Efficient spectrum sensing approaches based on waveform detection

A. Nasser*†, A. Mansour†, K. C. Yao*, H. Charara‡, and M. Chaitou ‡

* LABSTICC, UBO, 3 Rue des Archives, 29238 Brest, France
† LABSTICC, ENSTA Bretagne, 2 Rue François Verny, 29200 Brest, France
‡ Lebanese University, Faculty of Science, Beirut, Lebanon
Email: abbas.nasser@univ-brest.fr, mansour@ieee.org, koffi-clement.yao@univ-brest.fr
hussein.charara@ul.edu.lb, mohamad.chaitou@ul.edu.lb
Web Page: www.lab-sticc.fr; www.ul.edu.lb; ali.mansour.free.fr

***Abstract*—Spectrum Sensing is widely used in smart or cognitive radio transmission system in order to allocate unused bandwidth by a primary user to a secondary user. The allocation scheme depends on determining a threshold reflecting the existence or not of the primary user. This manuscript deals with this problem by proposing two major contributions: the first one is a novel mechanism to calculate the threshold based on a known distribution of the correlation function between the pilot and the received signal. Our main finding is that the threshold could be, in some circumstances, independent from the SNR which relieves the detector from processing threshold updates in case when the SNR frequently varies. In the second contribution we use the Waveform technique in order to detect the existing or not of Primary user signals while a secondary user is transmitting without interrupting the detection mechanism of the primary user. Contrary to existing methods, which require a silence period of secondary users in order to sense the activity of the primary user, our approach does not need this period which enhances the total transmission rate.**
Our simulation results corroborate the two proposed approaches. Simulation results are presented and discussed.

***Index Terms*—Cognitive Radio; Spectrum Sensing; Waveform Detection.**

I. Introduction

Spectrum sensing is an essential part of the Cognitive Radio (CR) system, it deals with the status of the transmission: when the PU is absent, then a SU may transmit.

In fact, many techniques have been proposed in order to perform the Spectrum Sensing, such that Energy Detection (ED), Cyclo-Stationary Detection (CSD), Waveform Detection (WF),... [1]. These methods take a decision about the presence of PU basing on the comparison of a metric derived by each method, to a pre-defined threshold, this threshold depends on the noise and the signal model and the signal to noise ratio (SNR). In addition, conventional methods such as ED, WF, CSD etc., cannot sense the channel while SU being in operation. As the SU should pause during examination step, the transmission rate decreases. The Blind source separation (BSS) [2], was introduced as solution of such problem [3], since the BSS can separate the mixtures of independent signals by applying Independent Component Analysis [4] [5].

Being the focused technique in this paper, the WF is an optimal method [1], it achieves good performance even at a low SNR. Hence, the SU should know the waveform of PU signal or pilot [1] [6] [7] and correlate it with the received signal. The pilot is a simple signal transmitted by the PU as a signature signal. The pilot could be a simple sine wave tone [6]. However, in many technologies, a pilot tone is sent from the PU transmitter, so SU can use this pilot to detect the PU activity. In this case, SU should know the form of this tone signal.

In this article, we present two approaches based on WF, in the first, the comparison threshold used in WF is set by admitting a new criterion test, called Range Decision Test, to detect the presence of the PU signal, this threshold does not need to be updated according to the SNR. By admitting this threshold, the detector achieves a good probability of detection p_d with a very small probability of false alarm p_{fa}. The second approach is introduced to allow the SU to sense the status of channel while he is transmitting by using WF, so, the SU's rate increases. A study is developed in order to maintain the optimal threshold of comparison, this approach leads to a good detection without performance loss with respect to classical WF methods.

