BLIND DECONVOLUTION ALGORITHMS FOR MIMO-FIR SYSTEMS DRIVEN BY FOURTH-ORDER COLORED SIGNALS

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ABSTRACT

In this paper, we propose a new iterative algorithm to solve the blind deconvolution problem of MIMO-FIR channels driven by source signals which are temporally second-order uncorrelated but fourth-order correlated and spatially second- and fourth-order uncorrelated. To achieve our goal, we extend the super-exponential deflation algorithm proposed by Inouye and Tanebe [2] to the case of the blind deconvolution problem of MIMO-FIR channels driven by the source signals which possess fourth-order auto-correlations. In our new approach, to recover one source signal, there are two stages: First, by using our proposed super-exponential algorithm, a cascaded integrator-comb (CIC) filter is acquired. It implies that one filtered source signal is separated from the mixtures of the source signals. Next, by making the filtered source signal uncorrelated, one source signal is recovered from the filtered source signal. To show the validity of the proposed algorithm, some simulation results are presented.

1. INTRODUCTION

The blind deconvolution problem consists of extracting source signals from their convolutive mixtures observed by sensors without knowledge about the source signals and about the transfer function (transmission channel) between the sources and the sensors. The blind deconvolution has drawn an attention in diverse fields, for example, digital communications, speech processing, image processing, and array signal processing, etc.

The blind deconvolution problem has been studied by many researchers [1, 2, 3, 5, 6]. Almost all of the proposed methods to date have been developed under the assumption that the source signals are temporally independent and identically distributed (i.i.d.) and spatially independent [1, 2, 5]. However, in some applications, the i.i.d. assumption for the source signals becomes very strong. As an example, in digital communications, the information bearing sequences are coded in order to reduce noise corruption and channel distortion. These codes implicitly are not mutually independent among sequences and hence it is unlikely that they are i.i.d. signals. On the other hand, these code sequences are interleaved to avoid burst errors when the codes are transmitted [4]. These interleaved sequences are usually considered to be uncorrelated. To solve the blind deconvolution problem for such an application, therefore, one can assume that the source signals have a weaker condition than the i.i.d. condition, for example, the source signals are temporally second-order uncorrelated but higher-order correlated.

Here we propose an iterative algorithm to achieve the blind deconvolution of MIMO-FIR channel systems driven by source signals which are temporally high-order colored signals (but temporally second-order white and spatially second- and fourth-order uncorrelated signals). This condition for the source signals is weaker than the i.i.d. condition. To do that, we consider a deflation approach. Algorithms based on deflation approaches have been used to achieve blind deconvolution under the assumption that the source signals are i.i.d. and spatially independent [2, 5]. However, it is not clear whether the deflation approach can be applied to the MIMO-FIR channels in the case that the sources are fourth-order colored signals. Our new algorithm is an extension of the super-exponential deflation algorithm proposed by Inouye and Tanebe [2] to the case of the blind deconvolution problem of an MIMO-FIR channel driven by the fourth-order colored signals. In our approach, we should consider two stages to recover one source signal from the output of a multiple-input single-output finite impulse response (MISO-FIR) system: First, a linear phase filter with identical tap coefficients which is called a *cascaded integrator-comb* (CIC) $filter$ is acquired. It implies that one filtered source signal is separated from the mixtures of the source signals (i.e. the MISO-FIR channel driven by the source signals is reduced to an SISO-FIR channel driven by one of the source signals). Secondly, by making the output signal of the SISO-FIR channel white in the sense of second-order statistics, the source signal can be recovered from the filtered source signal. Simulation examples are presented to illustrate the performance of the proposed algorithm.

2. PROBLEM FORMULATION

Let us consider the following MIMO-FIR system:

$$\boldsymbol{x}(t) = \sum_{k=0}^{K} \boldsymbol{H}^{(k)} \boldsymbol{s}(t-k), \tag{1}$$

where $\boldsymbol{x}(t)$ represents an m-column output vector called the *observed signal*, $\boldsymbol{s}(t)$ represents an n-column input vector called the *source signal*, $\{\boldsymbol{H}^{(k)}\}$ is an $m \times n$ matrix sequence representing the impulse response of the transmission channel, and the number K denotes its order. Equation (1) can be written as

$$\boldsymbol{x}(t) = \boldsymbol{H}(z)\boldsymbol{s}(t), \tag{2}$$

where $\boldsymbol{H}(z)$ is the z-transform of the transfer function, i.e.

