HOS Criteria & ICA Algorithms Applied to Radar Detection

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ABSTRACT
This paper deals with radar detection and identification problems. Nowadays, To improve radar detection capability, engineers use high resolution methods (i.e. ESPRIT or MUSIC *etc*). Recently, some methods based on High Order Statistics (HOS) have been used for the same purpose. Here, a comparaison among different methods is proposed. In addition, the application of ICA algorithms in this field is discussed.

KEY WORDS: BSS, ICA, MUSIC, ARMA, ESPRIT, High Resolution Methods, Crosstalk, SNR, Real World Applications.

1. Introduction

The first patent and practical radar (RAdio Detection And Ranging system) is credited to the German, Hulsmeyer, in 1904. His 'telemobiloscope' was designed as an anti-collision device for ships. Since that time, radar has been used in many applications such as navigation control, military surveillance and intelligence, and so on.

The first radar detection techniques were based on the spectrum and Fourier-based-methods. Later on, high-resolution methods have been proposed such as ARMA modeling, Prony methods, MUSIC (Multiple Signal Classification) or ESPRIT (Estimation of Signal Parameters via Rotational Invariance Technique).

Actually, blind identification, equalization and separation are the most up-to-date methods in signal processing. Since the beginning of 80th, such methods have been used in many interesting applications and in different fields [1, 2] (as: robotics, telecommunication, radar, sonar, mobil phone, free hand phone, control of nuclear reactor as well as the surveillance and the control of airport traffic airlines, *etc*).

In radar context, few approaches based on these methods can be founded in the literature [3, 4]. As for exemple, in [5], the authors describe the case of complex linear mixture of complex signals. In [6], the estimation of some parameters are based on the extension of the minimum norm principal eigenvectors method. Fourth order cumulants are used to estimate harmonic frequency of the received signal [7]. The extension of MUSIC to high order statistics was described by Cardoso in [8]. To estimate the Direction Of Arrival (DOA), one can use the contracted Quadricovariance as in [9, 10].

In our project, we would like to apply some blind identification and separation methods along with high resolution and classical methods to improve performances for radar detection of moving targets. To reach our goal, we should mention that radar detection and identification mean: an estimation of the DOA, frequency spectres, amplitudes, echo delays and Doppler frequencies among others. In the following, some of these features are estimated.

2. Signal model

Let us consider an antenna composed of m sensors, equidistant of d, and n ($n \leq m$) narrow-band sources around f_0 (wave frequency). Let $\mathbf{a}(\theta_i)$ be the antenna response for a narrow-band source at a direction (DOA) θ_i:

$$\mathbf{a}(\theta_i) = [1,\ e^{j\phi_i},\ \cdots,\ e^{j(m-1)\phi_i}]^T. \quad (1)$$

Here $\phi_i = \frac{2\pi d}{\lambda}\sin\theta_i$ and λ is the wave length. The received signal $y(t)$ at an instant t is given by:

$$\mathbf{y}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) = \sum_{i=1}^{n} \mathbf{a}(\theta_i) s_i(t) + \mathbf{n}(t), \quad (2)$$

where $\mathbf{n}(t)$ stands for the $m \times 1$ vector of Additive zero-mean White Gaussian Noise (AWGN) and $\mathbf{s}(t) = [s_1(t), ..., s_n(t)]^T$ is the source vector which its ith component is stationary complex zero-mean random variable. Finally, $\mathbf{A} = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_n)]$ stands for the $m \times n$ mixing matrix.

In the following, we assume that the signals emitted by different sources are statistically independent from each other and they are independent from the AWGN $\mathbf{n}(t)$.

3. Statistical approaches

For narrow band signals, most of the proposed methods are based on second order statistics. At first, we emphasis some criteria based on fourth order statistics to analysis mono-dimensional signal. Later on, some array processing methods based on second and higher order statistics are discussed.

3.1 Frequency estimation using fourth-ordre cumulant

Usually, spectral analysis of the signals are done using Fourier transform, parametric approaches (i.e. ARMA, AR, *etc*) or second order statistics and subspace based approaches (as MUSIC, ESPIRT or Prony method). In this subsection, we emphasis some algorithms based on fourth-order statistics criteria to estimate the signal frequencies.

