Sensing process, and the performance of the proposed technique comparing to other mutli-antenna techniques. In real applications, it is hard to know perfectly the covariance matrix. For that, we can estimate C according to (4). Figure (2), shows the ROC curve of ED when C is estimated using (4). The channel is assumed to be Gaussian and the the number of samples is fixed to $N = 500$ samples. Figure (2) shows the ROC curve of ED with PCA when C is perfectly known and when C is estimated. Furthermore, ED with SCS and HCS is presented as well as ED which is performed at single antenna. It is shown that the estimation process slightly affects the detection performance. Nevertheless, PCA techniques leads ED to be more efficient than the situations where SCS and HCS are used.

To show the efficiency of the PCA on various detectors. Widely used methods sush as ED, CSD and ACD are considered to perform the Spectrum Sensing. The proposed PCA technique is compared with SCS under various situations in order to show its efficiency. For the upcoming simulations, we assume that the channel between the PU base station and the

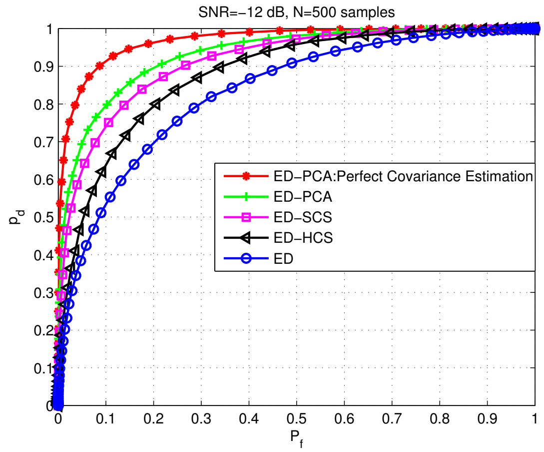


Fig. 2. ROC curve obtained by using the Covariance matrix according to (4)

ith receiving SU antenna is Raleigh flat-fading and the number of samples is $N = 1000$ samples.
For ED, ACD and CSD, we evaluate the three corresponding test statistics, T_{ed}, T_{acd} and T_{csd} respectively as follows:

$$T_{ed} = \frac{1}{N}\mathbf{p}_{val}\mathbf{p}_{val}^{H} \tag{18}$$

$$T_{acd} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{p}_{val}(n)\mathbf{p}_{val}(n-\tau)^{*} \tag{19}$$

$$T_{csd} = \frac{1}{N^2}\left|\sum_{n=1}^{N}\mathbf{p}_{val}(n)\mathbf{p}_{val}(n-\tau)^{*}e^{-j2\pi\alpha n}\right|^{2} \tag{20}$$

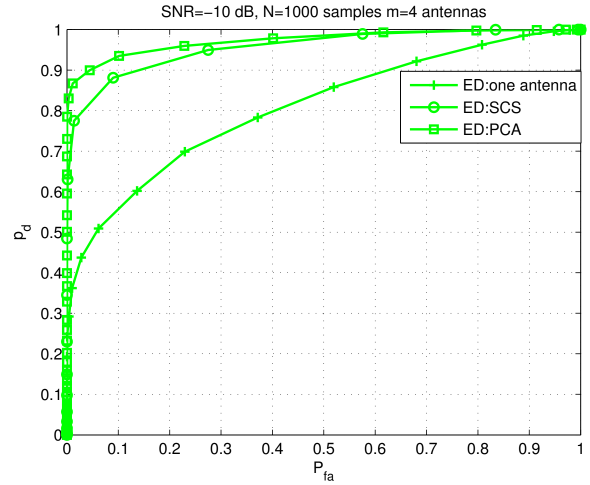
Where $\mathbf{p}_{val}(n)$ is the nth component in $\mathbf{p}_{val}$, τ is the lag value and it should be non-zero for ACD [4], and α is a non-zero cyclic frequency of $\mathbf{s}$.

Figure (3) shows the ROC curves for ED, ACD and CSD using $\hat{C}$ for a SNR of -10 dB and $m = 4$ antennas. As shown in figure (3), PCA enhances the performance of ED, ACD and CSD more than SCS. For $p_{fa} = 0.1$, CSD reaches $p_d = 0.5$ when SCS is used, while the probability of detection of this detector becomes more than 0.7 when PCA is used.

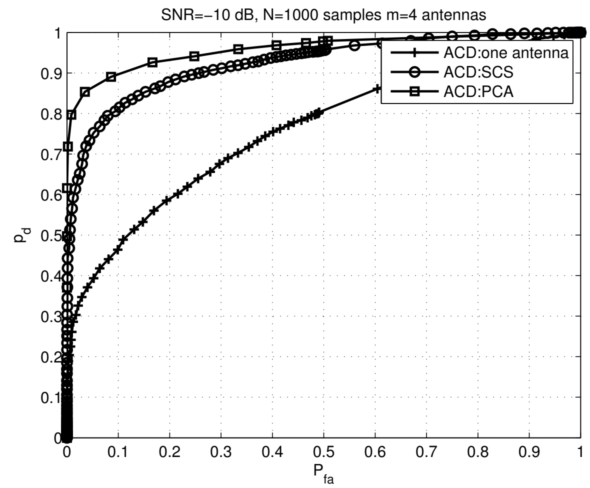
Figure (4) shows the variation of p_d with respect to the number of SU receiving antennas, for $p_{fa} = 0.1$ and SNR=-12 dB. For the different used detectors, PCA technique outperforms slightly SCS. p_d of ACD exceeds 0.9 at $m = 4$ antennas with PCA, while it reaches this values for $m = 5$ antennas with SCS. Similarly, for ED and CSD, where the performance with PCA becomes more efficient than that with SCS.