$$\boldsymbol{H}(z) = \sum_{k=0}^{K} \boldsymbol{H}^{(k)} z^k.$$

To solve the blind deconvolution problem, let us consider the following FIR system called the *equalizer*.

$$\boldsymbol{y}(t) = \sum_{k=0}^{L} \boldsymbol{W}^{(k)} \boldsymbol{x}(t-k), \tag{3}$$

where $\boldsymbol{y}(t)$ is an n-column vector representing the output signal of the equalizer, $\{\boldsymbol{W}^{(k)}\}$ is an $n \times m$ matrix sequence, and the number L is the order of the equalizer. Equation (3) can be written as

$$\boldsymbol{y}(t) = \boldsymbol{W}(z)\boldsymbol{x}(t), \tag{4}$$

where $\boldsymbol{W}(z)$ is the equalizer transfer function defined by

$$\boldsymbol{W}(z) = \sum_{k=0}^{L} \boldsymbol{W}^{(k)} z^k.$$

Substituting (2) into (4), we have

$$\boldsymbol{y}(t) = \boldsymbol{W}(z)\boldsymbol{H}(z)\boldsymbol{s}(t) = \boldsymbol{G}(z)\boldsymbol{s}(t), \tag{5}$$

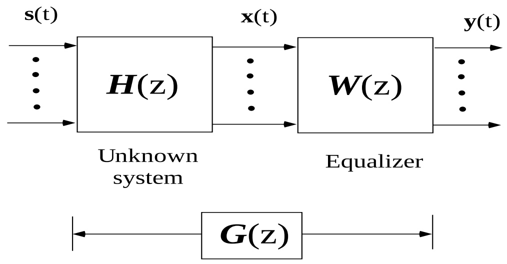

Figure 1: The cascade system of an unknown system and an equalizer.

where

$$\boldsymbol{G}(z) = \boldsymbol{W}(z)\boldsymbol{H}(z) = \sum_{k=0}^{K+L} \boldsymbol{G}^{(k)} z^k. \tag{6}$$

Figure 1 shows the cascade system of an unknown system followed by an equalizer. All variables can be complex-valued (this is required for such an application using quadrature amplitude modulation (QAM) signals [4]). In order to solve the blind deconvolution problem, as the first stage, we consider the blind generation problem mentioned below, in which CIC *filtered source signals* are generated from the observed signals.

The cascade system can be written in scalar form as

$$y_i(t) = \sum_{j=1}^{n} \sum_{k=0}^{K+L} {g_{ij}}^{(k)} s_j(t-k), \quad i = 1, 2, \cdots, n, \tag{7}$$

where

$${g_{ij}}^{(k)} = \sum_{l=1}^{m} \sum_{\tau=0}^{L} {w_{il}}^{(\tau)} {h_{lj}}^{(k-\tau)}, \ k = 0, 1, \cdots, K+L, \tag{8}$$

Here $i = 1, \cdots, n$, and $j = 1, \cdots, n$. The set of equations (7) can be written in vector notation as

$$y_i(t) = \tilde{\boldsymbol{g}}_i^T \tilde{\boldsymbol{s}}(t), \tag{9}$$

where the superscript T denotes the transpose of a vector, and $\tilde{\boldsymbol{s}}(t)$ is the column vector defined by

$$\tilde{\boldsymbol{s}}(t) := [\tilde{\boldsymbol{s}}_1(t)^T, \tilde{\boldsymbol{s}}_2(t)^T, \cdots, \tilde{\boldsymbol{s}}_n(t)^T]^T, \tag{10}$$
$$\tilde{\boldsymbol{s}}_i(t) := [s_i(t), s_i(t-1), \cdots, s_i(t-K-L)]^T \tag{11}$$

and $\tilde{\boldsymbol{g}}_i$ is the column vector consisting of the ith output impulse response of the cascade system defined by

$$\tilde{\boldsymbol{g}}_i := [\tilde{\boldsymbol{g}}_{i1}^T, \tilde{\boldsymbol{g}}_{i2}^T, \cdots, \tilde{\boldsymbol{g}}_{in}^T]^T, \tag{12}$$
$$\tilde{\boldsymbol{g}}_{ij} := [{g_{ij}}^{(0)}, {g_{ij}}^{(1)}, \cdots, {g_{ij}}^{(K+L)}]^T. \tag{13}$$

