

A DEFLATION ALGORITHM FOR THE BLIND DECONVOLUTION OF MIMO-FIR CHANNELS DRIVEN BY FOURTH-ORDER COLORED SIGNALS

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ABSTRACT

In this paper, we propose a new iterative algorithm to solve the blind deconvolution problem of MIMO-FIR channels driven by source signals which are temporally second-order uncorrelated but fourth-order correlated and spatially second- and fourth-order uncorrelated. In our new approach, to solve the blind deconvolution problem, we consider two stages: First, filtered source signals are extracted from the mixtures of source signals. Second, the source signals are recovered from the filtered source signals.

1. INTRODUCTION

The blind deconvolution problem consists of extracting source signals from their convolutive mixtures observed by sensors without knowledge about the source signals and about the transfer functions (transmission channels) between the sources and the sensors.

The blind deconvolution problem has been studied by many researchers (e.g., [1, 2, 3, 5, 6]). Almost all of the proposed methods to date have been developed under the assumption that the source signals are temporally independent and identically distributed (i.i.d.) and spatially independent (e.g., [1, 2, 6]). However, in some applications, the i.i.d. assumption for the source signals becomes very strong (e.g., applications in digital communications [4]). To solve the blind deconvolution problem for such applications, therefore, one must assume that the source signals have a weaker condition than the i.i.d. condition, for example, the source signals are temporally second-order uncorrelated but higher-order correlated [3, 5].

Here we propose a new iterative algorithm to achieve the blind deconvolution of MIMO-FIR channel systems driven by source signals which are temporally high-order colored signals (but temporally second-order white and spatially second- and fourth-order uncorrelated signals). To do that, we consider a deflation approach. Algorithms based on deflation approaches have been

used to achieve blind deconvolution under the assumption that the source signals are i.i.d. and spatially independent [2, 6]. However, it is not clear whether the deflation approach can be applied to the MIMO-FIR channels in the case that the sources are fourth-order colored signals. It has been shown by Simon et al. [5] that the deflation approach can be applied to MIMO-IIR channels in the case that source signals are colored signals (but white signals in the sense of second-order statistics). Their proposed method cannot solve blind deconvolution problem but solve a blind signal generation problem in which filtered source signals are extracted from the mixtures of the sources.

Our new deflation algorithm is a modification of the super-exponential deflation algorithm proposed by Inouye and Tanebe [2] to the case of the blind deconvolution problem of an MIMO-FIR channel driven by the fourth-order colored signals. In our approach, we should consider two stages to recover one source signal from the output of an multiple-input single-output finite impulse response (MISO-FIR) system: First, a *cascaded integrator-comb (CIC) filter* is acquired. It implies that one filtered source signal is generated from the mixtures of the source signals. Secondly, by making the filtered signal be white in the sense of second-order statistics, the source signal can be recovered from the CIC filtered source signal.

2. PROBLEM FORMULATION

We consider the following MIMO-FIR system:

$$\mathbf{x}(t) = \sum_{k=0}^K \mathbf{H}^{(k)} \mathbf{s}(t-k), \quad (1)$$

where $\mathbf{x}(t)$ represents an m -column output vector called the *observed signal*, $\mathbf{s}(t)$ represents an n -column input vector called the *source signal*, $\{\mathbf{H}^{(k)}\}$ is an $m \times n$ matrix sequence representing the impulse response of the transmission channel, and the number K denotes its

order. Equation (1) can be written as

$$\mathbf{x}(t) = \mathbf{H}(z)\mathbf{s}(t), \quad (2)$$

where $\mathbf{H}(z)$ is the z -transform of the transfer function, i.e.

$$\mathbf{H}(z) = \sum_{k=0}^K \mathbf{H}^{(k)} z^k.$$

In the above, we note that we use variable z instead of variable z^{-1} in the z -transform.

