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# Automatic modulation recognition of MPSK signals using constellation rotation and its 4th order cumulant

Maciej Pedzisz *, Ali Mansour

*École Nationale Supérieure des Ingénieurs des Études et Techniques d'Armement, Laboratory "Extraction et Exploitation de l'Information en Environements Incertains" ($E^3I^2$), 2 rue François Verny, 29806 Brest cedex 9, France*

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**Abstract**

We derive and analyze a new pattern recognition approach for automatic modulation recognition of MPSK (2, 4, and 8) signals in broad-band Gaussian noise. Presented method is based on constellation rotation of the received symbols, and a 4th order cumulant of a 1D distribution of the signal's in-phase component. Using Fourier series expansion of this cumulant as a function of the rotation angle, we extract invariant features which are then used in a neural classifier. Discrimination power of the proposed set of features is verified through extensive simulations, and the performance of the suggested algorithm is compared to the maximum-likelihood (ML) classifiers. Corresponding results show that our technique is comparable to the coherent ML classifier and outperforms the non-coherent pseudo-ML method for all considered signal-to-noise ratio (SNR) without the computational overhead of the latter.


*Keywords:* Automatic modulation recognition; Signal classification; Constellation identification; Higher-order statistics

* Corresponding author.
*E-mail addresses:* pedzisma@ensieta.fr (M. Pedzisz), mansour@ieee.org (A. Mansour).

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## 1. Introduction

Automatic classification of an applied modulation type has received international scientific attention for over a two decades now. It can be considered as an intermediate step between signal interception and information recovery. When the modulation scheme is identified, an appropriate demodulator can be selected to demodulate the signal and then to recover the information.

Applications of such algorithms are primary military ones—in COMINT (communication intelligence), one need to recognize the applied modulation type to more reliably identify the source of an emission (ESM—electronic support measure). It is also useful in choosing the appropriate method of jamming (ECM—electronic counter measure), or to protect oneself (ECCM—electronic counter counter measure). Another group of applications can be found in the communications systems—spectrum monitoring, interference identification, universal demodulator, or software radio are the most promising ones.

Many modulation recognition techniques have been published in the literature. Some authors solve the problem using the decision-theoretic approach [1–4], others [5–9] by using pattern recognition algorithms. An extensive study of different modulation recognition methods can be found in a book written by Azzouz and Nandi [10].

Different order statistical moments and cumulants are the base for algorithms proposed in [6] (moments of the instantaneous phase), [11] (nonlinear combination of 2nd and 4th order moments of the signal), [12] (different statistics of the instantaneous amplitude, phase and frequency), [13] (eigendecomposition of spatial moments arranged in a symmetric positive definite matrix), [14] (different order cumulants and a hierarchical classifier), or [15] (4th, 6th, and 8th order cyclic cumulants).

In this contribution, we propose a new pattern recognition approach based on statistical properties of the constellation of the received signal. We overcome dimensionality problem in case of 2D distributions by rotating the constellation of the received symbols, and then analyzing 1D distribution of the in-phase component. Based on Fourier series expansion of the 4th order cumulant, we extract a set of invariant features which are then used in a neural classifier.

## 2. Signal model

Let us assume that signal's carrier frequency is known to the receiver or it can be correctly estimated [16–20]. After quadrature downconversion, the received MPSK ($M$-ary phase shift keying) signal can be written as

$$s(t) = Ae^{i\Theta} \sum_{k=1}^{K} e^{i\varphi_k} h(t - kT) + z(t), \tag{1}$$

$$\varphi_k \in \left\{ \tfrac{2\pi}{M}(m-1),\ m = 1, 2, \ldots, M \right\}, \tag{2}$$

where $A$ is a carrier amplitude, $\Theta$ is a carrier phase, $K$ denotes the number of observed symbols, $\varphi_k$ describes constellation of the signal, $h(t)$ is a pulse shaping function, $T$ is a symbol duration, and $z(t)$ corresponds to a complex, zero-mean, additive white Gaussian noise (AWGN).

Sampling the output of a blind channel equalizer [21–28] at a symbol rate, the $k$th received symbol can be expressed as

$$s[k] = \big(p[k] + z_p[k]\big) + i\big(q[k] + z_q[k]\big), \tag{3}$$

where $p[k]$ and $q[k]$ are signals's in-phase and quadrature components

$$p[k] = A\cos(\varphi_k + \Theta) \quad \text{and} \quad q[k] = A\sin(\varphi_k + \Theta) \tag{4}$$

and $z_p[k]$ and $z_q[k]$ are equivalent baseband noise components.