Using (12), (8) can be written in vector notation as

$$\tilde{\boldsymbol{g}}_i = \tilde{\boldsymbol{H}} \tilde{\boldsymbol{w}}_i, \qquad i = 1, 2, \cdots, n, \tag{14}$$

where $\tilde{\boldsymbol{w}}_i$ is an $(L+1)m$ column vector consisting of the coefficients (corresponding to the ith output) of the equalizer defined by

$$\tilde{\boldsymbol{w}}_i := [\tilde{\boldsymbol{w}}_{i1}^T, \tilde{\boldsymbol{w}}_{i2}^T, \cdots, \tilde{\boldsymbol{w}}_{im}^T]^T, \quad (15)$$

$$\tilde{\boldsymbol{w}}_{ij} := [{w_{ij}}^{(0)}, {w_{ij}}^{(1)}, \cdots, {w_{ij}}^{(L)}]^T, \quad (16)$$

and $\tilde{\boldsymbol{H}}$ is an n × m block matrix defined by

$$\tilde{\boldsymbol{H}} := \begin{bmatrix} \boldsymbol{H}_{11} & \boldsymbol{H}_{12} & \cdots & \boldsymbol{H}_{1m} \\ \boldsymbol{H}_{21} & \boldsymbol{H}_{22} & \cdots & \boldsymbol{H}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{H}_{n1} & \boldsymbol{H}_{n2} & \cdots & \boldsymbol{H}_{nm} \end{bmatrix} \quad (17)$$

whose (i,j)th block element $\boldsymbol{H}_{ij}$ is a $(K+L+1)\times(L+1)$ matrix with the (l,r)th element $[\boldsymbol{H}_{ij}]_{lr}$ defined by

$$[\boldsymbol{H}_{ij}]_{lr} := h_{ji}(l-r),\ l = 0, \cdots, K+L;\ r = 0, \cdots, L. \quad (18)$$

If $\tilde{\boldsymbol{g}}_i$'s become such $\tilde{\boldsymbol{g}}_{i_0}$'s that there exist $\tilde{\boldsymbol{w}}_{i_0}$'s satisfying

$$[\tilde{\boldsymbol{g}}_{1_0}, \cdots, \tilde{\boldsymbol{g}}_{n_0}] = \tilde{\boldsymbol{H}}[\tilde{\boldsymbol{w}}_{1_0}, \cdots, \tilde{\boldsymbol{w}}_{n_0}] = [\tilde{\boldsymbol{\delta}}_1, \cdots, \tilde{\boldsymbol{\delta}}_n]\boldsymbol{P}, \quad (19)$$

then a filtered version of each component of $\boldsymbol{s}(t)$ can be recovered from the observed signals $x_i(t)$'s. Here $\boldsymbol{P}$ is an $n \times n$ permutation matrix and $\tilde{\boldsymbol{\delta}}_i$ is the n-block column vector defined by

$$\tilde{\boldsymbol{\delta}}_i = [\boldsymbol{0}, \cdots, \boldsymbol{0}, \boldsymbol{g}_{ii}^T(ith\ \ vector), \boldsymbol{0}, \cdots, \boldsymbol{0}]^T, \quad (20)$$

where $\boldsymbol{g}_{ii}$ is a $(K+L)$-column vector whose elements take nonzero values. Hence, the ith component of $\boldsymbol{y}(t)$ is expressed as

$$\begin{aligned} y_i(t) &= \tilde{\boldsymbol{\delta}}_{p_i}^T \tilde{\boldsymbol{s}}_{p_i}(t), \qquad i = 1, 2, \cdots, n, \\ &= g_{i_0}(z) s_{p_i}(t) \qquad i = 1, 2, \cdots, n, \quad (21) \end{aligned}$$

where $\{p_1, \cdots, p_n\}$ is an arbitrary permutation of $\{1, \cdots, n\}$ and $g_{i_0}(z) = g_{i_0 p_i}\ (1 + z + \cdots + z^{K+L})$ which is a CIC filter. Therefore, we call $g_{i_0}(z)s_{p_i}(t)$ (or $\tilde{\boldsymbol{\delta}}_{p_i}^T \tilde{\boldsymbol{s}}_{p_i}(t)$) a *CIC filtered source signal*. Without knowing the block matrix $\tilde{\boldsymbol{H}}$ along with the source signals $s_i(t)$, one can solve the blind generation problem by finding a matrix $\tilde{\boldsymbol{W}}_0 := [\tilde{\boldsymbol{w}}_{1_0}, \cdots, \tilde{\boldsymbol{w}}_{n_0}]$ satisfying (19).

To find a matrix $\tilde{\boldsymbol{W}}_0$, we need the following assumptions:

(**A1**) The transfer function $\boldsymbol{H}(z)$ in (2) is *irreducible*, that is, $rank\ \boldsymbol{H}(z) = n$ for any $z \in C$ (this implies that the unknown system has less inputs than outputs, that is, $n \leq m$).