II. Modified waveform approach

In this section, modified WF approaches are presented. Usually, the WF method consists of evaluating the projection of the received signal over a known pilot signal. The pilot signal being orthogonal to the data signal, the signal transmitted by the PU, $y(n)$ can be written as:

$$y(n) = y_t(n) + y_d(n) \tag{1}$$

where $y_t(n)$ stands for the pilot, $y_d(n)$ represents the data signal, $n = 1, 2, ..., M$ is the sample index, and M denotes the total number of observed samples.
The detection problem of the PU can be reduced to a binary detection problem as follows:

$$\begin{cases} H_0 & : x(n) = w(n) \\ H_1 & : x(n) = y_t(n) + w(n). \end{cases}$$

Where $x(n)$ is the received signal (In fact, $y_d(n)$ can be ignored thanks to the projection of $x(n)$ over $y_t(n)$ as $y_d(n)$ and $y_t(n)$ are orthogonal.), and $w(n)$ is an additive white Gaussian noise (AWGN), with a zero mean, and a variance of σ^2. Under the hypothesis H_0, the PU is absent and the SU can be activated. When, PU becomes active, SU must immediately vacate the channel, in order to avoid any interference.

The projection, R, of $x(n)$ onto $y_t(n)$ under the two hypotheses can be evaluated as follows:

$$\begin{cases} H_0 & : R = \mathrm{Re}\{\sum_n y_t^*(n)w(n)\} \\ H_1 & : R = \mathrm{Re}\{\sum_n |y_t(n)|^2 + \sum_n y_t^*(n)w(n)\} \end{cases}$$

Where x^* and $\mathrm{Re}\{x\}$ stand respectively for the conjugate and the real part of X. When $y_t(n)$ is a real signal, R can be simplified as $R = \sum_n y_t(n)x(n)$.

R should be compared to a threshold γ in order to make a decision about the existing of PU.

$$\begin{cases} R < \gamma & : PU \nexists \\ R \geq \gamma & : PU \exists \end{cases}$$

In the following, the pilot $y_t(n)$ is assumed to be a sine wave tone, without any loss of generality. In fact, the sine wave tone pilot was studied in [6], where $y_t(n)$ is affected by AWGN. Under the two hypotheses H_0 and H_1, it is easy to prove that R follows a normal distribution [6].

$$\begin{cases} H_0 & : R \backsim \mathcal{N}(0, \alpha\sigma^2) \\ H_1 & : R \backsim \mathcal{N}(\alpha, \alpha\sigma^2) \end{cases}$$

where $\alpha = \sum_n (y_t)^2$ stands for the estimated energy of the pilot signal. As R has a normal distribution, the probability of false alarm, p_{fa}, and the probability of missed detection, p_{md}, can be given using the Pearson-Neyman detection technique, as follows [6]:

$$P_{fa} = Q(\frac{\gamma}{\sqrt{\alpha\sigma^2}}) \tag{2}$$

$$P_{md} = 1 - Q(\frac{\gamma - \alpha}{\sqrt{\alpha\sigma^2}}) \tag{3}$$

Where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{\frac{-t^2}{2}} dt$ is the Q-function.

A. Range Decision Test

Normally, the presence of a PU can be achieved by comparing a projection index to a pre-selected threshold as shown in the previous section. Hereinafter, a new threshold is presented, basing on new decision test. This criterion is called Range Decision Test based WF (RDT-WF). In fact, our approach is based on the knowledge of the mean and the variance of the distribution of R. Indeed, instead of using a classical threshold-based WF (CT-WF) to make a decision, our criterion consists in that R belongs under H_0 to $D_0 = [B_{0l}; B_{0u}]$ and under H_1 to $D_1 = [B_{1l}; B_{1h}]$, where B_{il} is the lower bound of D_i, and B_{iu} is the upper bound, $i = 0, 1$. The values of B_{il} and B_{iu}, $i = 0, 1$, should be set using the variance of R under the hypotheses H_0 and H_1:

$$B_{0l} = -a\alpha\sigma^2; \; B_{0u} = a\alpha\sigma^2 \tag{4}$$

$$B_{1l} = (1 - a\sigma^2)\alpha; \; B_{1u} = (1 + a\sigma^2)\alpha \tag{5}$$

Where a is a positive real number.

