

# Multichannel Blind Separation of Sources Algorithm For Instantaneous Mixture.

Ali MANSOUR, Member IEEE and Noboru OHNISHI, Member IEEE.

Bio-Mimetic Control Research Center (RIKEN),  
2271-130, Anagahora, Shimoshidami, Moriyama-ku, Nagoya 463 (JAPAN).

Tél: +81 - 52 - 736 - 5867, Fax: +81 - 52 - 736 - 5868.

email: mansour@nagoya.riken.go.jp, ohnishi@nagoya.riken.go.jp.

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## Abstract

The algorithms of blind separation of sources, in the general case and for instantaneous mixtures, are based on high-order statistics; most of them use the fourth-order statistics. For an instantaneous mixture of only two sources, we proposed in [14] an algorithm of blind separation of sources. The separation was achieved by minimizing the cross-cumulant (2x2) of the two output signals. The minimization of that cross-cumulant was achieved using a gradient algorithm. In this paper, we derive a new cost function which is more general than the first one, also based on the cross-cumulant (2x2) of the output signals. This new algorithm deals with Multiple Inputs and Multiple Outputs (MIMO) and uses a Levenberg-Marquardt method for the minimization of the cost function. The actual algorithm is very fast; the criterion convergence is attained in less than 50 iterations. In addition, it yields good results even in the case of about 300 signal samples. Good experimental results were obtained even with five stationary signals.

# 1 Introduction

The blind-separation-of-sources problem involves in retrieving the sources from the observations of unknown mixtures of unknown sources. In general case, authors assume that the sources are non-Gaussian signals and statistically independent of one another.

The blind separation of sources was initially proposed by Héroult *et al.* [10, 11]. It has been one of the recent and important signal processing problems. Most of the blind separation algorithms deal with two kinds of mixtures: instantaneous (memoryless) mixtures [2, 12, 4, 15] and convolutive mixtures (the channel effects can be modeled by a matrix of filters) [22, 8].

In the general case and in instantaneous mixtures, the fourth-order statistics are required to separate the sources (see [5, 15]). Recently, for convolutive mixtures and by using a subspace method [1], it was proven in [9, 16] that the separation of sources can be achieved by using only second-order statistics. These subspace algorithms are very elegant from a theoretical point of view, but are very slow due to the minimization of large size matrices [17].

In this paper, a new cost function for instantaneous mixtures, based on the cross-cumulant (2x2) of all the output signals, is proposed. A Levenberg-Marquardt method is adopted for the minimization of the cost function. So, even though the new algorithm deals with Multiple Inputs and Multiple Outputs (MIMO), the convergence of the algorithm is obtained within a small number of iterations.

To avoid some possible spurious solutions, we must assume that the sources have the same sign of kurtosis<sup>1</sup>. In fact, if the sources do not have the same sign of kurtosis, then separation-solutions may not be obtained by minimizing the cross-cumulant (2x2)[14]. Similar assumptions regarding the sign of kurtosis, have already been made by many authors in [21, 13, 7]. On the other hand, it was proven in [14] that minimization or cancellation of a cost function based on the cross-cumulant (2x2) leads to a set of solutions whose spurious ones can be simply canceled by using a decorrelation. In the following, let us assume that this assumption is satisfied.

## 2 Channel model

As shown in figure 1, at any time  $n$ , and with the help of  $N$  sensors,  $N$  instantaneous mixtures  $y_i(n)$  of the  $N$  unknown zero-mean sources  $x_i(n)$ , assumed to be statistically independent, are observed. In addition, we assume that the unknown mixture matrix is a  $N \times N$  regular matrix.

Taking  $\mathbf{M}$  as the mixture matrix,  $Y$  as the mixture vector with additive noise,  $X$  as the source vector, and  $N_g$  as the noise vector (see figure 1), we can

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<sup>1</sup>the kurtosis of a signal is its fourth-order cumulant normalized by the square of its second-order cumulant.

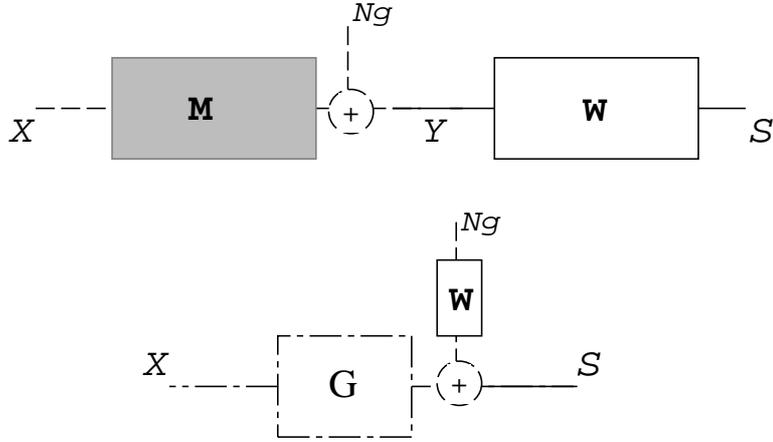


Figure 1: Channel model.