(**A2**) The input sequence $\{\boldsymbol{s}(t)\}$ is a zero-mean stationary vector process whose component processes $\{s_i(t)\}$ $(i = 1, \cdots, n)$ are temporally second-order white and spatially second- and fourth-order uncorrelated. At most, one component of $\{\boldsymbol{s}(t)\}$ can be Gaussian, and all the others should be non-Gaussian with unit variance and nonzero different K_i, where K_i is the sum of all the fourth-order auto-cumulants of the ith component signal:

$$K_i = \sum_{\tau_1,\tau_2,\tau_3 \in Z} Cs_i(\tau_1, \tau_2, \tau_3) \neq 0 \quad (< \infty), \quad (22)$$

$$K_i \neq K_j, \qquad i, j = 1, \cdots, n; i \neq j. \quad (23)$$

Here Z denotes the set of all integers and $C\nu(\tau_1,\tau_2,\tau_3)$ is the fourth-order auto-cumulant function of signal $\nu(t)$ defined by

$$C\nu(\tau_1, \tau_2, \tau_3) \equiv Cum\{\nu(t), \nu(t-\tau_1)^*, \nu(t-\tau_2), \nu(t-\tau_3)^*\},$$

where the superscript $*$ denotes the complex conjugate. The sum of the fourth-order auto-cumulants, K_i is assumed to be unknown for $i = 1, \cdots, n$.

At the first stage, our first objective is to generate CIC filtered source signals from the observed signals. In order to achieve the blind deconvolution, as the second stage, we consider of recovering the original source signals from the CIC filtered source signals. In the subsection 3.2, we will show how to recover a source signal from $\tilde{\boldsymbol{\delta}}_{p_i}^T \tilde{\boldsymbol{s}}_{p_i}(t)$.

3. THE NEW ALGORITHM

3.1. First stage of the Proposed Super-Exponential Algorithm

To generate the CIC filtered source signals, we consider the following two-step algorithm based on the Inouye-Tanebe algorithm [2] of adjusting the elements ${g_{ij}}^{(k)}$ for the cascade system,

$$\begin{aligned} {g_{ij}}^{(k)[1]} &= \Gamma_j (\sum_{l=0}^{K+L} {g_{ij}}^{(l)})^p (\sum_{l=0}^{K+L} {g_{ij}}^{(l)*})^q, \\ & \qquad k = 0, \cdots, K+L, \quad (24) \\ {g_{ij}}^{(k)[2]} &= {g_{ij}}^{(k)[1]} \frac{1}{\sqrt{\sum_{j=1}^{n} \sum_l |g_{i,j}{}^{(l)[1]}|^2}}, \\ & \qquad k = 0, \cdots, K+L, \quad (25) \end{aligned}$$

where where $(\cdot)^{[1]}$, $(\cdot)^{[2]}$ stand for the result of the first step and the result of the second step per iteration, p and q are nonnegative integers such that $p+q \geq 2$,

$\Gamma_j =$

$$\sum_{\underbrace{\tau_{p_1}, \cdots, \tau_{p_p}}_{p}, \underbrace{\tau_{q_1}, \cdots, \tau_{q_{q+1}}}_{q+1} \in Z} Cum\{s_j(t), s_j(t-\tau_{p_1}), \cdots,$$

$$s_j(t-\tau_{p_p}), s_j(t-\tau_{q_1})^*, \cdots, s_j(t-\tau_{q_{q+1}})^*\}$$
$$for \quad j=1,\cdots,n. \qquad (26)$$

We should note from (24) that the elements $g_{ij}{}^{(k)}$'s (where $k = 0, \cdots, K + L$) take an identical value for fixed i and j.

Using the similar way as in [2], it can be easily proved that the following iterative algorithm with respect to $\tilde{\boldsymbol{w}}_i$ can be derived from (24) and (25):

$$\tilde{\boldsymbol{w}}_i{}^{[1]} = \tilde{\boldsymbol{R}}^{\dagger}\tilde{\boldsymbol{D}}_i, \qquad i=1,2,\cdots,n, \quad (27)$$
$$\tilde{\boldsymbol{w}}_i{}^{[2]} = \frac{\tilde{\boldsymbol{w}}_i{}^{[1]}}{\sqrt{\tilde{\boldsymbol{w}}_i{}^{[1]*T}\tilde{\boldsymbol{R}}\tilde{\boldsymbol{w}}_i{}^{[1]}}}, \quad i=1,2,\cdots,n, \quad (28)$$