Here, let us consider the following FIR system called a *filter* which is driven by the observed signals.

$$\mathbf{y}(t) = \sum_{k=0}^L \mathbf{W}^{(k)} \mathbf{x}(t-k), \quad (3)$$

where $\mathbf{y}(t)$ is an q -column vector representing the output signal of the filter, $\{\mathbf{W}^{(k)}\}$ is an $q \times m$ matrix sequence, and the number L is the order of the filter. Equation (3) can be written as

$$\mathbf{y}(t) = \mathbf{W}(z)\mathbf{x}(t), \quad (4)$$

where $\mathbf{W}(z)$ is the transfer function of the filter defined by

$$\mathbf{W}(z) = \sum_{k=0}^L \mathbf{W}^{(k)} z^k.$$

Substituting (2) into (4), we have

$$\mathbf{y}(t) = \mathbf{G}(z)\mathbf{s}(t), \quad (5)$$

where

$$\mathbf{G}(z) := \mathbf{W}(z)\mathbf{H}(z) = \sum_{k=0}^{K+L} \mathbf{G}^{(k)} z^k. \quad (6)$$

In this paper, we consider the two types of filters: $q = n$ and $q = 1$. When $q = n$, we can formulate the blind deconvolution as follows: Find a filter $\mathbf{W}(z)$, called an equalizer, satisfying the following condition, without the knowledge of $\mathbf{H}(z)$,

$$\mathbf{W}(z)\mathbf{H}(z) = \mathbf{P}\mathbf{D}\mathbf{\Lambda}(z), \quad (7)$$

where \mathbf{P} is an $n \times n$ permutation matrix, \mathbf{D} is an $n \times n$ regular diagonal matrix, and $\mathbf{\Lambda}(z)$ is an $n \times n$ regular diagonal matrix with diagonal entries being monic monomials. We consider the type $q = 1$ when we want to extract one filtered source signal from the mixtures of the source signals.

In order to solve the blind deconvolution problem, as the first stage, we consider the blind signal generation problem mentioned below, in which CIC filtered source signals are generated from the observed signals.

The composite system (5) can be written in scalar form as

$$y_i(t) = \sum_{j=1}^n \sum_{k=0}^{K+L} g_{ij}^{(k)} s_j(t-k), \quad (8)$$

where

$$g_{ij}^{(k)} = \sum_{l_1=1}^m \sum_{\tau=0}^L w_{il_1}^{(\tau)} h_{l_1j}^{(k-\tau)}, \quad (9)$$

Here $i = 1, \dots, q$, $j = 1, \dots, n$, and $k = 0, 1, \dots, K+L$. The set of equations (8) can be written in vector notation as

$$y_i(t) = \tilde{\mathbf{g}}_i^T \tilde{\mathbf{s}}(t), \quad (10)$$

where the superscript T denotes the transpose of a vector, and $\tilde{\mathbf{s}}(t)$ is the column vector defined by

$$\tilde{\mathbf{s}}(t) := [\tilde{\mathbf{s}}_1(t)^T, \tilde{\mathbf{s}}_2(t)^T, \dots, \tilde{\mathbf{s}}_n(t)^T]^T, \quad (11)$$

$$\tilde{\mathbf{s}}_i(t) := [s_i(t), s_i(t-1), \dots, s_i(t-K-L)]^T, \quad (12)$$

and $\tilde{\mathbf{g}}_i$ is the column vector consisting of the i th output impulse response of the cascade system defined by

$$\tilde{\mathbf{g}}_i := [\tilde{\mathbf{g}}_{i1}^T, \tilde{\mathbf{g}}_{i2}^T, \dots, \tilde{\mathbf{g}}_{in}^T]^T, \quad (13)$$

$$\tilde{\mathbf{g}}_{ij} := [g_{ij}^{(0)}, g_{ij}^{(1)}, \dots, g_{ij}^{(K+L)}]^T. \quad (14)$$