The values of bounds of the range D_0 are chosen to make D_0 centred at zero, which is the mean of R under H_0, and for a similar reason, the bounds of D_1 are set as this form. In addition, this is to make the two ranges symmetric. In fact, a must satisfy the following condition to avoid the overlapping between the interval D_0 and D_1 (see figure 1):

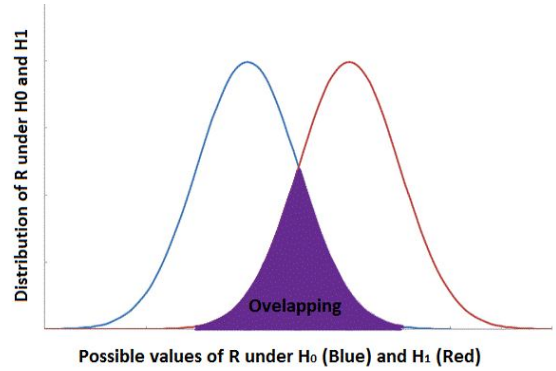

Fig. 1. Overlapping between D_0 and D_1.

$$(1 - a\sigma^2)\alpha > a\alpha\sigma^2 \Rightarrow a < \frac{1}{2\sigma^2} \tag{6}$$

Hereinafter, P_1 is defined as the probability that R belongs to D_1 under H_1.

$$P_1 = Pr((1 - a\sigma^2)\alpha \leq R \leq (1 + a\sigma^2)\alpha) \tag{7}$$

By applying the change of variable : $T = \frac{R - \alpha}{\sqrt{\alpha}\sigma}$, $T \sim \mathcal{N}(0, 1)$ is obtained. Then,

$$P_1 = 1 - 2Q(\epsilon) \tag{8}$$

Where $\epsilon = a\sqrt{\alpha}\sigma$. With a similar procedure, P_0 is the probability that R is in D_0 under H_0:

$$P_0 = 1 - 2Q(\epsilon) \tag{9}$$

is obtained.

For a given $P_0 = p$, (or $P_1 = p$), $\epsilon_0 = Q^{-1}(\frac{1-p}{2})$ is found. It is simple to obtain:

$$a = \frac{\epsilon_0}{\sqrt{\alpha}\sigma} \tag{10}$$

However, a must satisfy the conditions of equation (6). It is clear that a is always positive, then a should be less than $\frac{1}{2\sigma^2}$ to avoid the overlapping between D_0 and D_1.

This condition leads us to a relation between the number of samples M and the SNR.

$$M > \frac{4\epsilon^2}{SNR} \tag{11}$$

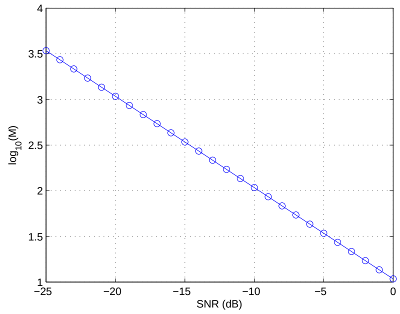

Fig. 2. Variation of the required number of samples in terms of SNR.

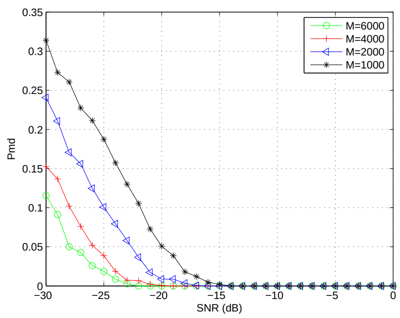

Fig. 3. p_{md} for RDT-WF approach for various number of samples

Figure 2 shows the minimum values should be taken by M for $p = 0.9$, in terms of SNR.

At low SNR, M should be of high value, this is because M is inversely proportional to the SNR.