write:

$$Y = \mathbf{M}X + N_g. \quad (1)$$

The separation is achieved by estimating a  $N \times N$  matrix  $\mathbf{W}$  satisfying  $\mathbf{W}\mathbf{M} = \mathbf{\Delta}\mathbf{P}$ , where  $\mathbf{P}$  is any permutation matrix and  $\mathbf{\Delta}$  is any regular diagonal matrix [6]. Let  $S$  be the vector of the output signals, then:

$$S = \mathbf{W}Y = \mathbf{G}X + \mathbf{W}N_g, \quad (2)$$

where  $G = (g_{(i,j)})$  is the global matrix, i.e.  $\mathbf{G} = \mathbf{W}\mathbf{M}$ . The separation will be performed when  $\mathbf{G}$  becomes a general permutation matrix<sup>2</sup>. At first we consider the mixture signals without noise ( $N_g = 0$ ) and will discuss the case with noise, in section 5.

### 3 The cost function

In this section, we prove that the  $N$  sources can be separated by minimizing the following cost function:

$$\min_{\mathbf{W}} \left\{ \sum_{m>n}^N \text{Cum}_{22}^2(s_m, s_n) \right\} \quad (3)$$

where  $s_m$  is the  $m$ th output signal. The canceling<sup>3</sup> of this cost function, when the sources have the same sign of kurtosis, is equivalent to  $\frac{N(N-1)}{2}$  independent equations (see appendix A):

$$\text{Cum}_{22}(s_m, s_n) = 0 \iff g_{(m,j)}g_{(n,j)} = 0 \quad (1 \leq j \leq N) \quad \text{and} \quad m \neq n \quad (4)$$

<sup>2</sup>i.e.  $\mathbf{G} = \mathbf{\Delta}\mathbf{P}$ , where  $\mathbf{P}$  is a permutation matrix and  $\mathbf{\Delta}$  is a regular diagonal matrix.

<sup>3</sup>The cost function (3) is a positive function and its minimum is zero.

The canceling of this cost function retains the following properties of the global matrix  $\mathbf{G}$ :

- **Proposition 1: All the column vectors of the global matrix  $\mathbf{G}$  have at most one coefficient not equal to zero.**

In fact, let us suppose that one coefficient  $g_{(m,j)}$  in the  $j$ th column of  $\mathbf{G}$  is nonzero. From the equation (4), we can prove that  $g_{(n,j)} = 0, \forall n \neq m$ . So all the coefficients (except the  $m$ th one) of the  $j$ th column of  $\mathbf{G}$  are equal to zero.

- **Proposition 2: All the row vectors of the global matrix  $\mathbf{G}$  have at least one coefficient not equal to zero.**

Let us assume that all the principal diagonal elements of the weight matrix  $\mathbf{W}$  are equal to one<sup>4</sup> ( $w_{(i,i)} = 1, 1 \leq i \leq N$ ):

$$w_{(i,i)} = 1 \implies W_i \neq 0 \quad (1 \leq i \leq N) \quad (5)$$

where  $W_i$  is the  $i$ th row of the weight matrix  $\mathbf{W}$ . Now, we can prove that the global matrix cannot have a zero row. In fact, let us denote by  $M_j$  the  $j$ th column of the mixture matrix  $\mathbf{M}$  and let us assume that the  $i$ th row of  $\mathbf{G}$  is equal to zero, so:

$$g_{(i,j)} = W_i M_j = 0 \quad 1 \leq j \leq N, \quad (6)$$

or the vectors  $M_j$  ( $1 \leq j \leq N$ ) are linear independent vectors and nonzero because the mixture matrix  $\mathbf{M}$  is a regular matrix and on the other hand, all the vectors  $W_i$  are not equal to zero (see equation (5)). So it is impossible to satisfy equation (6), because we cannot find a real vector which is simultaneously nonzero and orthogonal to  $N$  independent linear vectors in  $\mathbb{R}^N$  at the same time. Finally, **the global matrix cannot have a zero row.**

- **Proposition 3: The global matrix is a general permutation matrix.**

By using propositions 1 and 2, we find that the global matrix  $\mathbf{G}$  can have at most one nonzero coefficient in the same column and at least one nonzero coefficient in the same row. These two propositions make the global matrix a general permutation matrix.