VI. Conclusion

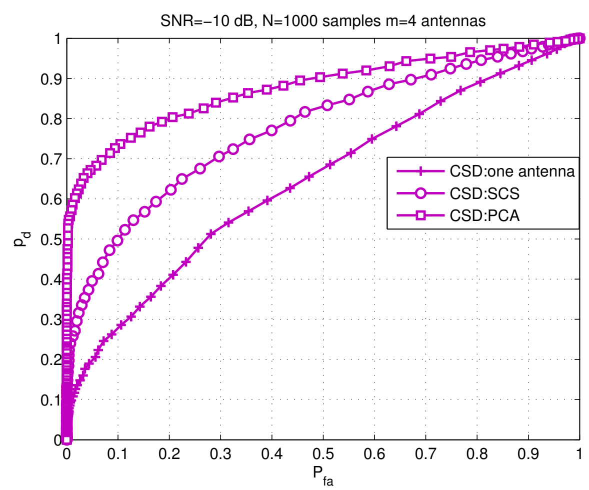
In this paper, Principal Component Analysis (PCA) is proposed to enhance Spectrum Sensing performance. With PCA, the Spectrum Sensing process is divided into two steps: in the first one, PCA is applied on the collected observations on mutli-antenna. PCA yields a filtered copy of the PU signal with an improved SNR which increases linearly with the number of observations. In the second step, a Spectrum Sensing method is applied on the filtered copy found by PCA. Simulation results



(a) ED

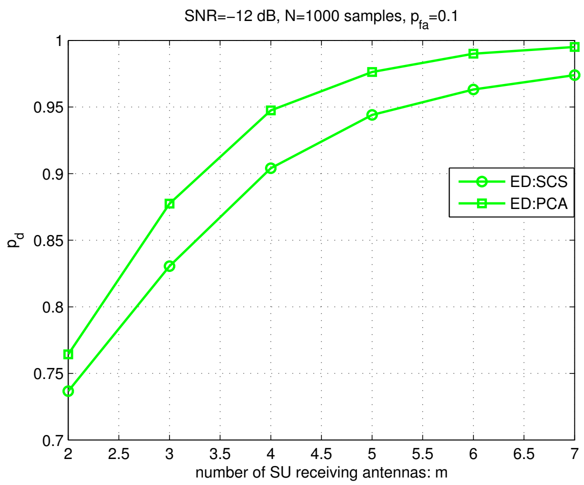


(b) ACD

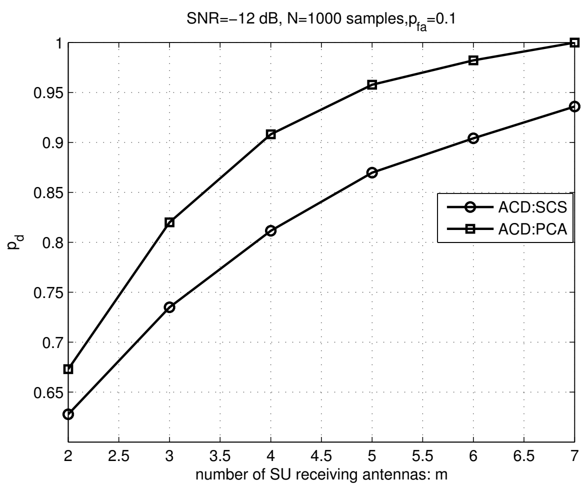


(c) CSD

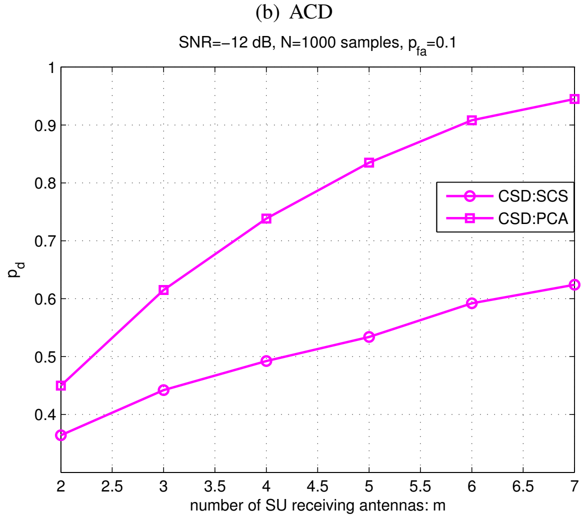
Fig. 3. ROC curve of ED, ACD and CSD using PCA and SCS



(a) ED



(b) ACD



(c) CSD

Fig. 4. The variation of p_d in terms of the number of SU receiving antennas under SNR=-12 dB and $p_{fa} = 0.1$ using PCA and SCS

show the efficiency of our method which ameliorates the performance of various Spectrum Sensing method considered in this manuscript.

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