Fifth International Symposium on Signal Processing and its Applications, ISSPA'99, Brisbane, Australia, 22-25 August, 1999
Organized by the Signal Processing Research Center, QUT, Brisbane, Australia

The blind separation of non stationary signals by only using the second order statistics.

Ali MANSOUR

BMC Research Center (RIKEN),
2271-130, Anagahora, Shimoshidami, Moriyama-ku, Nagoya 463 (JAPAN)
email: mansour@nagoya.riken.go.jp
http://www.bmc.riken.go.jp/sensor/Mansour/

Abstract *"For the last 10 years, source separation has raised an increasing interest, partly because it has been discovered that space-time approaches will play an essential role in future radio communications" Lathauwer and Comon [1].*

In the case of instantaneous mixture (memoryless mixture or channel), many algorithms are proposed to solve the blind separation problem. In general case (where no special assumption is assumed), the high order statistics (i.e fourth order) are used [2]. By adding special assumptions, algorithms and criteria can be simplified [3].

In this paper, we discuss and shortly present how the separation of non stationary signal can be done using only second order statistic.

Keywords: Decorrelation, Second order Statistics, Whiteness, Blind separation of sources, natural gradient, Kullback-Leibler divergence.

1 Discussion

Recently in the signal processing field, a new and important problem has been introduced by Hérault *et al.* [4, 5]. That problem involves retrieving unknown sources from the observation of unknown mixtures of these sources.

Generally, the sources and the channel are assumed unknown and the authors assume two fundamental assumptions [3]:

- **H1**: The sources are assumed to be unknown and statistically independent from each other.
- **H2**: The channel model is known. So, the mixture can be linear mixture (i.e. instantaneous mixture or "memoryless channel" and convolutive mixture) or non linear mixture [6].

Generally, the number of sensors q is assumed to be equal or great than the number of sources p, $1 < p \leq q$. In subspace approaches the number of sensors must be great than the number of sources, $q > p$. But, for BPSK and MSK sources, Comon and Grellier [7] propose an approach based on the adding of virtual sensor measurements to solve the under-determined mixtures ($p > q$).

For the general case of the instantaneous mixture, many algorithms and approaches have been proposed, using: a maximum likelihood [8, 9], Kullback-Leibler divergence properties [10], the natural gradient [11, 12], deflation approach [13], higher order statistics [14, 15, 16, 17], Nonlinear PCA [18], Lococode (Low complexity coding and Decoding [19], Renyi's quadratic entropy [20] and many other criteria. All these algorithms, in the general case, were based on the high order statistics (most of the cases, the fourth order cumulant or moment are used).

By adding more assumptions, the algorithms can be simplified or the criteria can be based only on second order statistics. Concerning the criteria based on second order statistics, one can find different approaches using:

- The subspace properties of the channel [21, 22],
- The correlation properties of the sources (i.e. the samples of each source are correlated) [23, 24],
- The non stationary properties of the sources [25].

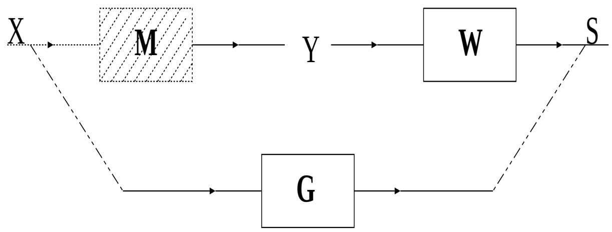

Figure 1: Mixture Model.

In this paper, we will discuss and shortly present that the separation of non-stationary signal can be done using only second order statistic. In the following, we assumed that **H1** is satisfied, the mixture is instantaneous and $p = q$.