Using (13), (9) can be written in vector notation as

$$\tilde{\mathbf{g}}_i = \tilde{\mathbf{H}} \tilde{\mathbf{w}}_i, \quad i = 1, 2, \dots, q, \quad (15)$$

where $\tilde{\mathbf{w}}_i$ is an $(L+1)m$ -column vector consisting of the coefficients (corresponding to the i th output) of the filter defined by

$$\tilde{\mathbf{w}}_i := [\tilde{\mathbf{w}}_{i1}^T, \tilde{\mathbf{w}}_{i2}^T, \dots, \tilde{\mathbf{w}}_{im}^T]^T, \quad (16)$$

$$\tilde{\mathbf{w}}_{ij} := [w_{ij}^{(0)}, w_{ij}^{(1)}, \dots, w_{ij}^{(L)}]^T, \quad (17)$$

and $\tilde{\mathbf{H}}$ is an $n \times m$ block matrix defined by

$$\tilde{\mathbf{H}} := \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} & \cdots & \mathbf{H}_{1m} \\ \mathbf{H}_{21} & \mathbf{H}_{22} & \cdots & \mathbf{H}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{n1} & \mathbf{H}_{n2} & \cdots & \mathbf{H}_{nm} \end{bmatrix} \quad (18)$$

whose (i, j) th block element \mathbf{H}_{ij} is a $(K+L+1) \times (L+1)$ matrix with the (i_1, j_1) th element $[\mathbf{H}_{ij}]_{i_1 j_1}$ defined by

$$[\mathbf{H}_{ij}]_{i_1 j_1} := h_{ji}(i_1 - j_1), \quad i_1 = 0, \dots, K+L; \quad j_1 = 0, \dots, L. \quad (19)$$

Now we consider the generation of filtered source signals from the observed signal $\mathbf{x}(t)$. If $\tilde{\mathbf{g}}_i$'s become $\tilde{\mathbf{g}}_{i_0}$'s such that there exist $\tilde{\mathbf{w}}_{i_0}$'s satisfying

$$[\tilde{\mathbf{g}}_{1_0}, \dots, \tilde{\mathbf{g}}_{q_0}] = \tilde{\mathbf{H}}[\tilde{\mathbf{w}}_{1_0}, \dots, \tilde{\mathbf{w}}_{q_0}] = [\tilde{\mathbf{\delta}}_1, \dots, \tilde{\mathbf{\delta}}_q] \mathbf{P}, \quad (20)$$

then a filtered version of each component of $\mathbf{s}(t)$ can be recovered from the observed signals $x_i(t)$'s. Here $\tilde{\mathbf{\delta}}_i$ is the n -block column vector defined by

$$\tilde{\mathbf{\delta}}_i = [\mathbf{0}, \dots, \mathbf{0}, \mathbf{g}_{ii}^T(i\text{th vector}), \mathbf{0}, \dots, \mathbf{0}]^T, \quad (21)$$

where $\mathbf{g}_{ii} = [g_{ii}^1, g_{ii}^2, \dots, g_{ii}^{K+L+1}]^T$ is a non-zero $(K+L+1)$ -column vector and $\mathbf{0}$ is a $(K+L+1)$ -row zero vector. We note the each \mathbf{g}_{ii} can be chosen to be any non-zero vector in order to generate filtered source signals. However, in order to devise a new super-exponential deflation algorithm, we should choose each \mathbf{g}_{ii} to be a non-zero vector whose elements all g_{ii}^j ($j = 1, \dots, K+L+1$) take a non-zero identical value $g_{ii} \neq 0$. This constitutes a novel key point in the following development of this paper. Hence, the i th component of $\mathbf{y}(t)$ is expressed as

$$\begin{aligned} y_i(t) &= \tilde{\mathbf{d}}_{p_i}^T \tilde{\mathbf{s}}(t), & i = 1, 2, \dots, q, \\ &= g_{p_i}(z) s_{p_i}(t), & i = 1, 2, \dots, q, \end{aligned} \quad (22)$$