where † denotes the pseudo-inverse operation of a matrix, $\tilde{\boldsymbol{R}}$ is the $m \times n$ block matrix defined by

$$\tilde{\boldsymbol{R}} := \begin{bmatrix} \tilde{\boldsymbol{R}}_{11} & \tilde{\boldsymbol{R}}_{12} & \cdots & \tilde{\boldsymbol{R}}_{1n} \\ \tilde{\boldsymbol{R}}_{21} & \tilde{\boldsymbol{R}}_{22} & \cdots & \tilde{\boldsymbol{R}}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\boldsymbol{R}}_{m1} & \tilde{\boldsymbol{R}}_{m2} & \cdots & \tilde{\boldsymbol{R}}_{mn} \end{bmatrix} \qquad (29)$$

whose (i, j)th block element $\tilde{\boldsymbol{R}}_{ij}$ is the $(L+1) \times (L+1)$ matrix with the (l, r)th element $[\tilde{\boldsymbol{R}}_{ij}]_{lr}$ defined by

$$[\tilde{\boldsymbol{R}}_{ij}]_{lr} = Cum\{x_j(t-r), x_i(t-l)^*\}, \qquad (30)$$

and $\tilde{\boldsymbol{D}}_i$ is the n-block vector defined by

$$\tilde{\boldsymbol{D}}_i := [\boldsymbol{d}_{i1}^T, \boldsymbol{d}_{i2}^T, ..., \boldsymbol{d}_{in}^T]^T \qquad (31)$$

where $\boldsymbol{d}_{ij}$th is an $(L + 1)$-column vector with the rth element $[\boldsymbol{d}_{ij}]_r$ given by

$$[\boldsymbol{d}_{ij}]_r = \sum_{\tau_1,\tau_2,\tau_3 \in Z} Cum\{y_i(t), y_i(t-\tau_2), y_i(t-\tau_1)^*, x_j(t-r-\tau_3)^*\}. \qquad (32)$$

As for (32), under assumption (A2), we confine ourselves to the case of $p = 2$ and $q = 1$. (27) and (28) are the main iterative steps in our algorithm.

Theorem 1 Infinite iterations of two steps (24) and (25) can yield an SISO cascade system $g_i(z) = \sum_{j=1}^{n}\sum_{k=0}^{K+L} g_{ij}{}^{(k)}z^k$ such that its impulse response vector defined by (12) and (13) satisfies

$$\begin{array}{lll} \tilde{\boldsymbol{g}}_{ij} = \boldsymbol{g}_{jj} & for\ some\ j = j_0, \\ \tilde{\boldsymbol{g}}_{ij} = \boldsymbol{0} & for\ all\ j \neq j_0, \end{array} \qquad (33)$$

where $j_0 = \max_j |\Gamma_j|| \sum_{k=0}^{K+L} g_{ij}{}^{(k)}(0)|$ and $j \in \{1, \cdots, n\}$.

From Theorem 1, it can be proved that the algorithms (24) and (25) (or (27) and (28)) can be used to acquire a CIC filtered source signal $\tilde{\boldsymbol{\delta}}_{j_0}^T\tilde{\boldsymbol{s}}_{j_0}(t)$.

3.2. Second stage

In this subsection, we show how to obtain a source signal $s_{j_0}(t - k_i)$. There are two cases:
Case 1: If the order K in (1) is known,

$$u_i(t) = (1/g_i(z))y_i(t) \qquad (34)$$

is calculated. Because $g_i(z) = g_{ij_0}(1 + z + \cdots + z^{K+L})$ can be obtained by (27) and (28), where g_{ij_0} is a complex constant.

Case 2: If the order K is unknown, we use the following theorem.
Theorem 2(Liu-Dong Theorem[6]) Let $y(t)$ be the output of an SISO-FIR system; $y(t) = \sum_{k=0}^{M} g^{(k)}s(t-k)$. Assume that $s(t)$ is white in the sense of second-order statistics; $E[s(t)s(t)^*] = 1$ and $E[s(t)s(t-\tau)^*] = 0$ $(\tau \neq 0)$. The filter $g(z)$ becomes dz^l, if and only if the output $y(t)$ is second-order uncorrelated signal, that is, $E[y(t)y(t)^*] = 1$ and $E[y(t)y(t-\tau)^*] = 0$ $(\tau \neq 0)$.