## 4 The new algorithm

It is clear that the cost function (3), proposed in this paper, is not a simple square error function. As a consequence, this function may not be minimized using a LMS algorithm, a stochastic gradient or a conjugate gradient<sup>5</sup> method.

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<sup>4</sup>It is easy to satisfy this constraint, by putting the coefficients  $w_{(i,i)} = 1$  ( $1 \leq i \leq N$ ) in all the iterations of our algorithm.

<sup>5</sup>The conjugate gradient method is used to find the generalized eigenvector corresponding to the minimum generalized eigenvalue of a semidefinite Hermitian matrix [3]

For that reason, the cost function (3) will be minimized using the Levenberg-Marquardt method. On the other hand, by choosing the Levenberg-Marquardt method, the convergence of our criterion will be faster.

The Levenberg-Marquardt method consists in minimizing an error function with respect to some vector  $V$  [20, 24] (see appendix B). In this section, we discuss how we can apply the Levenberg-Marquardt method to our case where the cost function (3) should be minimized with respect to the  $N \times N$  matrix  $\mathbf{W}$ .

Let  $V = \text{Col}(\mathbf{W})$ , where  $\text{Col}$  is the operator that corresponds the vector  $V$  to the matrix  $\mathbf{W}$ . In other words, the relationship between the components of  $V$  and the components of  $\mathbf{W}$  becomes:

$$w_{(m,n)} = v_{(m-1)N+n} \quad 1 \leq m \leq N \text{ and } 1 \leq n \leq N. \quad (7)$$

It is clear that the cost function (3) is composed of  $\frac{N(N-1)}{2}$  different cross-cumulants ( $\text{Cum}_{22}(s_m, s_n)$ ,  $1 \leq n < m \leq N$ ). Let us denote by  $\Phi = (\phi_1, \phi_2, \dots, \phi_{N(N-1)/2})^T$  the function vector (see appendix B), each component of  $\Phi$  must correspond to one different cross-cumulant. In the Levenberg-Marquardt method, the vector  $V$  (or indirectly the matrix  $\mathbf{W}$ ) is adapted by using the Jacobian matrix  $\mathbf{J}(V) = (J_{(i,j)}) = \frac{\partial \Phi}{\partial V}$  of the vector  $\Phi$  with respect to  $V$  (or  $\mathbf{W}$ , see appendix B). After some calculations, we can easily link the components of  $\Phi$  to the different cross-cumulants using the following relationship:

$$\begin{aligned} \phi_{(m^2-3m+2n+2)/2} &= \text{Cum}_{22}(s_m, s_n) \\ &= E(s_m^2 s_n^2) - E(s_m^2)E(s_n^2) - 2E^2(s_m s_n), \end{aligned} \quad (8)$$

where  $E(\cdot)$  denotes the mathematical expectation and  $1 \leq n < m \leq N$ . By using equation (2), equation (8) can be rewritten as:

$$\begin{aligned} \phi_{(m^2-3m+2n+2)/2} &= \sum_{ijkl} w_{(m,i)} w_{(m,k)} E(y_i y_j y_k y_l) w_{(n,j)} w_{(n,l)} \\ &\quad - W_m \mathbf{R}_Y W_m^T W_n \mathbf{R}_Y W_n^T - 2W_m \mathbf{R}_Y W_n^T W_m \mathbf{R}_Y W_n^T \end{aligned} \quad (9)$$

where  $W_m$  is the  $m$ th row of  $\mathbf{W}$  and  $\mathbf{R}_Y$  is the covariance matrix of  $Y$  ( $\mathbf{R}_Y = E(YY^T)$ ). To calculate the Jacobian matrix  $\mathbf{J}(V)$ , we derive at first equation (9) with respect to the row of  $\mathbf{W}$ :

$$\begin{aligned} \frac{1}{2} \frac{\partial \phi_{(m^2-3m+2n+2)/2}}{\partial W_l} &= \delta_{lm} [W_m E(s_n^2 Y Y^T) - W_m \mathbf{R}_Y (W_n \mathbf{R}_Y W_n^T) - 2W_n \mathbf{R}_Y (W_m \mathbf{R}_Y W_n^T)] \\ &\quad + \delta_{ln} [W_n E(s_m^2 Y Y^T) - W_n \mathbf{R}_Y (W_m \mathbf{R}_Y W_m^T) - 2W_m \mathbf{R}_Y (W_n \mathbf{R}_Y W_m^T)] \end{aligned} \quad (10)$$

where  $\delta_{lm}$  is the Kronecker symbol<sup>6</sup> and  $1 \leq n < m \leq N$ . Let us denote by  $J_{(i,j)}$  the general component of the  $\frac{N(N-1)}{2} \times N^2$  Jacobian matrix  $\mathbf{J}(V)$ , then by using equations (7) and (10) we obtain:

<sup>6</sup>The Kronecker symbol  $\delta_{lm} = 1$  if and only if  $l = m$ , otherwise  $\delta_{lm} = 0$ .

$$\begin{aligned}
J_{((m^2-3m+2n+2)/2, (l-1)N+k)} &= \delta_{lm} [W_m \rho_{(n,k)} - (W_n \mathbf{R}_Y W_n^T) W_m R_{(Y,k)} - 2(W_m \mathbf{R}_Y W_n^T) W_n R_{(Y,k)}] \\
&+ \delta_{ln} [W_n \rho_{(m,k)} - (W_m \mathbf{R}_Y W_m^T) W_n R_{(Y,k)} - 2(W_n \mathbf{R}_Y W_m^T) W_m R_{(Y,k)}] \quad (11) \\
&1 \leq n < m \leq N, 1 \leq k \leq N \text{ and } 1 \leq l \leq N,
\end{aligned}$$

where  $\rho_{(m,k)}$  (resp.  $R_{(Y,k)}$ ) is the  $k$ th column of  $E(s_m^2 Y Y^T)$  (resp.  $\mathbf{R}_Y$ ). Finally the weight matrix should be adapted, for minimizing the cost function, by:

$$V_{k+1} = V_k - [\mathbf{J}(V_k)^T \mathbf{J}(V_k) + \lambda_k \mathbf{I}]^{-1} \mathbf{J}(V_k)^T \Phi \quad (12)$$

where  $V_k = \text{Col}(\mathbf{W}_k)$  is the weight vector at the  $k$ th iteration,  $\mathbf{I}$  is the identity matrix, and  $\lambda_k$  is a parameter in the Levenberg-Marquardt method (see appendix B).

## 5 With Gaussian noise

Suppose that the noise  $N_g = (n_1, n_2, \dots, n_N)^T$  is a Gaussian zero-mean signal and it is statistically independent of the sources. By using equation (2), the cross-cumulant (2x2) of any two output signals can be evaluated as:

$$\begin{aligned}
\text{Cum}_{22}(s_m, s_n) &= \text{Cum}(s_m, s_m, s_n, s_n) \\
&= \text{Cum}\left(\sum_i (g_{(m,i)} x_i + w_{(m,i)} n_i), \sum_j (g_{(m,j)} x_j + w_{(m,j)} n_j), \right. \\
&\quad \left. \sum_k (g_{(n,k)} x_k + w_{(n,k)} n_k), \sum_l (g_{(n,l)} x_l + w_{(n,l)} n_l)\right) \quad (13)
\end{aligned}$$

By using the multilinearity properties of the cumulant [23] and the independence between the sources and the noise, equation (13) can be rewritten as:

$$\begin{aligned}
\text{Cum}_{22}(s_m, s_n) &= \sum_{i,j,k,l} g_{(m,i)} g_{(m,j)} g_{(n,k)} g_{(n,l)} \text{Cum}(x_i, x_j, x_k, x_l) \\
&+ \sum_{i,j,k,l} w_{(m,i)} w_{(m,j)} w_{(n,k)} w_{(n,l)} \text{Cum}(n_i, n_j, n_k, n_l).
\end{aligned}$$

The second term on the right-hand side becomes zero because the fourth-order cross-cumulant of the Gaussian noise  $N_g$  is equal to zero (i.e.  $\text{Cum}(n_i, n_j, n_k, n_l) = 0$ ). Thus we have the same cost function as the one without noise. Therefore, (as proven in section 3) by canceling the cost function (3), the weight matrix  $\mathbf{W}$  becomes  $\mathbf{W} = \mathbf{\Delta P M}^{-1}$  (i.e. the global matrix  $\mathbf{G}$  is a general permutation matrix).

Each output signal consists of a source signal and a Gaussian noise both of which are permuted and scaled by a constant value. We can summarize as follows:

$$\begin{aligned}
\text{Cost} &= 0 \Rightarrow \mathbf{W} = \mathbf{\Delta P M}^{-1} \\
S &= \mathbf{W} Y = \mathbf{\Delta P M}^{-1} \mathbf{M} X + \mathbf{\Delta P M}^{-1} N_g \\
&= \mathbf{\Delta P} X + \mathbf{\Delta P M}^{-1} N_g \quad (14)
\end{aligned}$$

Now, the problem becomes a classic identification problem of a signal with noise.