Without loss of generality, we assume that all modulation states ($\varphi_k$) are independent and identically distributed (i.i.d. processes), they are equiprobable (which is accomplished when source coding is applied), and the equivalent noise components ($z_p$ and $z_q$) are Gaussian, independent (so not correlated), centered and have the same variance ($\sigma_{z_p}^2 = \sigma_{z_q}^2 = \sigma_z^2$).

## 3. Distinctive features

To extract the information concerning applied modulation type, one can use a joint distribution of the received symbols $s[k]$. It can be modeled in terms of its in-phase and quadrature components as a mean of $M$ 2D probability density function (PDF) over all constellation points

$$f_{pq}(x, y) = \frac{1}{M}\sum_{k=1}^{M}\mathcal{N}_{2\mathrm{D}}\big(x, y; p[k], q[k], \sigma_z\big), \tag{5}$$

where

$$\mathcal{N}_{2\mathrm{D}}(x, y; \mu_x, \mu_y, \sigma) \triangleq \frac{1}{2\pi\sigma^2}\exp\left[-\frac{(x-\mu_x)^2 + (y-\mu_y)^2}{2\sigma^2}\right]. \tag{6}$$

The analyze of such distribution can be simplified by using a method based on 1D PDF of the rotated constellation[1] (multiplication the symbols $s[k]$ by a complex exponential $e^{i\alpha}$). Taking into consideration component $p$, the marginal PDF as a function of the rotation angle $\alpha$ can be expressed as

$$f_p(x; \alpha) = \frac{1}{M}\sum_{k=1}^{M}\mathcal{N}(x; \mu_k, \sigma_k), \tag{7}$$

where

$$\mu_k = A\cos(\varphi_k + \Theta + \alpha), \qquad \sigma_k = \sigma_z,$$

and

$$\mathcal{N}(x; \mu, \sigma) \triangleq \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right].$$

[1] It can be also seen as a radon transform of a 2D PDF.

Its moments can be calculated using the following formulae:

$$m_r(\alpha) = \int_{-\infty}^{\infty} x^r f_p(x;\alpha)\,dx = \frac{1}{M}\sum_{k=1}^{M}\int_{-\infty}^{\infty} x^r \mathcal{N}(x;\mu_k,\sigma_k)\,dx \tag{8}$$

and the cumulants using the relation [29]

$$\kappa_r(\alpha) = m_r(\alpha) - \sum_{l=1}^{r-1}\binom{r-1}{l-1}\kappa_l(\alpha)m_{r-l}(\alpha), \qquad \kappa_1(\alpha) = m_1(\alpha). \tag{9}$$

Using the facts that the constellations of MPSK signals are symmetric, zero-mean, and are formed as a sum of Gaussians, all odd moments are 0, and the low order even moments can be expressed as[2]

$$m_2(\alpha) = \frac{1}{M}\sum_{k=1}^{M}\mu_k^2 + \sigma_k^2, \tag{10}$$

$$m_4(\alpha) = \frac{1}{M}\sum_{k=1}^{M}\mu_k^4 + 6\mu_k^2\sigma_k^2 + 3\sigma_k^4. \tag{11}$$

Exploiting an expanded form of the 4th order cumulant for symmetric distributions

$$\kappa_4(\alpha) = m_4(\alpha) - 3m_2^2(\alpha) \tag{12}$$

one can write

$$\kappa_4(\alpha) = \frac{1}{M}\sum_{k=1}^{M}\left(\mu_k^4 + 6\mu_k^2\sigma_k^2 + 3\sigma_k^4\right) - 3\left(\frac{1}{M}\sum_{k=1}^{M}\mu_k^2 + \sigma_k^2\right)^2. \tag{13}$$

Taking into considerations a BPSK signal, one can write corresponding distribution as

$$f_p(x;\alpha) = \frac{1}{2}\mathcal{N}\big(x; A\cos(\Theta+\alpha),\sigma_z\big) + \frac{1}{2}\mathcal{N}\big(x; A\cos(\Theta+\pi+\alpha),\sigma_z\big). \tag{14}$$

Its 2nd and 4th order moments are

$$m_2(\alpha) = A^2\cos^2(\Theta+\alpha) + \sigma_z^2, \tag{15}$$

$$m_4(\alpha) = A^4\cos^4(\Theta+\alpha) + 6A^2\sigma_z^2\cos^2(\Theta+\alpha) + 3\sigma_z^4 \tag{16}$$

and the 4th order cumulant becomes

$$\kappa_4(\alpha) = -\tfrac{3}{4}A^4 - \tfrac{1}{4}A^4\cos(4\Theta+4\alpha) - A^4\cos(2\Theta+2\alpha). \tag{17}$$

It should be noted that among the even cumulants, calculating the 4th one is sufficient enough to identify 2, 4 and 8-PSK signals. The estimator is simple to calculate, effective in comparison to higher order cumulants (as far as the variance is concerned), and there are methods to make it unbiased and/or adaptive [31]. Besides, it is the first one

[2] For a Gaussian PDF [30], we have $m_2 = \mu^2 + \sigma^2$ and $m_4 = \mu^4 + 6\mu^2\sigma^2 + 3\sigma^4$.