In radar applications, observed or received signals can be considered as the sum of n finite sinusoids. Let $\mathbf{x}(t) = (a_1 e^{j2\pi f_1 t + j\Phi_1}, \cdots, a_n e^{j2\pi f_n t + j\Phi_n})^T$ denotes the vector of harmonics and $\mathbf{h} = (1, \cdots, 1)^T$. In this case, the received signal $y(t)$ is given by:

$$\begin{aligned} y(t) &= \mathbf{h}^T \mathbf{x}(t) + n(t) = \sum_{i=1}^{n} a_i \, e^{j2\pi f_i t + j\Phi_i} \; + \; n(t) \\ &= \mathbf{h}^T \, \mathbf{U} \, \mathbf{x}(t-1) + n(t), \end{aligned} \tag{3}$$

where $\mathbf{U} = \mathbf{diag}(e^{j2\pi f_1}, e^{j2\pi f_2}, \cdots, e^{j2\pi f_n})$ and a_i, f_i are respectively the amplitude and the frequency of the ith complex sinusoids.

In [7], an harmonic retrieval estimation method based on cumulant C_{13} was proposed:

$$\begin{aligned} C_{13,y}(h) &= cum(y(n), \, y^*(n), \, y(n), \, y^*(n+h)) \\ &= E\left\{y(n)y^*(n)y(n)y^*(n+h)\right\} \\ &\quad -2\, E\left\{y(n)y^*(n)\right\} E\left\{y(n)y^*(n+h)\right\} \\ &\quad -E\left\{y(n)^2\right\} E\left\{y^*(n)y^*(n+h))\right\}. \end{aligned}$$

Here E stands for the expectation and y^* is the complex conjugate of y. In our previous works [11, 12], we found that criteria based on cum_{22} can achieve better performance results than the ones based on cum_{13} or cum_{31}, especially in the blind separation context. Therefore, a criterion based on cum_{22} and the approach in [7] has been investigated. The cum_{22} is given by:

$$\begin{aligned} C_{22,y}(h) &= cum(y(n), \, y^*(n+h), \, y(n), \, y^*(n+h)) \\ &= E\left\{y(n)y^*(n+h)y(n)y^*(n+h)\right\} \\ &\quad -2\, E\left\{y(n)y^*(n+h)\right\} E\left\{y(n)y^*(n+h)\right\} \\ &\quad -E\left\{y(n)^2\right\} E\left\{y^*(n+h)^2)\right\}. \end{aligned}$$

Let $\mathbf{C}_{22}(m)$ be the $L \times L$ following matrix:

$$\mathbf{C}_{22} = \begin{bmatrix} C_{22}(0) & C_{22}(1) & \cdots & C_{22}(L-1) \\ C_{22}(-1) & C_{22}(0) & \cdots & C_{22}(L-2) \\ \vdots & \vdots & \ddots & \vdots \\ C_{22}(-L+1) & C_{22}(-L+2) & \cdots & C_{22}(0) \end{bmatrix}.$$

The harmonics of the signal can be estimated by a singular value decomposition (SVD) of the previous matrix. In fact, the SVD of $\mathbf{C}_{22}$ gives two orthogonal matrices $\mathbf{U}$ and $\mathbf{V}$ such that:

$$\mathbf{C}_{22} = \mathbf{U} \, \mathbf{S} \, \mathbf{V}^T,$$

where $\mathbf{S}$ is a diagonal matrix. Different signal harmonics can be estimated, by a simple projection of the signal into the noise subspace, as the minima of the function:

$$Q(w) = \mathbf{h}(w)^H \; \mathbf{V}_2 \; \mathbf{V}_2^H \; \mathbf{h}(w), \tag{4}$$

where $\mathbf{v}^H$ is the conjugate transpose of $\mathbf{v}$, $\mathbf{h}(w) = [1, e^{jw}, \cdots, e^{j(L-1)w}]^T$ and $\mathbf{V}_2$ is a part of $\mathbf{V}$ that characterizes the noise sub-space. Finally, we should mention that the actual version of the method with cum_{22} gives similar performance results as the original version proposed in [7].