## 6 Experimental results

The experimental study shows that the algorithm proposed in this paper, is very fast:

In the case of five stationary signals  $N = 5$ , good results are obtained (about -23 dB at most of the channels, see figure 3). The five sources used in this simulation are as follows:

- The first two sources are white noise, zero-mean signals.
- The third source is colored noise obtained by filtering white noise using a pass-band MA filter  $A(z) = 1 - 0.2z^{-1} - 0.4z^{-2} + 0.3z^{-3} + 0.5z^{-4} - 0.6z^{-5}$ .
- The fourth source is a triangular signal.
- The fifth source is a sine signal.

The cumulant is adaptively estimated according to [18] and using about 1000 samples. The convergence is obtained after 20 iterations (see figure 2).

In the general case, the convergence can be obtained in less than 50 iterations. Satisfactory results are obtained when the sources are stationary signals and the number of sources is less than 7.

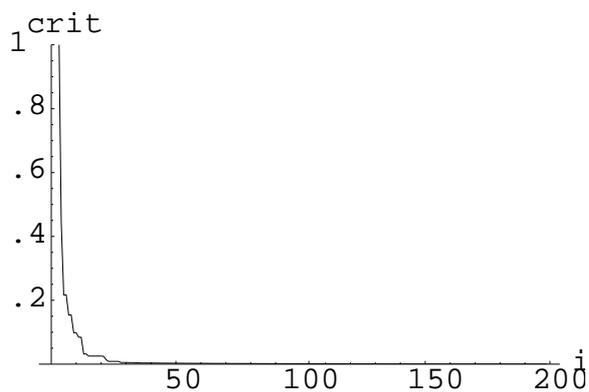


Figure 2: Criterion convergence.