Since D_0 and D_1 are separated, and symmetric, it is easy to set a threshold λ such as:

$$\lambda = \frac{B_{0u} + B_{1l}}{2} = \frac{\alpha}{2} \tag{12}$$

The decision about the PU follows the two hypotheses:

$$\begin{cases} R < \frac{\alpha}{2} & : PU \nexists \\ R \geq \frac{\alpha}{2} & : PU \exists \end{cases}$$

The p_d and p_{fa} can be evaluated as follow:

$$p_{fa} = Pr(R > \lambda | H0) = Q(\frac{\sqrt{\alpha}}{2\sigma}) \tag{13}$$

$$p_d = Pr(R > \lambda | H0) = Q(\frac{\frac{\alpha}{2} - \alpha}{\sqrt{\alpha}\sigma}) = Q(\frac{-\sqrt{\alpha}}{2\sigma}) = 1 - p_{fa} \tag{14}$$

Figure 3 shows the results of our simulations for the probability of missed detection $p_{md} = 1 - p_d = p_{fa}$, under the threshold $\lambda = \frac{\alpha}{2}$, for several values of the number of samples.

It is clear that the p_{md} tends quickly to zero when M is increasing, and it satisfies the inequality of equation (11). Moreover, this threshold is fixed even if SNR varies, so, the detector do not need to update it.

B. Proposed WF

The new proposed approach allows us to detect the status of channel even if SU is in operation. At first, a unique PU, and SU are assumed to cooperate. In this case, if SU is active, we can distinguish between following hypotheses:

$$\begin{cases} H_0^s & : x(n) = s(n) + w(n) \\ H_1^s & : x(n) = y_t(n) + s(n) + w(n). \end{cases}$$

where s is the secondary user signal, which is assumed to be zero mean with a variance equal to σ_s^2. In addition, $s(n)$ and $y_t(n)$ are assumed to be statistically independent.

In this new scenario, H_0^s and H_1^s stand for the no existing and the existing of the PU, when the SU is transmitting, respectively. Under H_1^s, SU should immediately stop the transmission, in order to avoid the interference. Under these two new hypotheses, let us define the correlation R_s between $x(n)$ and $y_t(n)$ as follows:

$$\begin{cases} H_0^s & : R_s = \sum_n \{y_t(n)s(n) + y_t(n)w(n)\} \\ H_1^s & : R_s = \sum_n \{(y_t(n))^2 + y_t(n)s(n) + y_t(n)w(n)\}. \end{cases}$$

The new term $\sum_n y_t(n)s(n)$, in the two above equations, requires a new threshold γ_s to be compared with R_s. To determine the new threshold γ_s under H_0^s, the mean μ_0 and the variance σ_0^2 of R_s should be evaluated.

Using the independence and the zero mean assumptions, we can show that:

$$\begin{aligned} \mu_0 = E[R_s] = E[\sum_n y_t(n)s(n) + \sum_n y_t(n)w(n)] = \\ \sum_n E[y_t(n)s(n)] + \sum_n E[y_t(n)w(n)] = \\ \sum_n E[y_t(n)]E[s(n)] + \sum_n E[y_t(n)]E[w(n)] = 0. \end{aligned} \tag{15}$$

As $\mu_0 = 0$, the variance σ_0^2 of R_s becomes:

$$\begin{aligned} \sigma_0^2 = E[{R_s}^2] - E^2[R_s] = \\ = E[(\sum_n y_t(n)s(n) + y_t(n)w(n))^2] \\ = \sum_{n,m} E[y_t(n)y_t(m)]E[s(n)s(m)] + \\ \sum_{n,m} E[y_t(n)y_t(m)]E[w(n)w(m)] + \\ 2\sum_{n,m} E[y_t(n)y_t(m)]E[s(n)]E[w(m)] \end{aligned} \tag{16}$$

Suppose that $s(n)$ is a white signal [8]: $E[s(n)s(m)] = \sigma_s^2 \delta_n^m$, where δ_n^m is the Kronecker symbol. In this case, equation (II-B) can be simplified as follows:

$$\begin{aligned} \sum_{n,m} E[y_t(n)y_t(m)]\sigma_s^2\delta_n^m + \sum_{n,m} E[y_t(n)y_t(m)]\sigma^2\delta_n^m \\ = \sum_n \frac{\alpha}{M}\sigma_s^2 + \sum_n \frac{\alpha}{M}\sigma^2 = \alpha(\sigma^2 + \sigma_s^2) \end{aligned} \tag{17}$$