3.2 Second-order array processing

The main idea consists on the use of covariance matrix of the received signals. When the number of the sources is strictly less than the number of sensors (i.e. $n < m$), one can use the subspace noise to establish a projector operator $\underline{\Pi}_2$ as in MUSIC method. It is well known that the DOA of the sources can be estimated by using the following localization function:

$$f(\theta) = \mathbf{a}(\theta)^H \; \underline{\Pi}_2 \; \underline{\Pi}_2^H \; \mathbf{a}(\theta) \tag{5}$$

We should mention that similar approaches are proposed in the literature as ESPIRT (where the signal subspace is considered) and Prony (where we estimate the harmonic frequencies of the sources which are assumed to be periodical stationary signals).

3.3 Fourth-order array processing

Recently, some methods based on high order statistics have been proposed. In fact, the Quadricovariance tensor can be considered as the natural extension of the covariance matrix which allows us to extend signal sub-space.

3.3.1 Quadricovariance

Hereinafter, we are considering tensorial notations. Let us denote the 4th and 2nd order moment by:

$$\begin{aligned} \mu_i^j &= E\{y_i y^j\} \\ \mu_{i\,l}^{j\,k} &= E\{y_i y^j y^k y_l\}. \end{aligned}$$

When the sources are circular complex signals, the 4th-order cumulants or the Quadricovariance tensor can be simplified:

$$Q_{i\,l}^{j\,k} = Cum(y_i y^j y^k y_l) = \mu_{i\,l}^{j\,k} - \mu_i^k \mu_l^j - \mu_i^j \mu_l^k. \tag{6}$$

Thanks to the multi-linear and the additive properties of the cumulants, we obtain :

$$Q_{i\ l}^{j\ k} = a_i^\alpha a_\beta^j a_\gamma^k a_k^\delta K_{\alpha\ \delta}^{\beta\ \gamma}, \quad (7)$$

where $\mathcal{K} = (K_{\alpha\ \delta}^{\beta\ \gamma})$ is the source signal Quadricovariance, and the noise is assumed to be an AWGN, i.e. his Quadicovariance tensor is zero.

It is clear that the Quadricovariance is a 4-dimensional tensor. On the other side, most of the numerical algorithms are optimized for matrix computations. In order to use such numerical algorithms and to simplify the complexity of the mathematical operations, some authors suggest to use a contracted version of the Quadricovariance as in [13]:

$$c_i^j = E\{y_i\ y^j\ y^k\ y_l\} - E\{y_i y^j\}E\{y_l y^k\} - E\{y_i y^k\}E\{y_l y^j\}.$$

In normal matrix notations, the previous expression can be written as:

$$\mathbf{C} = \mathbf{R}_c - \ \mathbf{R}\,\mathbf{R} - \ \mathbf{R}\,Tr(\mathbf{R}), \quad (8)$$

with $\mathbf{R}_c = E\{\mathbf{y}^H\mathbf{y}\ \mathbf{y}\mathbf{y}^H\}$, $\mathbf{R} = E\{\mathbf{y}\mathbf{y}\}^{\mathbf{H}}$ and $Tr(\mathbf{A})$ is the trace of the matrix **A**.

3.3.2 MUSIC-4th order

The standard MUSIC algorithm can be generalized to deal with fourth order statistics, see [8]. In addition, it was proved that the DOA of the signals can be determined by the application of MUSIC method with 4-order statistics, see [14]. The MUSIC-4th ordre algorithm is similar to MUSIC-2nd ordre where the Quadricovariance tensor is used instead of the covariance matrix.