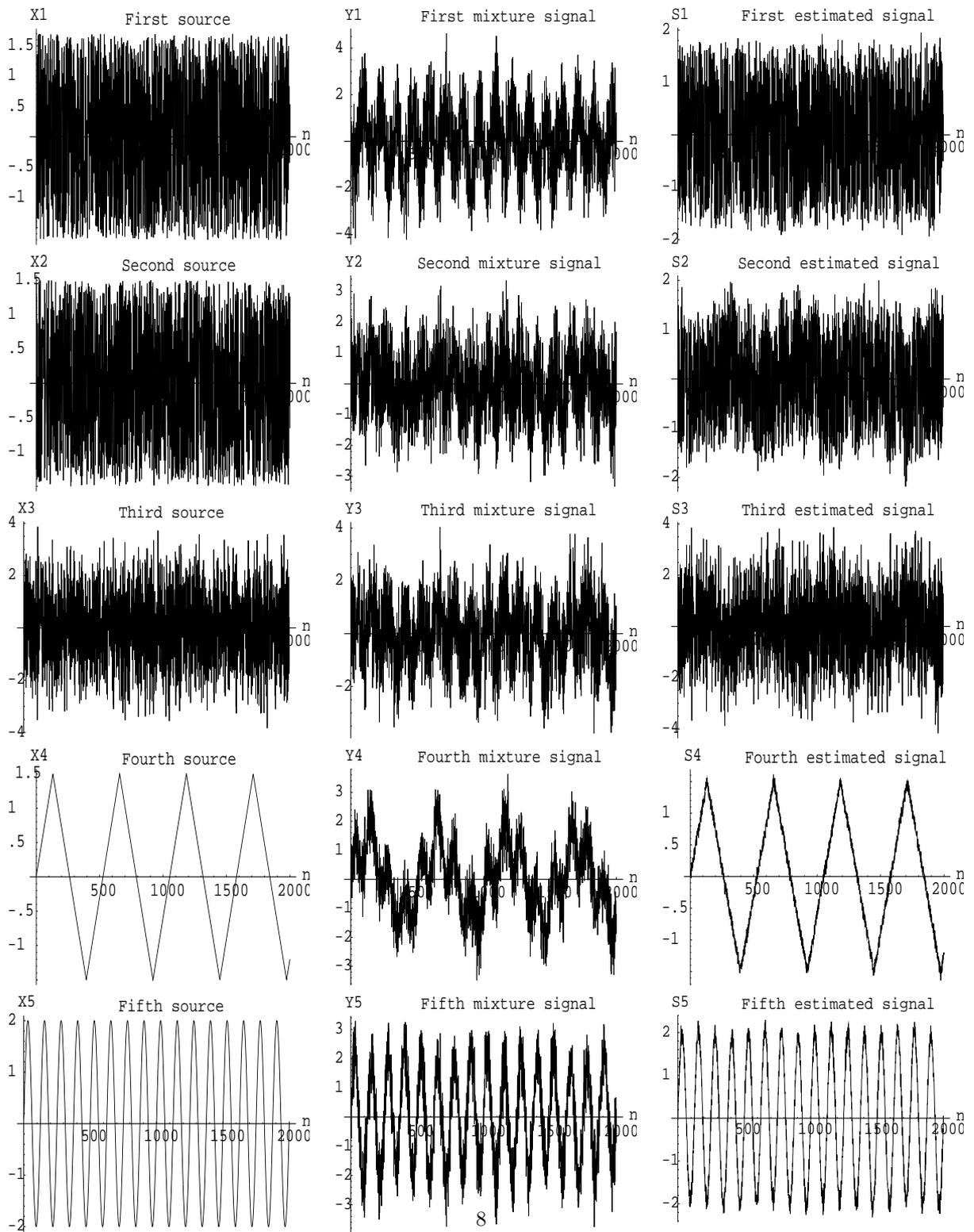


Figure 3: The separation of five stationary signals: First column contains the sources, the second column contains the mixture signals and the last one contains the estimated signals.

Unfortunately, in the case of nonstationary signals, the performance of the actual version of our algorithm is not satisfactory when the number of sources is greater than 3. However, satisfactory results are obtained (the crosstalk is about -20 dB) in the case of two nonstationary sources (see figures 4 and 5).

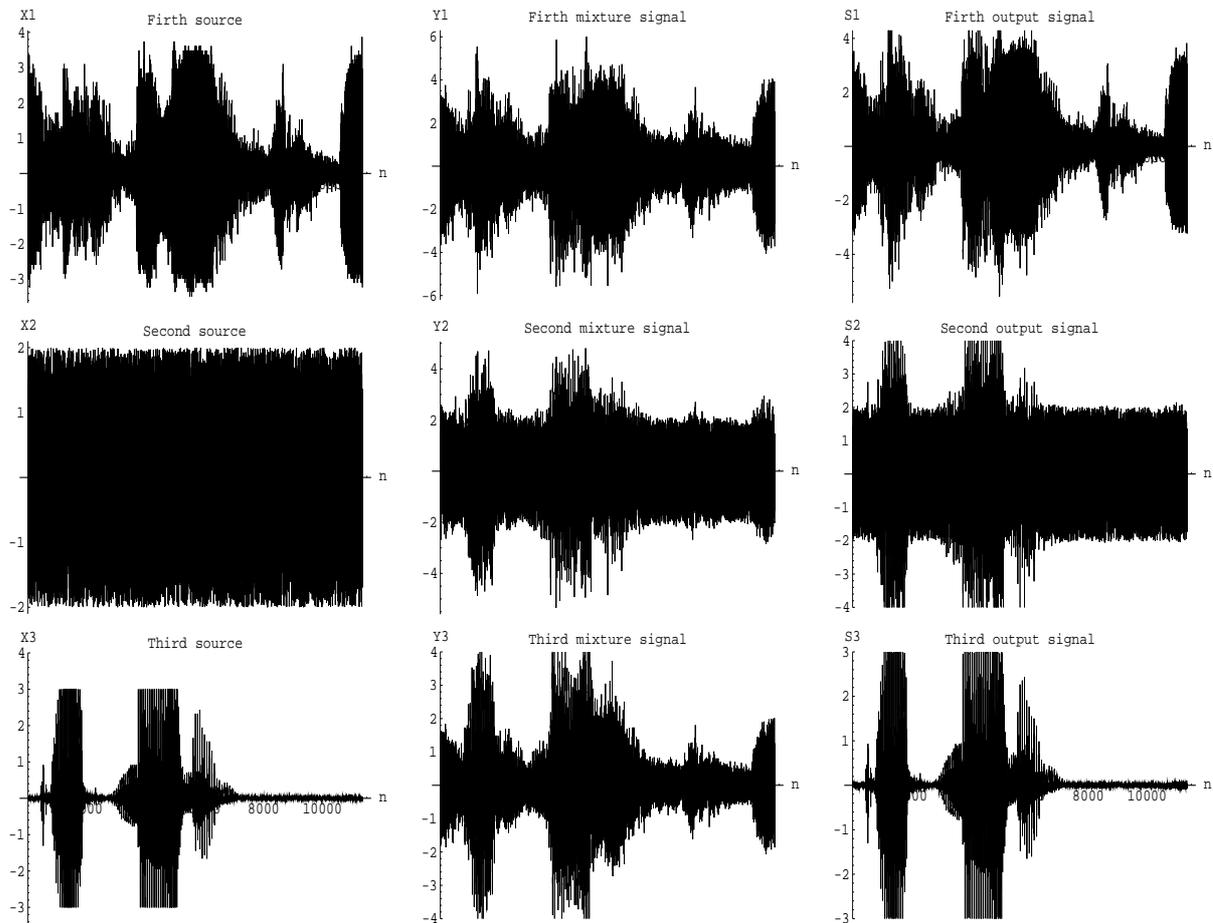


Figure 4: Nonstationary signals: First column contains the sources, the second column contains the mixture signals and the last one contains the estimated signals.