The mean μ_1 and the variance σ_1^2 of R_s under H_1^s are

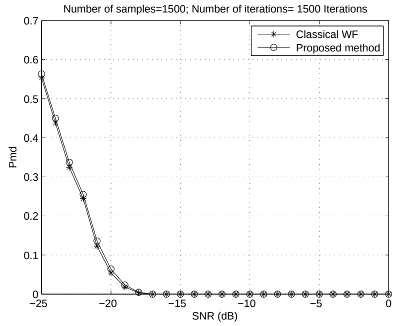

Fig. 4. New approach Vs. classical WF under $p_{fa} = 0.05$

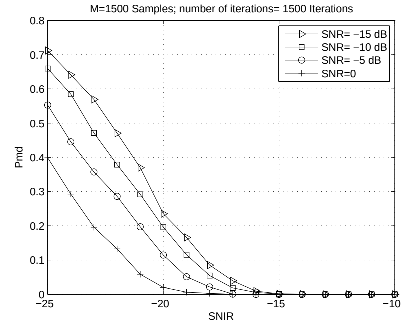

Fig. 5. Performance of proposed approach for various SINR (dB)

calculated as well as H_0^s. In fact, by following the same process, we obtain: $\mu_1 = \alpha$ and $\sigma_1^2 = \alpha(\sigma_s^2 + \sigma^2)$

Therefore R_s has under H_0^s and H_1^s a normal distribution:

$$\begin{cases} H_0^s & : R_s \backsim \mathcal{N}(0, \alpha(\sigma_s^2 + \sigma^2)) \\ H_1^s & : R_s \backsim \mathcal{N}(\alpha, \alpha(\sigma_s^2 + \sigma^2) \end{cases}$$

Then P_{fa} and P_d can be evaluated (See Appendix A):

$$P_{fa} = Q(\frac{\gamma_s}{\sqrt{\alpha(\sigma_s^2 + \sigma^2)}}) \tag{18}$$

$$P_d = Q(\frac{\gamma_s - \alpha}{\sqrt{\alpha(\sigma_s^2 + \sigma^2)}}) \tag{19}$$

III. Simulation

In this section, our simulation results of of the modified WF approach under various conditions are presented. This approach is compared to classical WF when no SU signal exist.

Figure 4 shows that there is no degradation of performance for p_{md} when the new approach is applied. Add to the necessity of knowing the pilot of PU, SU should measure the variance of s, that is easy to be realized.

The problem of an interference for PU, and its impact to the detection of y_t is discussed. The signal to interference and noise ratio (SINR) is defined as SINR $= \frac{P_{y_t}}{P_s + \sigma^2}$, where P_{y_t} and P_s stand for the pilot signal power and the SU signal power respectively. Figure 5 presents the results of simulations for change of SINR between -25 dB and -10 dB, for different SNR (-15 dB, -10dB; -5 dB, and 0 dB), The number of samples is set to be 1500 samples, with 1500 iterations.

As shown in figure 5, p_{md} decreases when SINR increases, once the effect of the interference is reduced, the decision test becomes more reliable. In addition, for the different values of SNR, this approach is exposed to degradation for SINR $\leq -17dB$. But at same time, this approach maintains its performance even at very low SINR, wherein p_{md} near zeros when SINR equal to -15 dB.

IV. Conclusion

In this paper, we presented two approaches based on the waveform detection method in spectrum sensing. The first approach is called "Range Decision Test", in which a new criterion is used to establish the comparison threshold; it is based on the distribution of the correlation function between the pilot and the received signal, i.e. R, under H_0 (i.e. PU does not exist) and H_1 (i.e. PU exists). Indeed, we suppose that the projection of the received signal over the pilot has two possible ranges: in the first range the PU is absent, while in the second range the PU is active. Under certain number of samples, the separation of these two ranges is achieved, which helps to define a fixed threshold, which is not dependent on SNR, in order to make the decision about the presence of PU.