The eigenvalues decomposition (EVD) of a $m \times m$ covariance matrix gives m eigenvalues and m orthonormal eigenvectors. By similar way [14], the Quadricovariance can be considered as an operator in m^2 dimensional space, which its EVD is written with m^2 real eigenvalues and m^2 eigen-matrices:

$$\mathcal{Q} = \sum_{i=1}^{m^2} \mu_i \mathbf{M}_i * \mathbf{M}_i^H, \quad (9)$$

where $*$ represents the tensorial product. The signal subspace is orthogonal to the noise subspace. Therefore, one can obtain the DOA by similar relationship as in (5) where the projector operator $\underline{\mathbf{\Pi}}_2$ should be replaced with fourth order projector $\underline{\mathbf{\Pi}}_4$:

$$\underline{\mathbf{\Pi}}_4 = \sum_i \mathbf{M}_i \mathbf{M}_i^H; \qquad \forall i \in \{1, m^2\} \ | \ \mu_i = 0. \quad (10)$$

The main advantage of MUSIC-4th consists on the possibility to identify up to $2(m-1)$ different DOA instead of $n < m$ for previous MUSIC-2 algorithm. Further details about this projector can be found in [8].

3.3.3 Contracted Quadricovariance

To minimize computational efforts, the authors of [10] propose a projector operator based on a contracted Quadricovariance **C**. Let us assume that the matrix **C** can be decomposed as following:

$$\mathbf{C} = \mathbf{A}\ \mathbf{Z}\ \mathbf{A}^H, \quad (11)$$

where **Z** is a $(n \times n)$ hermitian matrix. Let us denote:

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \end{bmatrix}, \quad (12)$$

where $\mathbf{C}_1$ (resp. $\mathbf{C}_2$) is a $n \times m$ (resp. $(m-n) \times m$ matrix). Now, the propagator **P** is defined in [10] as a $(m-n) \times n$ matrix given by:

$$\mathbf{P}^H = \mathbf{C}_2\ \mathbf{C}_1^H\ (\mathbf{C}_1\ \mathbf{C}_1^H)^{-1}. \quad (13)$$

Let **Q** denotes a $(m-n) \times m$ matrix defined by $\mathbf{Q}^H = [\mathbf{P}^H \ \ -\mathbf{I}]$, where **I** stands for a $(m-n) \times (m-n)$ identity matrix. In this case, one can prove that $\mathbf{Q}^H\mathbf{A} = \mathbf{0}$ and the direction of arrival can be obtained by the minimization of the following localization function:

$$F(\theta) = \mathbf{a}(\theta)^H\ \mathbf{Q}\ \mathbf{Q}^H\ \mathbf{a}(\theta). \quad (14)$$

Finally, to evaluate their projector, they choose the contracted Quadricovariance **C** of equation (11) proposed in [9] and estimated as:

$$\mathbf{C} = \frac{1}{N}\sum_{t=1}^{N} y(t)y(t)^H y(t)y(t)^H - \mathbf{R}^2 - \mathbf{R}\,Tr(\mathbf{R}), \quad (15)$$

where Tr stands for the matrix trace and N is the number of the samples. Performance analysis can be found in [9].

4. Estimation of DOA by blind separation

In this section, we assume that the received signal is given by Equation (2). The blind separation of sources problem (BSS) involves retrieving unknown statistically independent sources from their observed mixed signals.

4.1 Blind separation problem

Generally, blind separation of sources can be achieved up to a factor scale and up to a permutation, i.e. the estimated or separation matrix **B** is given by:

$$\mathbf{B} = \mathbf{A}\mathbf{P}\mathbf{\Delta}, \quad (16)$$

where $\mathbf{\Delta} = \mathbf{diag}(z_1, ..., z_n)$ is a full rank diagonal matrix and $\mathbf{P} = \{\mathbf{e}_1, \mathbf{e}_2, \cdots, \mathbf{e}_n\}$ is a permutation matrix, where $\mathbf{e}_i$ are n orthonormal vectors. It is clear that the mixing matrix **A** of equation (2) can be written as:

$$\mathbf{A} = \begin{bmatrix} 1 & \cdots & 1 \\ x_1 & \cdots & x_n \\ x_1^2 & \cdots & x_n^2 \\ \vdots & \vdots & \vdots \\ x_1^{m-1} & \cdots & x_n^{m-1} \end{bmatrix}, \quad (17)$$

where $x_i = e^{j2\pi(d/\lambda)\sin\theta_i}$ and $i \in \{1, \cdots, n\}$. In this case, the estimated matrix $\mathbf{B}$ has the following form, up to a column permutation:

$$\mathbf{B} = \begin{bmatrix} z_1 & \cdots & z_n \\ z_1\, x_1 & \cdots & z_n\, x_n \\ z_1\, x_1^2 & \cdots & z_n\, x_n^2 \\ \vdots & \vdots & \vdots \\ z_1\, x_1^{m-1} & \cdots & z_n\, x_n^{m-1} \end{bmatrix}. \qquad (18)$$