## 7 Conclusion

In this paper, a fast algorithm for blind separation of sources is proposed. This algorithm minimizes a cost function based on the cross-cumulant (2x2), using

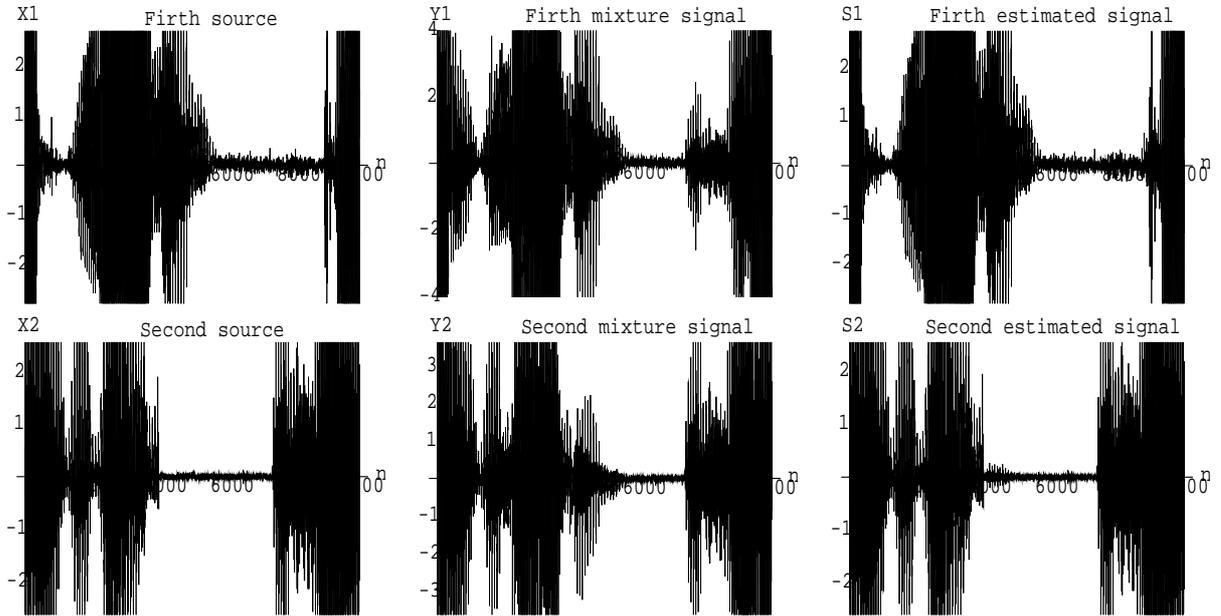


Figure 5: Two nonstationary signals (the sources are: an English word "Good Morning" and a Japanese Word "Ohayagosaimas").

the Levenberg-Marquardt method. The experimental study proves that the convergence is very fast (in general cases less than 50 iterations are needed to attempt the convergence).

In many experiments, good results are obtained even if a small number of samples are used (for stationary signals, the algorithm may converge by using only about 300 samples). Also in the case of stationary signals, this algorithm can separate more than two sources.

Up to now, we obtained satisfactory results in the case of two nonstationary signals: the crosstalk is about -20 dB. In the general case, when the sources are more than two signals, this algorithm may not converge due to the facts that

1. There may be silent periods in the speech signal, and
2. The statistics of the speech signal depend on time and on the wide of the estimation windows.

Actually, we plan to improve this algorithm to separate more than two nonstationary sources.

## A Evaluation of the cross-cumulant (2x2)

The  $Cum_{22}(s_m, s_n)$  of two zero-mean signals is given by [19]:

$$Cum_{22}(s_m, s_n) = E(s_m^2 s_n^2) - E(s_m^2)E(s_n^2) - 2E(s_m s_n)^2. \quad (15)$$

Equation (2) without noise makes  $s_m = \sum_i g_{(m,i)} x_i$ . To evaluate the cross-cumulant (15), the mathematical expectation  $E(s_m^2 s_n^2)$  is calculated at first:

$$E(s_m^2 s_n^2) = E\left(\left(\sum_i g_{(m,i)} x_i\right)^2 \left(\sum_i g_{(n,i)} x_i\right)^2\right) = \sum_{i,j,k,l} g_{(m,i)} g_{(m,j)} g_{(n,k)} g_{(n,l)} E(x_i x_j x_k x_l). \quad (16)$$