Table 1
Coefficients of Fourier series expansion for BPSK, QPSK, and 8PSK signals

| | $c_0$ | $c_2$ | $c_4$ | $\varphi_2$ | $\varphi_4$ |
|---|---|---|---|---|---|
| BPSK | $-\frac{3}{4}A^4$ | $A^4$ | $\frac{1}{4}A^4$ | $2\Theta$ | $4\Theta$ |
| QPSK | $-\frac{3}{8}A^4$ | 0 | $\frac{1}{8}A^4$ | 0 | $4\Theta$ |
| 8PSK | $-\frac{3}{8}A^4$ | 0 | 0 | 0 | 0 |

(except for the 3rd one which is zero) that is not dependent of $\sigma_z$ (invariant to Gaussian noise).

It is easy to show that for all MPSK modulation schemes, $\kappa_4(\alpha)$ is a periodic function of $\alpha$, it meets Dirichlet conditions, and there are maximum two harmonic components (2nd and/or 4th) and a constant. These properties make possible to express it as an even term Fourier series

$$\kappa_4(\alpha) = \frac{a_0}{2} + \sum_{l\in\{2,4\}} \big[a_l \cos(l\alpha) + b_l \sin(l\alpha)\big] = c_0 + c_2 \cos(2\alpha + \varphi_2) + c_4 \cos(4\alpha + \varphi_4), \tag{18}$$

where

$$a_l = \frac{1}{\pi}\int_{-\pi}^{\pi} \kappa_4(\alpha)\cos(l\alpha)\,d\alpha, \qquad b_l = \frac{1}{\pi}\int_{-\pi}^{\pi} \kappa_4(\alpha)\sin(l\alpha)\,d\alpha \tag{19}$$

and

$$c_0 = \frac{a_0}{2}, \qquad c_l = \sqrt{a_l^2 + b_l^2}, \qquad \varphi_l = \arctan\left(-\frac{b_l}{a_l}\right). \tag{20}$$

Using the relations (13), (19), and (20), it is straightforward to find the coefficients of Fourier series expansion for different MPSK signals. Corresponding results are provided in Table 1.

It is evident that the information concerning the shape of a constellation is contained in the coefficients $c_0$, $c_2$, and $c_4$; $\varphi_2$ and $\varphi_4$ carry only the information on initial carrier phase ($\Theta$).

To obtain the characteristics which are invariant to amplitude $A$, one can use the following normalization schemes:

$$C_l = \frac{|c_l|}{|c_0| + |c_2| + |c_4|}, \qquad C_l = \frac{c_l^2}{c_0^2 + c_2^2 + c_4^2}. \tag{21}$$

Both of them give different values of $C_l$ coefficients, and to determine which one is better, it is necessary to evaluate the final classifier. In Section 4 (Eqs. (25)–(27)) we describe the application of LDA transform to obtain the features which are used during classification. Using the classifier described there, we have made the simulations and obtained results (probability of error) were the same.

Using the simpler (and faster) method (normalization by sum of absolutes values), we obtained

$$\text{BPSK:} \quad C_0 = 3/8, \quad C_2 = 1/2, \quad C_4 = 1/8, \tag{22}$$

$$\text{QPSK:} \quad C_0 = 3/4, \quad C_2 = 0, \quad C_4 = 1/4, \tag{23}$$

$$\text{8PSK:} \quad C_0 = 1, \quad C_2 = 0, \quad C_4 = 0. \tag{24}$$

It is clear that the coefficients $C_l$ are normalized ($C_l \in [0, 1]$) real numbers, independent of the signal ($A$) and noise ($\sigma_z$) levels, as well as the initial carrier phase ($\Theta$).

In general, for $M$-ary PSK signals, only the $M$th harmonic of Fourier series expansion (except the constant value $a_0/2$) is non-zero. Knowing that this $M$th component can be seen at least in cumulants of order $M$, and the cumulant estimator variance grows fast with the cumulant order, it is clear that this method is limited to the case $M = 8$ (which is sufficient for the signals used in practice). The methods to get round with this limit, as well as the verification of this method for $M$-ary quadrature amplitude modulation (MQAM) signals, is our current topic of interest.