2 Model and Approach

Let X be a $p \times 1$ zero-mean random vector denotes the source vector at time t, Y be the observed signals obtained by an instantaneous mixture and let $\mathbf{M} = (m_{ij})$ be a $p \times p$ full-rank denotes the unknown mixture matrix. One can write (see fig. 1):

$$Y = \mathbf{M}X \tag{1}$$

Let us denote by $\mathbf{W} = (w_{ij})$ the weight matrix and by $\mathbf{G} = \mathbf{WM}$ the global matrix. The estimated sources are given by:

$$S = \mathbf{W}Y = \mathbf{WM}X = \mathbf{G}X, \tag{2}$$

The separation is considered achieved when the global matrix becomes [26]:

$$\mathbf{G} = \mathbf{P\Delta}, \tag{3}$$

where $\mathbf{P}$ is any $p \times p$ permutation matrix and Δ is any $p \times p$ full matrix.

In this section, it is proved that one can separate non-stationary signals using only the second order statistics (a simple decorrelation). To explain the geometrical solutions of this problem, let us consider, at first, the case of two sensors and two sources.

2.1 Simple Case: Two Sources

Let us consider $p = 2$ and let us cosidere[1] $w_{ii} = 1$. Suppose that one can achieve the decorrelation of the output signals $\mathbf{S}$ and by using assumption **H1**, one can prove that:

$$(m_{11} + m_{21}w_{12})(m_{21} + m_{11}w_{21})P_1 + (m_{21} + m_{11}w_{21})(m_{12}w_{21} + m_{22})P_2 = 0, \tag{4}$$

where $P_i = \mathrm{E}\{x_i^2\}$ is the power of the i-th source.

It is known that the power of a stationary signal is independent of time. Now, it is easy to observe that the equation (4) becomes the equation of an **hyperbola**, so it is clear that *the separation can not be achieved by only using a decorrelation.*

In the case of independent **non-stationary** sources, the power P_i is changing independently[2] with time. In this case, the equation (4) must be held for any value of $P_i > 0$, i.e. the weight matrix coefficient must satisfied the following condition:

$$(m_{11} + m_{21}w_{12})(m_{21} + m_{11}w_{21}) = 0 \tag{5}$$
$$(m_{21} + m_{11}w_{21})(m_{12}w_{21} + m_{22}) = 0 \tag{6}$$

After some algebraic equations, one can show that the precedent equations are the separating solutions (i.e $\mathbf{G}$ satisfied the condition (3). For more details, please see [27]).

Fig. 2 shows some hyperbolas (see equation (4)) corresponding to different signals with different P_i and it also shows the two intersection points corresponding to the separation points.

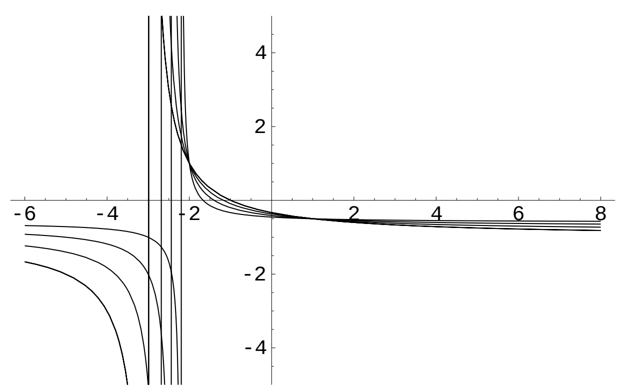

Figure 2: The set of hyperbola.

2.2 General Case

Let $\mathbf{\Lambda}$ denotes the covariance matrix of the non-stationary sources ($\mathbf{\Lambda}$ is changing with time). Using

[1]Using the fact that the separation is achieved up to a permutation and scale factor, see equation (3), one can write $w_{ii} = 1$ without any loss of generality.

[2]The P_i can not have a linear relationship among each others.

assumption **H1**, we deduce that $\mathbf{\Lambda}$ is a diagonal matrix, $\mathbf{\Lambda} = \text{diag}(P_1, \dots, P_p)$. Using the fact that the covariance matrix of the output signals becomes a diagonal matrix $\mathbf{D}$ when the decorrelation of the output signals is achieved. So, one can deduce that $\mathbf{G}$ is an orthogonal matrix and we can prove, see [27], that:

$$g_{il}^2 P_l = d_{ii} \tag{7}$$

$$\sum_l g_{il} g_{jl} P_l = 0 \quad \forall l, \text{ and } i \neq j \tag{8}$$

Using the the fact that $\mathbf{\Lambda}$ is changing with time, one can conclude that the equation (8) must hold for any value of P_i (i.e the P_i are assumed to be independently changing with time), and one can deduce that:

$$g_{il} g_{jl} = 0 \quad \forall l, \text{ and } i \neq j \tag{9}$$

The last equation (9) means that:

- P1: All columns of $\mathbf{G}$ have at most one non zero coefficient.
- P2: All the rows of $\mathbf{G}$ have at least one non zero coefficient.
- P1 and P2 means that: Each column of $\mathbf{G}$ has only one non zero coefficient or $\mathbf{G}$ satisfy the condition (3). That means the separation can be achieved using second order statistics.

3 Conclusion

In this paper, it has been proved that the second order statistics is enough to separate the instantaneous mixture of independent non-stationary signals.

For two signals, it has been shown that the decorrelation of the output signals make the weight matrix coefficients belong to a set of hyperbolas. And these hyperbolas have two intersection points which correspond to the blind separation solutions of non-stationary signals.

In the general case, it has been shown that the diagonalization of the auto-correlation matrix is enough to separate the non-stationary signals.

References

[1] L. D. Lathauwer and P. Comon, "Blind source separation and multichannel deconvolution," *Signal Processing*, vol. 73, no. 1-2, pp. 3–4, February 1999, Special issue on Blind source separation and multichannel deconvolution.

[2] J.-L. Lacoume, "A survey of source separation," in *First International Workshop on Independent Component Analysis and signal Separation (ICA99)*, J. F. Cardoso, Ch. Jutten, and Ph. loubaton, Eds., Aussois, France, 11-15 January 1999, pp. 1–5.

[3] C. Jutten and J. F. Cardoso, "Separation of sources: Really blind ?," in *International symposium on nonlinear theory and its applications*, Las Vegas, Nevada, U. S. A., December 1995.

[4] J. Hérault and C. Jutten, "Space or time adaptive signal processing by neural networks models," in *Intern. Conf. on Neural Networks for Computing*, Snowbird (Utah, USA), 1986, pp. 206–211.

[5] J. Hérault, C. Jutten, and B. Ans, "Détection de grandeurs primitives dans un message composite par une architecture de calcul neuromimétique en apprentissage non supervisé," in *Actes du Xème colloque GRETSI*, Nice, France, 20-24, May 1985, pp. 1017–1022.

[6] A. Taleb and Ch. Jutten, "Batch algorithm for source separation in postnonlinear mixtures," in *First International Workshop on Independent Component Analysis and signal Separation (ICA99)*, J. F. Cardoso, Ch. Jutten, and Ph. loubaton, Eds., Aussois, France, 11-15 January 1999, pp. 155–160.

[7] P. Comon and O. Grellier, "Non-linear inversion of underdetermined mixtures," in *First International Workshop on Independent Component Analysis and signal Separation (ICA99)*, J. F. Cardoso, Ch. Jutten, and Ph. loubaton, Eds., Aussois, France, 11-15 January 1999, pp. 461–465.

[8] D. T. Pham, P. Garat, and C. Jutten, "Separation of a mixture of independent sources through a maximum likelihood approach," in *Signal Processing VI, Theories and Applications*, J. Vandewalle, R. Boite, M. Moonen, and A. Oosterlinck, Eds., Brussels, Belgium, August 1992, pp. 771–774, Elsevier.

[9] D. T. Pham and P. Garat, "Blind separation of mixture of indepndent sources through a quasi-maximum likelihood approach," *IEEE Trans. on Signal Processing*, vol. 45, no. 7, pp. 1712–1725, July 1997.

[10] S. I. Amari, A. Cichoki, and H. H. Yang, "A new learning algorithm for blind signal separation," in *Neural Information Processing System 8*, Eds. D.S. Toureyzky et. al., 1995, pp. 757–763.

[11] J. F. Cardoso and B. Laheld, "Equivariant adaptive source separation," *IEEE Trans. on Signal Processing*, vol. 44, no. 12, pp. 3017–3030, December 1996.

