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A Simple ICA Algorithm Based On Geometrical Approach.

A. MANSOUR [1], C. G. PUNTONET [2] and N. OHNISHI [1,3]

1- BMC - RIKEN, Anagahora, Shimoshidami, Moriyama-ku, Nagoya 463-0003 (JAPAN)

2- Dept. of Architecture and Computer Technology.University of Granada. Granada 18071 (SPAIN)

3- Dept. of Information Eng., Nagoya Univ. Furo-cho, Chikusa-ku, Nagoya 464-8603 (JAPAN).

email: mansour@ieee.org, carlos@atc.ugr.es, ohnishi@ohnishi.nuie.nagoya-u.ac.jp.

Abstract

Here, a simple and new algorithm for blind separation of sources based on geometrical concepts is proposed. This algorithm deals with the instantaneous mixtures and it doesn't require the estimation of High Order Statistics (HOS).

Keywords: Blind Separation of Sources, ICA, PDF, Geometrical Methods, Decorrelation, Cholesky Factorization, Orthogonalization and Whitening process.

1 Introduction

The blind separation of sources (BSS) is a recent and important problem in signal processing and it has many applications [1]. Since 1984 [2], it has been studied by many authors whilst many algorithms have been proposed. The blind separation of sources problem consists in retrieving unknown sources $X(t)$ from only observing a mixture of them $Y(t)$ [3, 4] (see Fig. 1). In general case, authors assume that the sources are non-Gaussian signals (at most, one of the sources can be a Gaussian signal) and statistically independent of one another. Therefore, concepts of Independent Component Analysis (ICA) [5] have been widely used and developed to solve the BSS.

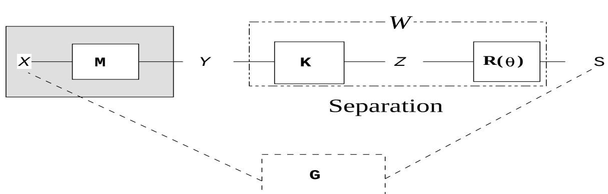


Figure 1: Channel Model

Concerning the transmission channel, one can find three principal models in the literature: Linear such as instantaneous mixture (i.e. Memoryless channel) or convolutive mixture (i.e. Memory channel) and nonlinear channel. The last model has been hardly described in the general case, however some algorithms have been proposed for specific non-linear models [6, 7, 8, 9]. Concerning the instantaneous or the convolutive model, different criteria and approaches have been proposed [10, 11, 12, 13, 14, 15, 16], most of them are based on HOS.

We should mention that the instantaneous mixture (i.e. the channel effect can be represented as the product of the source vector X by a scalar matrix $\mathbf{M}$) is very well understood and the literature contents a big number of approaches and criteria. Unfortunately, many of them are hard to be understood or implemented by non-specialist, or they need important computing efforts. On the other hand, some researchers proposed a very simple geometrical approach to dial with special signals (as binary or n-valued) [17]. That algorithm has been modified to separate analogical signals [18]. The basic idea is to use the independence concept from geometrical point of view. In fact, the scatter plot of two independent signals (i.e. $x_2(t)$ against $x_1(t)$ for every t) is a rectangular. on the other hand, the mixing effect becomes a geometrical transformation of that rectangular to a parallelogram shape (see Fig. 2).

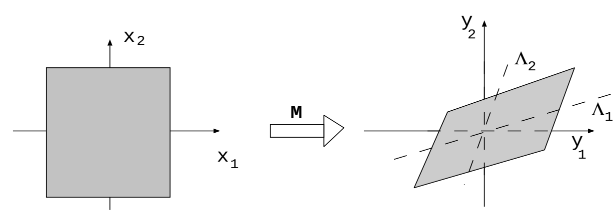


Figure 2: Geometrical Concepts

The algorithm proposed in [18] contents two steps (see Fig. 3):

- Translate the parallelogram to the first quadrant.
- Find the slopes of the edges Δ_1 and Δ_2 of the parallelogram. They proved that the slopes are equal to $\min_t(x_i(t)/x_j(t))$, for $i \neq j$. They showed how the mixing parameters can be easily obtained from the slopes.