To implement the whitening of $y_i(t)$, we consider applying the following AR model to the CIC filtered output $y_i(t)$:

$$y_i(t) = -\sum_{k=1}^{M} v_i{}^{(k)}y_i(t-k) + \beta u_i(t). \qquad (35)$$

where M is the order of the AR model and β is a constant. The whitening of $y_i(t)$ can be achieved by constructing the AR model (35) with a sufficiently large order M. From equation (35), we have

$$u_i(t) = V(z)y_i(t), \qquad (36)$$

where

$$V(z) = \beta^{-1}(1 + \sum_{k=1}^{M} v_i{}^{(k)}z^k). \qquad (37)$$

The parameters β and $v_i{}^{(k)}$ can be found using Yule-Walker equations and the Levinson algorithm.

3.3. The deconvolution algorithm

Our proposed algorithm can be summarized in the following steps:

Step 1. Set $i = 1$ (where i denotes the order of an input extracted).

Step 2. Take random initial values $w_{ij}{}^{(k)}(0)$ of $w_{ij}{}^{(k)}$, and then calulate $\tilde{\boldsymbol{w}}_i(0)/\sqrt{\tilde{\boldsymbol{w}}_i(0)^{*T}\tilde{\boldsymbol{R}}\tilde{\boldsymbol{w}}_i(0)}$, where $\tilde{\boldsymbol{w}}_i(0)$ is the initial value of $\tilde{\boldsymbol{w}}_i$. Set $l = 0$ (where l denotes the number of iterations).

Step 3. Calculate $\tilde{\boldsymbol{w}}_i(l)$ using (27) and (28). The expectionation can be estimated using a large sample of $y_i(t)$ and $x_j(t)$ (in our experimental studies, we used about 10,000 points)

Step 4. If $|\tilde{\boldsymbol{w}}_i{}^{*T}(l)\tilde{\boldsymbol{R}}\,\tilde{\boldsymbol{w}}_i(l-1)|$ is not close enough to 1, set $l = l + 1$ and go back to Step 3. Otherwise go to the next step.

Step 5. Calculate (34) or find the AR output using equation (36).

Step 6. At this stage, we assume that the source signal $s_{p_i}(t)$ has been recovered. Then we should compute the scale and the time-shift of the input $s_{p_i}(t)$ by using the following equations:

$$\begin{aligned} y_i(t) &= \sum_{j=1}^{m}\sum_{k} w_{ij}{}^{(k)} x_j(t-k), \\ u_i(t) &= \frac{1}{g_i(z)} y_i(t), or \\ u_i(t) &= \beta^{-1}(y_i(t) + \sum_{k=1}^{M} v_i{}^{(k)} y_i(t-k)), \end{aligned}$$

where $w_{ij}{}^{(k)}$, $g_i(z)$, and $v^{(k)}$ are obtained by the above five steps.

Step 7. Estimate the scale and the time-shift of $h_{qp_i}(\tau)$ by using the cross-correlation of the observed signals $x_q(t)$ and $u_i(t)$ as

$$\hat{h}_{qp_i}(\tau) = E[x_q(t)u_i(t-\tau)^*], \qquad q = 1, 2, \cdots, m. \quad (38)$$

Step 8. Estimate the contribution of $s_{p_i}(t)$ to the observed signals $x_q(t)$ $(q = 1, 2, \cdots, m)$, that is, $\sum_\tau h_{qp_i}(\tau)$ $s_{p_i}(t-\tau)$, using

$$\hat{x}_{qp_i}(t) = \sum_{\tau} \hat{h}_{qp_i}(\tau) u_i(t-\tau), \quad (39)$$

Step 9. Remove the above contribution using the following equation:

$$x_q{}^{(i)}(t) = x_q(t) - \hat{x}_{qp_i}(t), \quad (40)$$

where $x_q{}^{(i)}(t)$ $(q = 1, \cdots, m)$ are the outputs of a linear unknown multichannel system with m outputs and $n-1$ inputs.

Step 10. If the superscript (i) of $x_q{}^{(i)}(t)$ is less than n, then set $i = i + 1$ and $x_q(t) = x_q{}^{(i)}(t)$ $(q = 1, \cdots, m)$, and the procedures mentioned above are continued until $i = n$.

Note that the procedure of Step 6 to Step 10 are implemented to make it possible to extract the other source signals from the observed signals and the extracted source signals.