The sources are assumed zero-mean independent signals. As a consequence, we have:

$$E(x_i x_j x_k x_l) = \begin{cases} P_i P_k & \begin{cases} \text{if } i = j \neq k = l \\ \text{or } i = l \neq j = k \end{cases} \\ P_i P_j & \text{if } i = k \neq j = l \\ \gamma_i & \text{if } i = j = k = l \\ 0 & \text{otherwise.} \end{cases} \quad (17)$$

Where  $P_i = E(x_i^2)$ , and  $\gamma_i = E(x_i^4)$ . From equation (16) and (17), we prove that:

$$E(s_m^2 s_n^2) = \sum_i g_{(m,i)}^2 g_{(n,i)}^2 \gamma_i + \sum_{i \neq j} P_i P_j g_{(m,i)} g_{(n,j)} (g_{(m,i)} g_{(n,j)} + 2g_{(n,i)} g_{(m,j)}). \quad (18)$$

It is easy to prove that the second-order moment of the output signals are:

$$E(s_m^2) = \sum_i g_{(m,i)}^2 P_i \quad (19)$$

$$E(s_m s_n) = \sum_i g_{(m,i)} g_{(n,i)} P_i \quad (20)$$

Finally, we can prove, using equations (18), (19) and (20), that:

$$Cum_{22}(s_m, s_n) = \sum_i g_{(m,i)}^2 g_{(n,i)}^2 \beta_i. \quad (21)$$

Where  $\beta_i = Cum(x_i, x_i, x_i, x_i)$  is the fourth-order cumulant of the source signal. Let us assume that the source have the same sign of kurtosis (or the same sign of the fourth-order cumulant  $\beta_i$ ) and using the relation (21), it is easy to prove that:

$$Cum_{22}(s_m, s_n) = 0 \iff g_{(m,i)} g_{(n,i)} = 0 \quad (22)$$

where  $1 \leq i \leq N$ . Equation (22) is symmetrical with respect to  $m$  and  $n$ . As a consequence, we consider just the case which  $1 \leq n < m \leq N$ . Finally, using the definition of the cost function (3), we can prove:

$$Cost = 0 \iff g_{(m,j)} g_{(n,j)} = 0 \quad (1 \leq j \leq N) \quad \text{and } m \neq n \quad (23)$$

## B The Levenberg-Marquardt method

The Levenberg-Marquardt algorithm [20] can be thought of as a trust-region modification of the Gauss-Newton algorithm. It can minimize the non linear least squares problem:

$$0 = \min_V \{f(V) = \frac{1}{2} \sum_i \phi_i^2(V) \mid V \in \mathbb{R}^n\} \quad (24)$$

where  $V = (v_1, v_2, \dots, v_n)^T$ . And let  $\Phi = (\phi_1, \phi_2, \dots, \phi_m)^T$ , and  $\lambda_0 = 10^{-3}$ . Finally let us denote by  $\mathbf{J}(V)$  the  $m \times n$  Jacobian matrix of  $\Phi$ :

$$\mathbf{J}(V) = \begin{pmatrix} \frac{\partial \phi_1}{\partial v_1} & \frac{\partial \phi_1}{\partial v_2} & \cdots & \frac{\partial \phi_1}{\partial v_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \phi_m}{\partial v_1} & \frac{\partial \phi_m}{\partial v_2} & \cdots & \frac{\partial \phi_m}{\partial v_n} \end{pmatrix} \quad (25)$$

The Hessian of (24) is a combination of first and second-order terms [24]:

$$\nabla^2 f(V) = \mathbf{J}(V)^T \mathbf{J}(V) + \sum_j \phi_j \nabla^2 \phi_j(V). \quad (26)$$

In practice the Gauss-Newton approximation of the Hessian is used, i.e.  $\mathbf{H}(V) = \mathbf{J}(V)^T \mathbf{J}(V)$ .

The  $k$ th search direction is defined as the solution of:

$$[\mathbf{H}(V_{k-1}) + \lambda_{k-1} \mathbf{I}] D_k = -\mathbf{J}(V_{k-1})^T \Phi(V_{k-1}) \quad (27)$$

and  $V_k = V_{k-1} + D_k$ . Finally:

- If  $f(V_k) < f(V_{k-1})$  than  $\lambda_k = \lambda_{k-1}/2$  and  $V_k$  takes the place of  $V_{k-1}$  for the next iteration.
- Else  $\lambda_k = 2\lambda_{k-1}$  and we must use the same value of  $V_{k-1}$  for the next iteration.

The algorithm will be stopped when  $f(V_k)$  is very small or  $\lambda_k$  is very large.

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