The performances and the limitations of that algorithm were discussed in [19]. The main problem of that algorithm is the necessity of the translation step. On the other hand, it becomes very unlikely to find the edges of the parallelogram in the case of non-uniform pdf signals. For more than two sources, this algorithm can not be easily generalized.

Recently a new geometrical algorithm was proposed to deal with the speech signals or the signals with probability density function (pdf) close to Gamma pdf [20]. For these

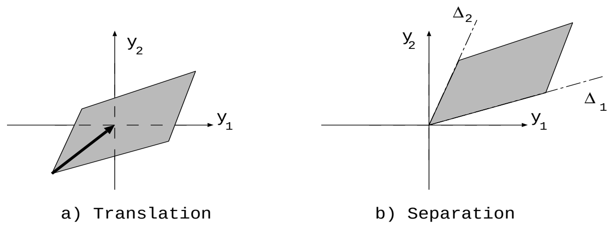


Figure 3: Previous Algorithm

kind of signals, it is very hard to find the edges of the parallelogram. Therefore, the authors suggest an algorithm to find the slopes of the principle axes of symmetry Λ_1 and Λ_2 of the parallelogram (see Fig. 2). To achieve this goal, the authors divide the mixture plan (the scatter plot) into many clusters and they correspond the symmetrical axes to the direction of the more popular cluster (i.e. the cluster which contents the max number of observation samples). One should mention that the performances of this new approach depends as well on the estimation of the slopes of the axes Λ_1 and Λ_2. The latter estimation depends on the number of the clusters and the decision to select the good one (if the number of clusters is small than the decision can be made easily but the estimation of the slopes becomes biased and vice versa).

2 Main Idea

The number of sources can be easily determined as the number of the strongest eigenvalue of the covariance matrix of the mixing signals [21, 22]. In the following, we consider only the case of instantaneous mixture with same number p of sources and sensors. On the other hand, it is known that the separation of instantaneous mixture can be achieved up to a permutation and a scale factor [5, 1] (i.e. the estimated sources $S(t)$ are equal to the sources up to a permutation and a scale factor). Therefore the global matrix $\mathbf{G} = \mathbf{WM}$ ($\mathbf{W}$ is the demixing matrix or the separating matrix) should satisfy the following:

$$\mathbf{G} = \mathbf{PD} \tag{1}$$

where $\mathbf{P}$ is any permutation and $\mathbf{D}$ is any full rank diagonal matrix.

It is obvious from the discussion of the previous section that the effect of the instantaneous mixture is a geometrical transformation in the observation space (or plane in the case of two sources). To separate the signals or to cancel the mixing effect, we propose a new algorithm consists on two steps, see Fig. 1 and 4. In the following we describe the algorithm in the case of two signals, later we discuss the general case.

2.1 Transformation

One can generate orthogonal signals Z from the mixing signals Y by a simple Cholesky decomposition [23]. In other words, we transform the mixing parallelogram to a square. Let $\mathbf{R}_Y = \mathrm{E}YY^T$ be the covariance matrix of $Y(t)$. Using Cholesky decomposition, one can obtain a square root $\mathbf{L}$ of $\mathbf{R}_Y$ such that $\mathbf{R}_Y = \mathbf{LL}^T$. When the sources are statistically independent and the number of sources is equal to the number of sensors, the mixing covariance matrix $\mathbf{R}_Y$ becomes a full rank matrix as well as its square root $\mathbf{L}$. Let $\mathbf{K} = \mathbf{L}^{-1}$. Finally, Z can be obtained as $Z(t) = \mathbf{K}Y(t)$. We should mention that the covariance matrix $\mathbf{R}_Z$ of Z is equal to the identity matrix $\mathbf{R}_Z = \mathrm{E}ZZ^T = \mathbf{K}\mathrm{E}YY^T\mathbf{K}^T = \mathbf{L}^{-1}\mathbf{R}_Y\mathbf{L}^{-T} = \mathbf{I}_p$, here $\mathbf{I}_p$ is a $p \times p$ identity matrix.