4. COMPUTER SIMULATIONS

To demonstrate the validity of our algorithm. Many computer simulations were conducted. Some results are shown in this section. We considered the following two-input and three-output FIR system.

$$\boldsymbol{H}(z) = \begin{bmatrix} 1.0 + 0.6z + 0.3z^2 & 0.6 + 0.5z - 0.2z^2 \\ 0.5 - 0.1z + 0.2z^2 & 0.3 + 0.4z + 0.5z^2 \\ 0.7 + 0.1z + 0.4z^2 & 0.1 + 0.2z + 0.1z^2 \end{bmatrix}.$$

The observed signals $\boldsymbol{x}(t) = [x_1(t), x_2(t), x_3(t)]^T$ $(t = 0, 1, 2, \cdots)$ were calculated by (2). The source signals $\{s_i(t)\}$ $(i = 1, 2)$ were generated using the following system:

$$\begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} = \begin{bmatrix} \frac{0.3+z}{1+0.3z} & 0 \\ 0 & \frac{0.1+z}{1+0.1z} \end{bmatrix} \begin{bmatrix} \nu_1(t) \\ \nu_2(t) \end{bmatrix}. \quad (41)$$

where $\{\nu_1(t)\}$ and $\{\nu_2(t)\}$ were non-Gaussian i.i.d. signals with zero mean and unit variance, but were independent with each other. Since the filters $(0.3+z)/(1+0.3z)$ and $(0.1+z)/(1+0.1z)$ in (41) were all-pass filters, the source signals $\{s_1(t)\}$ and $\{s_2(t)\}$ became temporally second-order white but temporally fourth-order colored signals. The values of τ_1, τ_2, and τ_3 in (32) were set to belong to the intervals [0,20], [0,20], and [0,20], respectively. To extract two source signals from the observed signals, we used two $3(L+1)$-column vectors $\tilde{\boldsymbol{w}}_1$ and $\tilde{\boldsymbol{w}}_2$, where L was set to 2, and $\tilde{\boldsymbol{w}}_1$ and $\tilde{\boldsymbol{w}}_2$ were used in the first deflation and second one, respectively. The parameters $w_{ij}{}^{(k)}$ $(i = 1, 2; j = 1, 2, 3; k = 0, 1, 2)$ were set to zero except for $w_{11}{}^{(0)} = 1$ and $w_{21}{}^{(1)} = 1$, where the initial values of $w_{ij}{}^{(k)}$'s were given by calculating $\tilde{\boldsymbol{w}}_i / \sqrt{\tilde{\boldsymbol{w}}_i{}^{*T}\tilde{\boldsymbol{R}}\tilde{\boldsymbol{w}}_i}$. The number M in (35) was set to 30. The expectations in (27) were estimated using 10000 data samples of $y(t)$ and $x_j(t)$.

The following $\tilde{\boldsymbol{w}}_1$ was found after 8 Monte Carlo runs by the procedure of Step 2.

$$\begin{aligned} \tilde{\boldsymbol{w}}_1 &= [0.33, 0.56, 0.63, -0.47, 0.14, 0.60, \\ &\quad -0.82, -1.85, -1.72]^T \quad (42) \\ (&= [w_{11}{}^{(0)}, w_{11}{}^{(1)}, w_{11}{}^{(2)}, w_{12}{}^{(0)}, w_{12}{}^{(1)}, w_{12}{}^{(2)}, \\ &\quad w_{13}{}^{(0)}, w_{13}{}^{(1)}, w_{13}{}^{(2)}]^T) \end{aligned}$$

From (42), we obtained the following $\tilde{\boldsymbol{g}}_1$;

$$\tilde{\boldsymbol{g}}_1 = [\mathbf{-0.48}, \mathbf{-0.50}, \mathbf{-0.46}, \mathbf{-0.40}, \mathbf{-0.38},$$

$$- 0.03, 0.003, -0.03, -0.01, 0.001]$$
$$(= [\boldsymbol{g}_{11}{}^{(0)}, \boldsymbol{g}_{11}{}^{(1)}, \boldsymbol{g}_{11}{}^{(2)}, \boldsymbol{g}_{11}{}^{(3)}, \boldsymbol{g}_{11}{}^{(4)}, \boldsymbol{g}_{12}{}^{(0)}, \boldsymbol{g}_{12}{}^{(1)}, \boldsymbol{g}_{12}{}^{(2)}, \boldsymbol{g}_{12}{}^{(3)}, \boldsymbol{g}_{12}{}^{(4)}]),$$

from which, it could been seen that the output signal $y_1(t)$ becames

$$y_1(t) \approx \tilde{\boldsymbol{\delta}}_1^T \tilde{\boldsymbol{s}}_1(t), \quad (43)$$

which means that the filtered source signal $\tilde{\boldsymbol{\delta}}_1{}^T \tilde{\boldsymbol{s}}_1(t)$ could be acquired by using (27) and (28). After calculating the AR output using (36), we obtained $u_1(t) \approx -s_1(t)$.