where $\{p_1, \dots, p_n\}$ is a permutation of $\{1, \dots, n\}$ and $g_{p_i}(z) = g_{p_i p_i}(1+z+\dots+z^{K+L})$ which is a CIC filter. Therefore, we call $g_{p_i}(z) s_{p_i}(t)$ (or $\tilde{\mathbf{d}}_{p_i}^T \tilde{\mathbf{s}}_{p_i}(t)$) a *CIC filtered source signal*. Without knowing the block matrix $\tilde{\mathbf{H}}$ along with the source signals $s_i(t)$, one can solve the blind signal generation problem by finding a matrix $\tilde{\mathbf{W}}_0 := [\tilde{\mathbf{w}}_{1_0}, \dots, \tilde{\mathbf{w}}_{q_0}]$ satisfying (20).

To find a matrix $\tilde{\mathbf{W}}_0$, we need the following assumptions:

(A1) The transfer function $\mathbf{H}(z)$ in (2) is *irreducible*, that is, $\text{rank } \mathbf{H}(z) = n$ for any $z \in C$ (this implies that the unknown system has less inputs than outputs, that is, $n \leq m$).

(A2) The input sequence $\{\mathbf{s}(t)\}$ is a zero-mean stationary vector process whose component processes $\{s_i(t)\}$ ($i = 1, \dots, n$) are temporally second-order white and spatially second- and fourth-order uncorrelated. At most, one component of $\{\mathbf{s}(t)\}$ can be Gaussian, and all the others should be non-Gaussian with unit variance and nonzero different K_i , where K_i is the sum of all the fourth-order auto-cumulants of the i th component signal:

$$K_i = \sum_{\tau_1, \tau_2, \tau_3 \in Z} C s_i(\tau_1, \tau_2, \tau_3) \neq 0 \quad (< \infty), \quad (23)$$

$$K_i \neq K_j, \quad i, j = 1, \dots, n; i \neq j. \quad (24)$$

Here Z denotes the set of all integers and $C\nu(\tau_1, \tau_2, \tau_3)$ is the fourth-order auto-cumulant function of signal $\nu(t)$ defined by

$$C\nu(\tau_1, \tau_2, \tau_3) \equiv \text{Cum}\{\nu(t), \nu(t-\tau_1)^*, \nu(t-\tau_2), \nu(t-\tau_3)^*\},$$

where the superscript $*$ denotes the complex conjugate. The sum of the fourth-order auto-cumulants, K_i is assumed to be unknown for $i = 1, \dots, n$.

Under the assumption (A1), we can show that there exists a matrix $\tilde{\mathbf{W}}_0$ satisfying (20), because $\mathbf{H}(z)$ has a causal left inverse.

At the first stage, our first objective is to generate CIC filtered source signals from the observed signals. In order to achieve the blind deconvolution, as the second stage, we consider of recovering the original source

signals from the CIC filtered source signals. In the subsection 3.2, we will show how to recover a source signal from the filtered one $\tilde{\mathbf{d}}_{p_i}^T \tilde{\mathbf{s}}_{p_i}(t)$.

3. A TWO-STAGE ALGORITHM

3.1. The first stage: A modified super-exponential deflation algorithm

To generate the CIC filtered source signals, we consider the following two-step algorithm adjusting the elements $g_{ij}^{(k)}$ for the cascade system,

$$g_{ij}^{(k)[1]} = \Gamma_j \left(\sum_{l=0}^{K+L} g_{ij}^{(l)} \right)^2 \left(\sum_{l=0}^{K+L} g_{ij}^{(l)*} \right), \quad (25)$$

$$g_{ij}^{(k)[2]} = g_{ij}^{(k)[1]} \frac{1}{\sqrt{\sum_{j=1}^n \sum_l |g_{ij}^{(l)[1]}|^2}}, \quad (26)$$

where $\Gamma_j =$

$$\sum_{\tau_1, \tau_2, \tau_3 \in Z} \text{Cum}\{s_j(t), s_j(t-\tau_1), s_j(t-\tau_2)^*, s_j(t-\tau_3)^*\} \quad \text{for } j = 1, \dots, n.$$