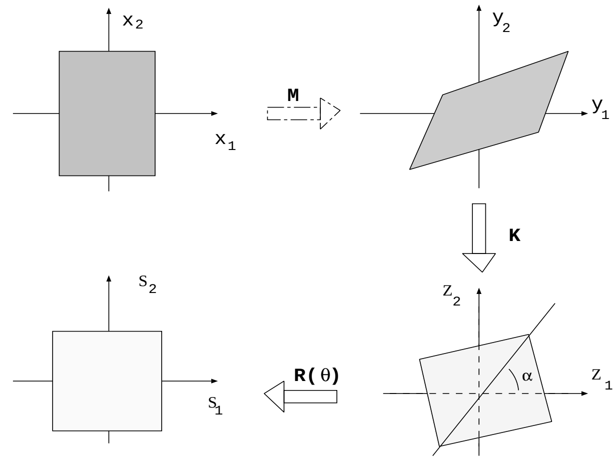


Figure 4: Two steps.

2.2 Rotation

The estimated signals $S(t)$ can be obtained from the orthogonal mixing signals $Z(t)$ via a rotation by an angle θ as $S(t) = \mathbf{R}(\theta)Z(t)$, here $\mathbf{R}(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$. Let α be the angle between the first diagonal and the horizontal axe (see Fig. 4). the angle α can be determined using the coordinate of the farthest point from the origin. It is clear that the determination of α is not unique and that is due principally to the permutation indeterminacy, see equation (1). Using the facts that all the physical signals are bounded and using the scale factor indeterminacy (1), one can consider that the sources have the same maximum amplitude or that the scatter plot of the sources is a square one. Finally, to separate the signals one should rotate the signals Z by an angle θ:

$$\theta_U = \pi/4 - \alpha. \tag{2}$$

We should mention that θ_U can be obtained from the previous equation when the sources have uniform pdf or close to uniform (as well the signals with an histogram similar to rectangular as sinusoidal signals). In the case of the signals with unimodal pdf as the symmetrical Gamma pdf, similar to symmetrical Gamma (as Cauchy pdf or Laplace's pdf) or close to symmetrical Gamma (as the speech signals), the scatter plot of these signals is more likely to be a cross shape

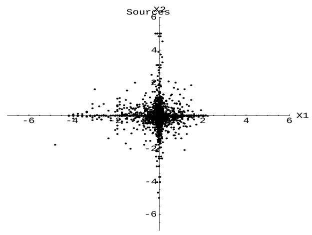


Figure 5: The scatter of two speech signals.

than a rectangular (see Fig. 5). In this case the farthest point (i.e the second step of our algorithm) will correspond to one of the two principle axes. Therefore θ should evaluated as:

$$\theta_G = -\alpha. \tag{3}$$

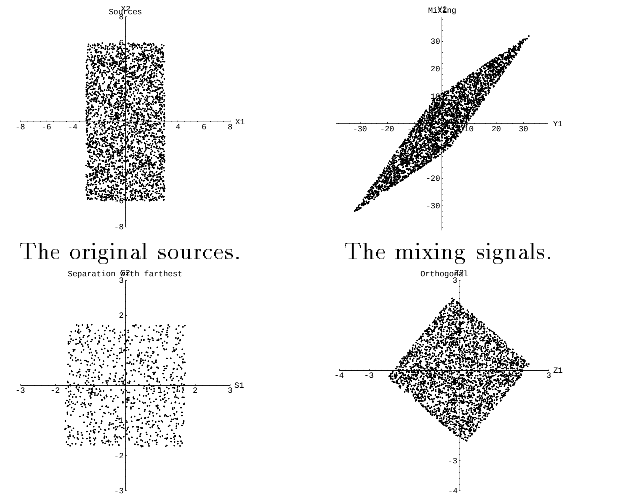
2.3 General case

It seems that the generalization of the algorithm is very simple since the first step (section 2.1) can be used without any modification. Concerning the second step (section 2.2), one should consider $p-1$ angles instead of one angle θ (more details can be found in [24]). These angles can be determined from the projection of the farthest point and the rotation can be conducted mainly using Givens rotation matrix [23]. Due to the limitation of the page number, we can not give here any further details neither experimental results concerning the generalization, nor studies on the performances of the algorithm.