It is clear that Matrix $\mathbf{B}$ can have similar structure to Matrix $\mathbf{A}$ by dividing each column $\mathbf{b}_i$ of $\mathbf{B}$ by its first component b_{1i}. Finally, the DOA can be obtained as follows:

$$\begin{aligned} x_i &= e^{j2\pi(d/\lambda)\sin\theta_i} = e^{j\alpha_i} \\ \theta_i &= \arcsin\left(\frac{\lambda}{2\pi d}\alpha_i\right), \end{aligned}$$

where $i \in \{1, \cdots, n\}$.

4.2 DOA estimated by JADE algorithm

To estimated DOA by using blind separation algorithms, one can find a wide selection of different algorithms. To clarify our ideas as well to conduct some simulated experiment, we select JADE (Joint Approximate Diagonalization of Eigen-matrices) algorithm proposed by Cardoso *et al.* [3]. One can briefly describe JADE algorithm by the following four steps:

1. Using the observation covariance matrix $\mathbf{R}_y$ of $\mathbf{y}(t)$, one can estimate a whitening matrix $\mathbf{W}$.

2. Let $\mathbf{z}(t) = \mathbf{W}\,\mathbf{y}(t)$, then the 4th cumulant $\mathcal{Q}_z$ can be estimated and the n most significant eigen pairs $\{\mu_r, \mathbf{M}_r,\ 1 \le r \le s\}$ can be selected.

3. One should jointly diagonalize the eigen set of $\mathcal{Q}_z$ by an unitary matrix $\mathbf{U}$: The joint Diagonalization can be obtained by minimizing the function $JOFF$:

$$\min_{\mathbf{U}} JOFF = \min_{\mathbf{U}} \sum_{k=1}^{s} Off(\mathbf{U}\,\mathbf{M}_k\,\mathbf{U}^H),$$

   where $\mathbf{U}$ is an unitary matrix. We should mention here that the function Off [15] of a matrix is defined by $Off(\mathbf{M}) = \sum_{i \neq j} |m_{ij}|^2$

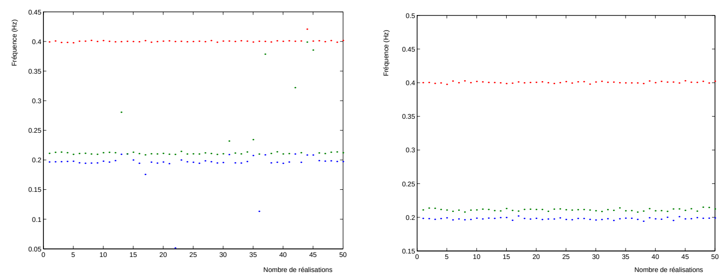
4. The estimate of $\mathbf{A}$ is $\hat{\mathbf{A}} = \hat{\mathbf{W}}^{\#}\,\hat{\mathbf{U}}$. (# pseudo inverse)

Finally, we should mention that some experiments have been conducted using different algorithms as the ones cited in [16, 17]. The latter algorithms need less computational effort and convergence time than JADE but they gives less attractive performing results. In the following, we only present the results obtained by JADE.

5. Experimental results

In our simulations, the second and the fourth order statistics are evaluated according to the estimators proposed in [18]. Many experiments have been carried out to study the performance results of the different algorithms studied in this paper. Some of our experimental results are illustrated in this section.

At first, 50 Monte Carlo simulations have been carried out to show the performance results of the frequency estimator based on fourth order cumulants and discussed in subsection (3.1). In these simulations, 64 samples of a signal with three harmonics ($f_1 = 0.2, f_2 = 0.21$ and $f_3 = 0.4$) are considered. Fig. 1 shows the estimation results of Music-2 (classic method) and the fourth order statistic estimator (in Fig. 1 (b)).