The second approach called "Modified Waveform Detection", deals with the possibility of detection of the PU signal even if the SU is in operation without the need of a silent period. A development was done in order to set the new threshold of comparison.The simulation results show that this approach can sense the channel even if SU is active, and then, it increases the transmission rate of SU, without loss of performance relatively to classical Wave Form.

Appendix A

p_{fa} and p_d under a normal distribution of R_s could be calculated as follow:

$$\begin{aligned} p_d &= Pr(R_s \geq \gamma_s | H_1) = \\ &Pr(\frac{R_s - \alpha}{\sqrt{\alpha(\sigma^2 + \sigma_s^2)}} \geq \frac{\gamma_s - \alpha}{\sqrt{\alpha(\sigma^2 + \sigma_s^2)}} | H_1) = \\ &1 - F_T(\frac{\gamma_s - \alpha}{\sqrt{\alpha(\sigma^2 + \sigma_s^2)}}) = Q(\frac{\gamma_s - \alpha}{\sqrt{\alpha(\sigma^2 + \sigma_s^2)}}). \end{aligned} \tag{20}$$

Where F_T is the cumulative distribution function.

$$\begin{aligned} p_{fa} &= Pr(R_s \geq \gamma_s | H_0) = \\ &Pr(\frac{R_s}{\sqrt{\alpha(\sigma^2 + \sigma_s^2)}} \geq \frac{\gamma_s}{\sqrt{\alpha(\sigma^2 + \sigma_s^2)}} | H_0) = \\ &1 - F_T(\frac{\gamma_s}{\sqrt{\alpha(\sigma^2 + \sigma_s^2)}}) = Q(\frac{\gamma_s}{\sqrt{\alpha(\sigma^2 + \sigma_s^2)}}). \end{aligned} \tag{21}$$

References

[1] T. Yucek and H. Arslan, *A Survey of Spectrum Sensing Algorithms for Cognitive Radio Applications*, IEEE COMMUNICATIONS SURVEYS & TUTORIALS, vol. 11, NO. 1, FIRST QUARTER 2009.

[2] A. Mansour, A. K. Barros, and N.Ohnishi, *Blind Separation of Sources: Methods, Assumptions and Applications*, IEICE TRANS. FUNDAMENTALS, VOL. E83-A, NO. 8 AUGUST 2000.

[3] S. S. Ivrigh, S. M. Sadough and S. A. Ghorashi, *A Blind Source Separation Technique for Spectrum Sensing in Cognitive Radio Networks Based on Kurtosis Metric,* ICCKE 2011, International Conference on Computer and Knowledge Engineering, p. 333 - 337, Ferdowsi University of Mashhad, Mashhad, Iran, october 2011.

[4] A. Mansour, J. Youssef, and K. C. Yao, *Underdetermined BSS of MISO OSTBC signals, Independent Component Analysis and Blind source Separation*, Lecture Notes in Computer Science 5441, p. 678-685, Paraty, Brazil, March 2009.

[5] A. Mansour, M. Kawamoto, *ICA Papers Classified According to their Applications and Performances*, IEICE TRANS. FUNDAMENTALS, VOL.E86-A, NO.3 MARCH 2003

[6] D. Cabric, A. Tkachenko and R. W. Brodersen, *Spectrum Sensing Measurements of Pilot, Energy, and Collaborative Detection, Military Communications Conference*, MILCOM 2006,p. 1 - 7, Washington DC, Oct. 2006.

[7] H. Tang, *Some Physical Layer Issues of Wide-band Cognitive Radio Systems*, DySPAN, p. 8-11, Baltimore, MD, USA, Nov. 2005.

[8] J. K. Tugnait, *ON Autocorrelation-based multiantenna spectrum sensing for cognitive radios in unknown noise*, IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), p. 2944 - 2947, Prague 2011.