3 Experimental Results

We conducted many experiments over different kind of signals. All of the conducted experiments show satisfactory results. In this section, we present two different experiment results:
First, the two sources are two zero-mean signals with uniform pdf ($-3 \leq x_1 \leq 3$ and $-6 \leq x_2 \leq 6$). The mixing matrix $\mathbf{M} = \begin{pmatrix} 5 & 3 \\ 7 & 2 \end{pmatrix}$. The estimated signals have been obtained with a global matrix $\mathbf{G} = \begin{pmatrix} -0.572954 & 0.0029111 \\ -0.00206487 & 0.286697 \end{pmatrix}$. We used 3000 samples, see Fig. 7. The experimental study shows satisfactory results even with few hundred of samples (in the case of uniform pdf, we obtained satisfactory results with 150 samples and very good results with more than a thousand of samples).

Second, we separate two speech signals. The mixing matrix $\mathbf{M} = \begin{pmatrix} 5 & 2 \\ -3 & 6 \end{pmatrix}$. The estimated signals have been estimated with a global matrix $\mathbf{G} = \begin{pmatrix} -0.0244912 & 1.35955 \\ -1.12395 & -0.0388072 \end{pmatrix}$. We used the first 3000 samples of the signals, see Fig. 7. With few thousand of samples, one can obtain good results. To separate some musical signals, we used up to 6000 samples and we obtained similar good results.



The original sources. The mixing signals.

The estimated signals. The orthogonal mixing signals.

Figure 6: The separation of two uniform pdf signals.

4 Conclusion

In this paper, we present a new simple algorithm for blind separation of sources based on geometrical concepts. The experimental results show that the performances of the algorithms are very satisfactory even for the separation of real non-stationary signals as the speech signals. The convergence time is very competitive and it can be obtained by few thousands of samples (less than 7000 samples). The number of samples can be reduce effectively to fewer hundred of samples when the signals become stationary signals with uniform pdf or close to uniform (as well the signals with an histogram similar to rectangular as sinusoidal signals).

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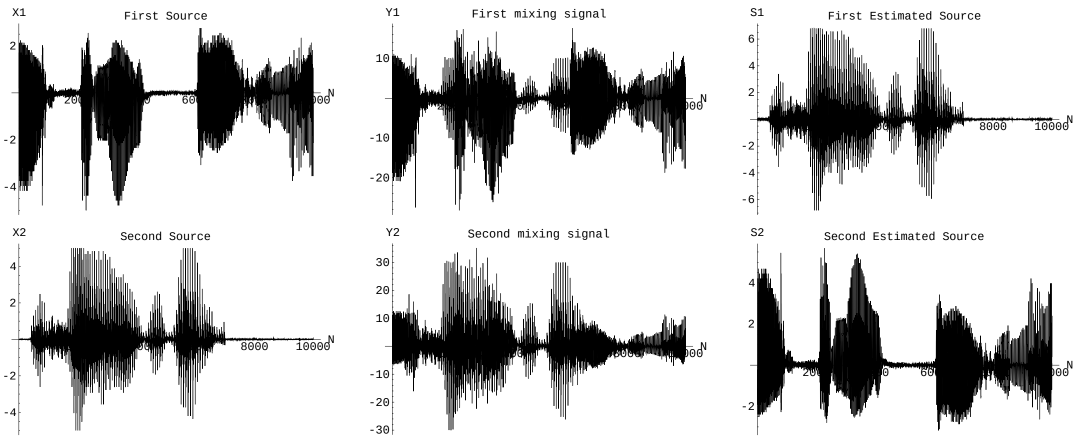

The original sources | The mixing signals | The estimated signals

Figure 7: The separation of speech signals.

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