To measure the accuracy of the result obtained by Step 1 to Step 5, we calculated the intersymbol interference (ISI) defined by

$$ISI = \frac{\sum_{j=1}^{n} \sum_{k=0}^{K+L+M} |g'_{ij}{}^{(k)}|^2 - |g'_{i\cdot}{}^{(\cdot)}|^2_{max}}{|g'_{i\cdot}{}^{(\cdot)}|^2_{max}}, \quad (44)$$

where $g'_{ij}{}^{(k)}$ denote the parameters of the filter between $u_i(t)$ and $s_i(t)$, and $|g'_{i\cdot}{}^{(\cdot)}|^2_{max}$ are defined by

$$|g'_{i\cdot}{}^{(\cdot)}|^2_{max} := \max_{j,k} |g'_{ij}{}^{(k)}|^2.$$

In that experiment, we found that $ISI = 0.0587$. If $g'_{ij}{}^{(k)} = \delta(k - k_i)$ for $k = 0, \cdots, K + L + M$, where $k_i \in \{0, ..., K + L + M\}$ is a nonegative integer, ISI becomes zero. The experimental results showed good performances of our algorithms .

After the procedures of Step 6 to Step 10, we applied the procedure of Step 3 to $y_2(t)$ ($= \sum_{j=1}^{3} \sum_{k=0}^{2} w_{2j}{}^{(k)} x_j(t-k)$) and calculted (36). And, we obtained $u_2(t) \approx s_2(t)$. In this case, ISI became 0.11.

From the results shown in the example, one can see that source signals can be successfully extracted from their convolutive mixtures using our proposed algorithms.

5. CONCLUSIONS

In this paper we proposed an iterative algorithm for the blind deconvolution problem in the case of temporally second-order white and spatially second- and fourth-order uncorrelated signals. Our algorithm is an extension of the the super-exponential deflation algorithm proposed by Inouye and Tanebe [2] to the case of the blind deconvolution problem of MIMO-FIR channels driven by fourth-order colored source signals. Our proposed super-exponential algorithm was used to generate CIC filtered source signals from the mixtures of source signals. To recover the original source signals from the filtered source signals, the Liu-Dong theorem have been used.

We have carried out computer simulations to demonstrate our proposed algorithms. The results have shown that the proposed algorithms can be used successfully to achieve the blind deconvolution.

Even if source signals possess high-order auto-correlations, our proposed algorithm can be extended to such a case by adjusting the degree of p and q in (26). It can be seen from (24) that if $|\sum_{k=0}^{K+L} g_{ij}{}^{(k)}(0)|$ for all j are equal to zero, $g_{ij}^{(k)}(1)$'s become zero. This corresponds to a pathological case. In this case, we consider that by resetting the initial values of $w_{ij}{}^{(k)}(0)$ to be appropriate values, one of $|\sum_{k=0}^{K+L} g_{ij}{}^{(k)}(0)|$ $(j = 1, ..., n)$ becomes at least nonzero. We expect that the deflation algorithm dealt with in this paper may exhibit global convergence, similar the result in [5]. Unfortunately, we have not solved this problem yet. If we can solve this problem, the solution obtained by the deflation algorithm can be applied to the initial values of the equalizer in the case of the non-deflation algorithm (e.g., [3]). This is because, in the existing deflation algorithm, the results of recovered source signals gradually degrade as the number of the deflation increases, and it is impossible to guarantee global convergence in the case of the non-deflation algorithm [3].

6. REFERENCES

[1] Special Issue on Blind System Identification and Estimation, IEEE Proc., October 1998.

[2] Y. Inouye and K. Tanebe, Super-exponential algorithms for multichannel blind deconvolution, IEEE Trans. Signal Processing, vol. 48, no. 3, pp. 881-888, March 2000.

[3] R.-W. Liu and Y. Inouye, "Direct blind deconvolution of multiuser-multichannel systems driven by temporally white source signals," to appear in IEEE Trans. Inform. Theory.

[4] J. G. Proakis, Digital Communications, New York, NY: McGraw-Hill, 1995.

[5] O. Shalvi and E. Weinstein, "Super-exponential methods for blind deconvolution," IEEE Trans. Inform. Theory, vol. 39, pp. 504-519, 1993.

[6] R.-W. Liu and G. Dong, "A Fundamental Theorem for Multiple-Channel Blind Equalization," IEEE Trans. Circuits and Systems-1; Fundamental Theory and Applications, vol. 44, no. 5, pp. 472-473, May, 1997.