## 4. Classifier and experimental results

It is obvious that limiting the number of features will make learning and testing faster and demanding less memory. Aside from this, feature space of a lower dimension may enable more accurate classifiers for a finite learning set. In this 3-class problem, we decided to reduce the feature space to two dimensions by applying linear discriminant analysis (LDA), and then to use a feed-forward neural Network as a classifier.

Based on the characteristics presented in Section 3, and using the Fisher's criterion [32]

$$J_{\mathrm{F}} = \operatorname{tr}\{\mathbf{T}\} = \operatorname{tr}\left\{\mathbf{S}_{\mathrm{w}}^{-1}\mathbf{S}_{\mathrm{b}}\right\}, \tag{25}$$

where $\mathbf{S}_{\mathrm{w}}$ is the within-class covariance matrix (the sum of covariance matrices computed for each class separately), and $\mathbf{S}_{\mathrm{b}}$ is the between-class covariance matrix (the covariance matrix of class means), one can reduce dimensionality of the feature vector

$$\mathbf{x} = [C_0, C_2, C_4]^{\mathrm{T}} \tag{26}$$

by means of a linear transform

$$\mathbf{y} = \mathbf{W}\mathbf{x}, \tag{27}$$

where eigenvectors corresponding to largest eigenvalues of $\mathbf{T}$ form the rows of the transformation matrix $\mathbf{W}$.

Using the feature vector in the transformed space $\mathbf{y} = [y_1, y_2]^{\mathrm{T}}$ as the input to the neural network with one nonlinear hidden layer (3 neurons) and a linear output layer (one neuron per class), we constructed and trained the classifier with the training set of 5000 trials of BPSK, QPSK, and 8PSK MATLAB generated signals containing 100 and 256 symbols. Neurons of the hidden layer had sigmoidal transfer function, and for the training purposes we used the Levenberg–Marquardt [33] algorithm. To verify the performance of the proposed set of features and the classifier itself, we used another, independent validating set of 5000 trials of the same signal types. Source signals were modeled as uniformly distributed on all constellation states, additive noise was modeled as Gaussian, and $\mathrm{SNR} = 10\log(A^2/2\sigma_z^2)$ was varying from $-4$ to 14 dB with the step of 2 dB.

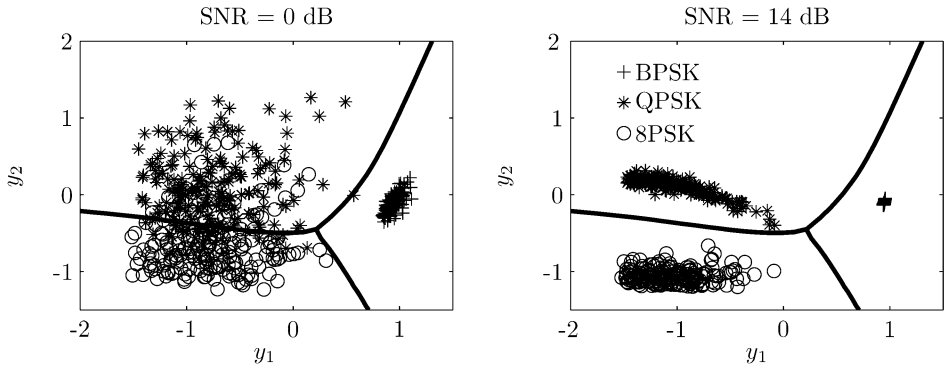


Fig. 1. Signals in a 2D space after dimensionality reduction.