We should note in (25) that the elements $g_{ij}^{(k)}$'s (where $k = 0, \dots, K+L$) take an identical value for fixed i and j . Moreover, we should note in (26) that the absolute value of the identical value is $1/\sqrt{K+L+1}$.

Using the similar way as in [2], one can easily prove that the following iterative algorithm with respect to $\tilde{\mathbf{w}}_i$ can be derived from (25) and (26):

$$\tilde{\mathbf{w}}_i^{[1]} = \tilde{\mathbf{R}}^\dagger \tilde{\mathbf{D}}_i, \quad i = 1, 2, \dots, q, \quad (27)$$

$$\tilde{\mathbf{w}}_i^{[2]} = \frac{\tilde{\mathbf{w}}_i^{[1]}}{\sqrt{\tilde{\mathbf{w}}_i^{[1]*T} \tilde{\mathbf{R}} \tilde{\mathbf{w}}_i^{[1]}}}, \quad i = 1, 2, \dots, q, \quad (28)$$

where \dagger denotes the pseudo-inverse operation of a matrix, $\tilde{\mathbf{R}}$ is the $m \times m$ block matrix defined by

$$\tilde{\mathbf{R}} := \begin{bmatrix} \tilde{\mathbf{R}}_{11} & \tilde{\mathbf{R}}_{12} & \cdots & \tilde{\mathbf{R}}_{1m} \\ \tilde{\mathbf{R}}_{21} & \tilde{\mathbf{R}}_{22} & \cdots & \tilde{\mathbf{R}}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\mathbf{R}}_{m1} & \tilde{\mathbf{R}}_{m2} & \cdots & \tilde{\mathbf{R}}_{mm} \end{bmatrix} \quad (29)$$

whose (i, j) th block element $\tilde{\mathbf{R}}_{ij}$ is the $(L+1) \times (L+1)$ matrix with the (i_1, j_1) th element $[\tilde{\mathbf{R}}_{ij}]_{i_1 j_1}$ defined by

$$[\tilde{\mathbf{R}}_{ij}]_{i_1 j_1} = \text{Cum}\{x_j(t-j_1), x_i(t-i_1)^*\}, \quad (30)$$

and $\tilde{\mathbf{D}}_i$ is the n -block vector defined by

$$\tilde{\mathbf{D}}_i := [d_{i1}^T, d_{i2}^T, \dots, d_{im}^T]^T \quad (31)$$

where d_{ij} th is an $(L+1)$ -column vector with the j_1 th element $[d_{ij}]_{j_1}$ given by

$$\sum_{\tau_1, \tau_2, \tau_3 \in Z} \text{Cum}\{y_i(t), y_i(t-\tau_2), y_i(t-\tau_1)^*, x_j(t-j_1-\tau_3)^*\}.$$

Theorem 1 Infinite iterations of two steps (25) and (26) can yield an SISO cascade system $g_i(z) = \sum_{j=1}^n \sum_{k=0}^{K+L} g_{ij}^{(k)} z^k$ such that its impulse response vector defined by (13) and (14) satisfies

$$\begin{aligned} \tilde{\mathbf{g}}_{ij} &= \mathbf{g}_{jj} & \text{for some } j = j_0, \\ \tilde{\mathbf{g}}_{ij} &= \mathbf{0} & \text{for all } j \neq j_0, \end{aligned} \quad (32)$$

where $j_0 = \max_j |\Gamma_j| \sum_{k=0}^{K+L} |g_{ij}^{(k)}(0)|$ and $j \in \{1, \dots, n\}$.