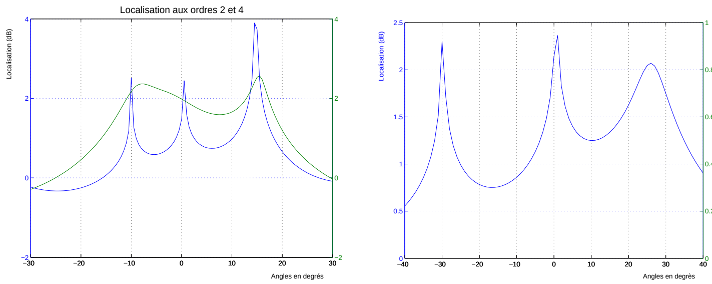
a- Classical MUSIC-2 b- 4th order cumulant estimator

Figure 1. Frequency estimation with AWGN and a SNR = 3dB

In these simulations, we found that MUSIC-2 needs less time and computational effort to converge, but the fourth order estimator can achieve better estimation of the frequency.

Concerning the fourth order MUSIC, it seems that MUSIC-4 is much more complicate to implement than MUSIC-2. In addition, MUSIC-2 needs less computation effort than MUSIC-4. On the other hand, MUSIC-4 can achieve better result than MUSIC-2 especially for a relatively small number of samples and a noisy data, see Fig. 2 (a). In that fig, we used 5 sensors and three sources with 300 samples, we find that MUSIC-4 is able to estimate the DOA of the three sources.

We should mention also that when the number of sources is equal to the number of sensors, MUSIC-2 is not able to estimate any angle (as the noise subspace becomes empty). Fig 2 (b) shows the estimation of three DOA with help of three sensors by using MUSIC-4. In this case, no results can be obtained by MUSIC-2.



a- 5 sensors and 3 sources b- 3 sensors and 3 sources.

Figure 2. DOA estimation by MUSIC-4 and MUSIC-2 with AWGN and a SNR = 0dB

In similar conditions as mentioned before, we found that the fourth order propagator (i.e. the contracted Quadricovariance estimator, see subsection 3.3.3) has similar performance results as MUSIC-4 and better performance results than MUSIC-2 algorithm. Fig. 3 shows the estimation of three DOA with help of 5 sensors by using fourth order propagator and Music-2.

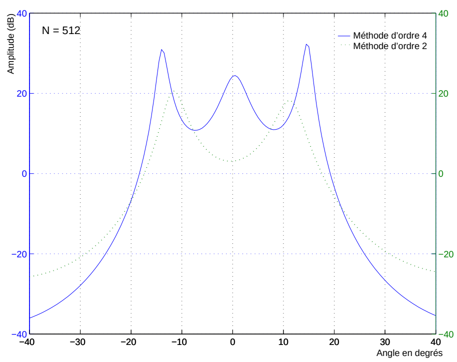


Figure 3. DOA estimation by the Contracted Quadricovariance and MUSIC-2 with AWGN and a SNR = 0dB

Concerning the BSS algorithms, some experiments have been conducted. From section 4, it is clear that BSS algorithms can estimate blindly (with out any specific model as in Prony or MUSIC-2) the features of the sources (frequency, DOA, *etc*). Fig 4 shows the estimation of the frequency obtained by Jade.

Up to now, we have just compared second and fourth order technics for the estimation of DOA. To conclude our experimental study, simulations were carried out to compare the performance of two fourth order technics: BSS with JADE and the fourth order propagator based on the contracted Quadricovariance. To reach our goal, a linear $\lambda/2$ equispaced array of $m = 5$ sensors are considered. Fig. 5 shows the deviation of the direction over $r = 150$ Monte carlo runs, the sample size is $N = 512$. For a SNR of 5dB, comparable performance results are obtained for