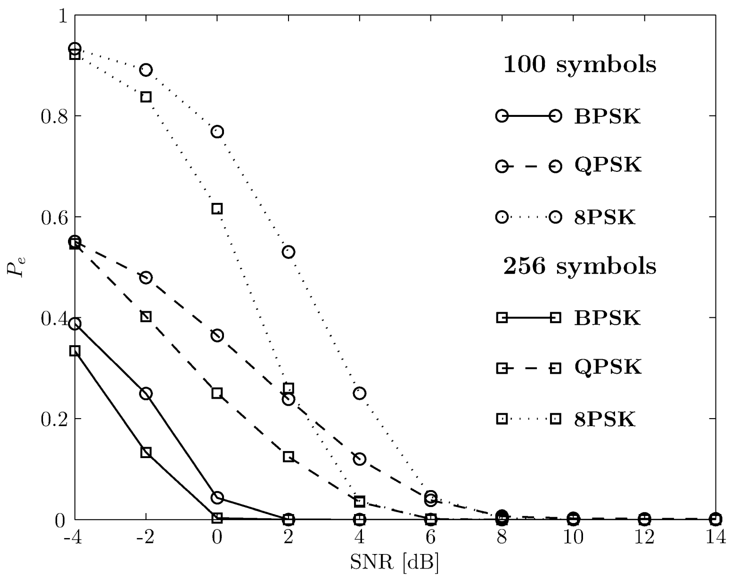


Fig. 2. Probability of error for BPSK, QPSK, and 8PSK signals.

In Fig. 1, we present BPSK, QPSK, and 8PSK signals in a 2D transformed space, as well as the decision regions of the trained classifier. There are 200 different realizations of each signal composed of 100 symbols, SNR was equal 0 (left figure) and 14 dB (right figure).

In Fig. 2, the probability of error (misclassification) ($P_e$) is shown for the 3 signals as a function of SNR. Whole validating set (5000 trials) was used during the simulation, and the SNR was varying from $-4$ to 14 dB. Finally, the confusion matrices for SNR $= 2$ dB are presented in Table 2.

It is clear that proposed set of distinctive features is very efficient even for low SNR. Perfect classification can be obtained for all considered signals when SNR $> 8$ dB (256

Table 2
Confusion matrices for SNR = 2 dB

| Input | Output | | | | | |
|---|---|---|---|---|---|---|
| | 100 symbols | | | 256 symbols | | |
| | BPSK | QPSK | 8PSK | BPSK | QPSK | 8PSK |
| BPSK | 0.9998 | 0 | 0.0002 | 1.0000 | 0 | 0 |
| QPSK | 0.0554 | 0.7616 | 0.1830 | 0.0032 | 0.8756 | 0.1212 |
| 8PSK | 0.0757 | 0.4546 | 0.4697 | 0.0095 | 0.2510 | 0.7395 |

symbols) and SNR > 10 dB (100 symbols). For a fixed probability of correct classification ($P_{cc} = 1 - P_e \geqslant 0.95$), classification can be assured when SNR $\approx$ 0 dB (BPSK), and SNR $\approx$ 4 dB (QPSK and 8PSK) for 256 symbols. When number of available symbols decreases to 100, there is a loss of performance of about 2 dB for all signals.

This loss of performance is due the fact that moments and cumulants are estimated using a finite number of samples. Knowing that these estimators are asymptotically effectives, one can anticipate that increasing the number of available symbols improves the classifier performance. From the practical point of view, this number cannot be too big because of the system inertia (rapid changes are not reflected) and the computational costs (calculation time and used memory). From the other side, if the number of available symbols is too small, the loss of performance is twofold: the variances of the estimators increase, and the assumption that all modulation states are equiprobable is no longer valid (extreme case: some of the constellation points are not observed).

The most confused modulation types are QPSK and 8PSK—at SNR = 2 dB (100 symbols), the output of the classifier is almost equally divided between the two modulation types.

## 5. Conclusion

Our new algorithm is targeted automatic modulation recognition of the most common used in practice modulation types (as far as MPSK signals are concerned). It is theoretically independent of the signal and noise levels, as well as the initial carrier and local oscillator phases. It is based on statistical properties of the received symbols, thus timing parameters (number of samples per symbol and a baud rate) do not influence the output of the classifier. It can be incorporated in a COMINT system (after carrier frequency recovery and blind channel equalizer blocs) as a part of a more general classifier (MPSK, MQAM, MFSK, OFDM, ...).

Proposed set of distinctive features is very efficient even for low SNR. Having at least 256 symbols available, perfect classification can be obtained for all considered signals when SNR > 8 dB. Allowing probability of error to be $P_e \approx 0.05$, classification can be assured when SNR $\approx$ 0 dB (BPSK) and SNR $\approx$ 4 dB (QPSK and 8PSK).

Comparing our algorithm to the ML classifiers proposed by Sills [34], we can conclude: for SNR > 4 dB and 256 symbols, the performance of our algorithm is comparable to that obtained with coherent (all signal parameters assumed known) ML approach, and is better than non-coherent pseudo-ML (all signal parameters are known except the carrier phase).

Extension of this method to MQAM signaling scheme, as well as searching for other, constellation based features is our current topic of interest. In the future, we will take into consideration other signal types (DQPSK, Π/4-QPSK, Π/8-8PSK), as well as the effects of inter-symbol and co-channel interferences.

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