From Theorem 1, it can be proved that the algorithms (25) and (26) (as well (27) and (28)) can be used to acquire a *CIC filtered source signal* $\tilde{\delta}_{j_0}^T \tilde{\mathbf{s}}_{j_0}(t)$.

3.2. The second stage

Here, we show how to obtain a source signal $s_{j_0}(t - k_i)$. Our approach is based on the fact that the source signal $s_{j_0}(t)$ is white but the obtained output $y_i(t)$ based on Theorem 1 is a colored signals. Therefore, the approach is to whiten the output $y_i(t)$ in the sense of second-order statistics. To implement the whitening of $y_i(t)$, we consider applying the following AR model to the CIC filtered output $y_i(t)$:

$$y_i(t) = - \sum_{k=1}^M v_i^{(k)} y_i(t - k) + \beta u_i(t). \quad (33)$$

where M is the order of an AR model and β is a constant. The whitening of $y_i(t)$ can be achieved by constructing the AR model (33) with a sufficiently large order M . The parameters β and $v_i^{(k)}$ can be found using the Yule-Walker equations and the Levinson algorithm.

3.3. The deconvolution algorithm

Our proposed algorithm can be summarized in the following steps:

Step 1. Set $i = 1$ (where i denotes the order of an input extracted).

Step 2. Choose random initial values $w_{ij}^{(k)}(0)$ of $w_{ij}^{(k)}$. Set $l = 0$ (l is the number of iterations).

Step 3. Calculate $\tilde{\mathbf{w}}_i(l)$ using (27) and (28).

Step 4. If $|\tilde{\mathbf{w}}_i^{*T}(l) \tilde{\mathbf{R}} \tilde{\mathbf{w}}_i(l - 1)|$ is not close enough to 1, set $l = l + 1$ and go back to Step 3. Otherwise go to the next step.

Step 5. Find the AR output using equation (33).

Step 6. At this stage, we assume that the source signal $s_{p_i}(t)$ has been recovered. Then we should compute the scale and the time-shift of the input $s_{p_i}(t)$ by using (22) and (33)

Step 7. Estimate the scale and the time-shift of $h_{jp_i}(\tau)$ by using $\hat{h}_{jp_i}(\tau) = E[x_j(t)u_i(t - \tau)^*]$, $j = 1, 2, \dots, m$.

Step 8. Estimate the contribution of $s_{p_i}(t)$ to the observed signals $x_j(t)$ ($j = 1, 2, \dots, m$), using $\hat{x}_{jp_i}(t) =$

$$\sum_{\tau} \hat{h}_{jp_i}(\tau) u_i(t - \tau).$$

Step 9. Remove the above contribution using $x_j^{(i)}(t) = x_j(t) - \hat{x}_{jp_i}(t)$, where $x_j^{(i)}(t)$ ($j = 1, \dots, m$) are the outputs of a linear unknown multichannel system with m outputs and $n - 1$ inputs.

Step 10. If the superscript (i) of $x_j^{(i)}(t)$ is less than n , then set $i = i + 1$ and $x_j(t) = x_j^{(i)}(t)$ ($j = 1, \dots, m$), and the procedures mentioned above are continued until $i = n$.

4. DISCUSSIONS

In this paper we proposed an iterative algorithm for the blind deconvolution problem in the case of temporally second-order white and spatially second- and fourth-order uncorrelated signals. The proposed algorithm is a modification of the the super-exponential deflation algorithm proposed by Inouye and Tanebe [11] to the case of the blind deconvolution problem of MIMO-FIR channels driven by fourth-order colored source signals. The proposed super-exponential algorithm was used to generate CIC filtered source signals from the mixtures of source signals. To recover the original source signals from the CIC filtered source signals, a whitening technique has been used.

We have carried out computer simulations to demonstrate the proposed method. The results have shown that the proposed algorithm can be used successfully to achieve the blind deconvolution.

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