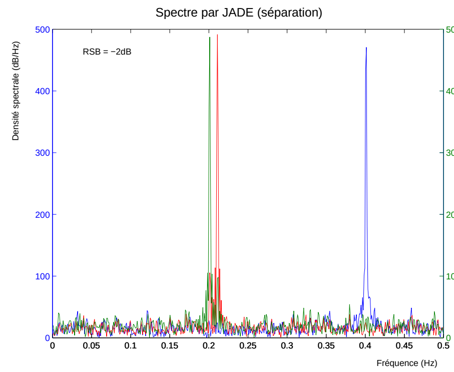


Figure 4. DOA estimation by Jade with AWGN and a SNR = -2dB

the two methods, see Fig. 5 (a) and (b). However, when the SNR decreases to -2dB, we observe better results with Jade than with the propagator method, see Fig. 5 (c) and (d). The latter method is limited in resolution (as shown in (d)). In fact, the estimation of direction can not be well determined if the differences among the DOA angles are less than $10°$ instead of $5°$ for Jade.

6. Conclusion

In this paper, we present and discuss some fourth order statistic approaches to estimate features from radar signals. The theoretical study can be considered as a survey for the up-to-date fourth order methods applied into radar fields. Our experimental study shows that the classical method are powerful estimation methods. In addition, these methods are much easier to be implemented than the methods based on fourth order statistics. However, the latter ones can achieve better results in various situations (with small number of samples, with a number of sources equal or great than the number of sensors, with an AGWN and a weak SNR, *etc*). Finally, the obtained simulated results are not enough to conclude definitively our study. However, they encourage us to continue our study and to test such methods on real world applications.

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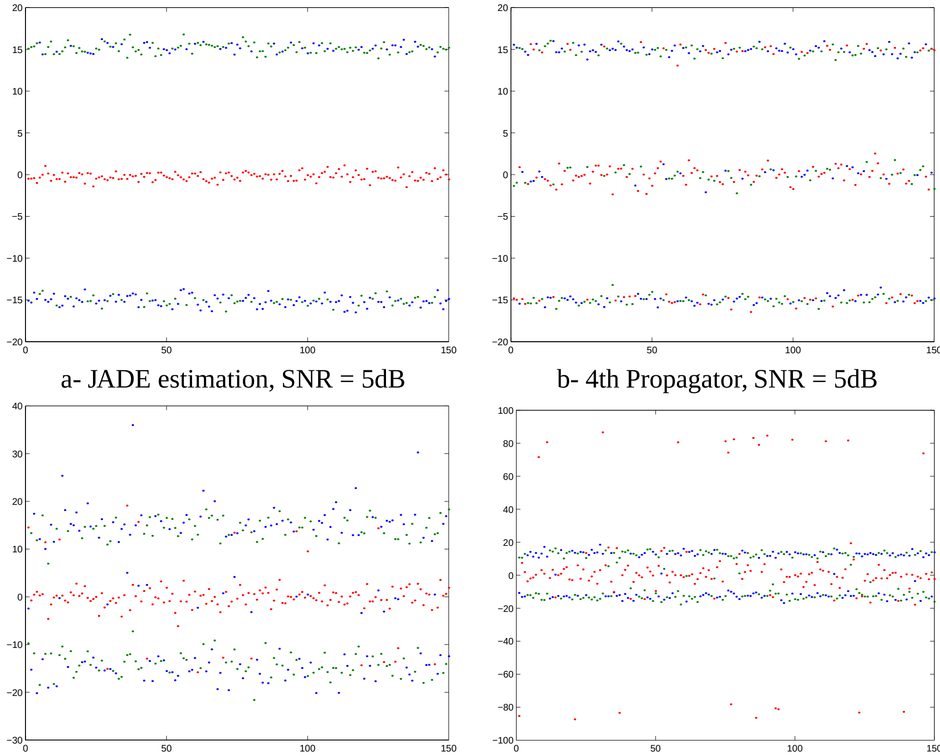

a- JADE estimation, SNR = 5dB

b- 4th Propagator, SNR = 5dB

c- JADE estimation, SNR = -2dB

d- 4th order Propagator, SNR = -2dB

Figure 5. DOA estimation by BSS algorithm and by fourth order propagator

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