[12] L. Q. Zhang, S. I. Amari, and A. Cichocki, "Natural gradient approach to blind separation of over- and under-complete mixtures," in *First International Workshop on Independent Component Analysis and signal Separation (ICA99)*, J. F. Cardoso, Ch. Jutten, and Ph. loubaton, Eds., Aussois, France, 11-15 January 1999, pp. 455–460.

[13] N. Delfosse and P. Loubaton, "Adaptive blind separation of independent sources: A deflation approach," *Signal Processing*, vol. 45, no. 1, pp. 59–83, July 1995.

[14] J. L. Lacoume, M. Gaeta, and W. Kofman, "Source separation using higher order statistics," *JATP*, vol. 54, no. 10, pp. 1217–1226, 1992.

[15] O. Macchi and E. Moreau, "Self-adaptive source separation by direct and recursive networks," in *in Proc. International Conference on Digital Signal Processing (DSP'95)*, Limasol, Cyprus, May 1993, pp. 1154–1159.

[16] A. Mansour and C. Jutten, "Fourth order criteria for blind separation of sources," *IEEE Trans. on Signal Processing*, vol. 43, no. 8, pp. 2022–2025, August 1995.

[17] A. Mansour and C. Jutten, "A direct solution for blind separation of sources," *IEEE Trans. on Signal Processing*, vol. 44, no. 3, pp. 746–748, March 1996.

[18] E. Oja, "Nonlinear pca criterion and maximum likelihood in independent component analysis," in *First International Workshop on Independent Component Analysis and signal Separation (ICA99)*, J. F. Cardoso, Ch. Jutten, and Ph. loubaton, Eds., Aussois, France, 11-15 January 1999, pp. 143–148.

[19] Se Hochreiter and J. Schmidhuber, "Lococode performs nonlinear ica withoutout knowing the number of sources," in *First International Workshop on Independent Component Analysis and signal Separation (ICA99)*, J. F. Cardoso, Ch. Jutten, and Ph. loubaton, Eds., Aussois, France, 11-15 January 1999, pp. 149–154.

[20] J. C. Principe and D. Xu, "Information-theoretic learning using renyi's quaratic entropy," in *First International Workshop on Independent Component Analysis and signal Separation (ICA99)*, J. F. Cardoso, Ch. Jutten, and Ph. loubaton, Eds., Aussois, France, 11-15 January 1999, pp. 407–412.

[21] A. Gorokhov and P. Loubaton, "Subspace based techniques for second order blind separation of convolutive mixtures with temporally correlated sources," *IEEE Trans. on Circuits and Systems*, vol. 44, pp. 813–820, September 1997.

[22] A. Mansour, C. Jutten, and P. Loubaton, "Subspace method for blind separation of sources and for a convolutive mixture model," in *Signal Processing VIII, Theories and Applications*, Triest, Italy, September 1996, pp. 2081–2084, Elsevier.

[23] A. Belouchrani, K. Abed-Meraim, J. F. Cardoso, and E. Moulines, "Second-order blind separation of correlated sources," in *Int. Conf. on Digital Sig.*, Nicosia, Cyprus, july 1993, pp. 346–351.

[24] S. I. Amari, "Ica of temporally correlated signals - learning algorithm," in *First International Workshop on Independent Component Analysis and signal Separation (ICA99)*, J. F. Cardoso, Ch. Jutten, and Ph. loubaton, Eds., Aussois, France, 11-15 January 1999, pp. 13–18.

[25] K. Matsuoka, M. Oya, and M. Kawamoto, "A neural net for blind separation of nonstationary signals," *Neural Networks*, vol. 8, no. 3, pp. 411–419, 1995.

[26] P. Comon, "Independent component analysis, a new concept?," *Signal Processing*, vol. 36, no. 3, pp. 287–314, April 1994.

[27] A. Mansour, A. Kardec Barros, and N. Ohnishi, "Blind separation of sources: Methods, assumptions and applications.," *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, vol. E83-A, no. 8, pp. 1498–1512, 2000, Special Section on Digital Signal Processing in IEICE EA.