Subspace Adaptive Algorithm For Blind Separation Of Convolutive Mixtures By Conjugate Gradient Method.

A. Mansour, A. Kardec Barros and N. Ohnishi
Bio-Mimetic Control Research Center (RIKEN),
2271-130, Anagahora, Shimoshidami, Moriyama-ku, Nagoya 463 (JAPAN)
email: mansour@nagoya.riken.go.jp, allan@nagoya.riken.go.jp,
ohnishi@sheraton.ohnishi.nuie.nagoya-u.ac.jp.
http://www.bmc.riken.go.jp/sensor/Mansour/mansour.html

Abstract

In this paper, a new subspace adaptive algorithm, for blind separation of convolutive mixture, is proposed. This algorithm can be decomposed into two steps: At first, the convolutive mixture will be reduced to an instantaneous mixture (memoryless mixture), using a second-order statistics criterion based on subspace approach. The second step consists on the separation of the residual instantaneous mixture.

The minimization of the criterion is achieved using a conjugate gradient method. The experimental results show that the convergence of our algorithm is improved thanks to the use of the conjugate gradient method. Finally, experimental results are shown.

1 Introduction

The problem of blind separation of independent sources consists in retrieving the sources from the observation of unknown mixtures of the unknown sources [7, 11, 13]. Since 1990, few methods of source separation have been proposed in the case of convolutive mixtures (i.e the channel effect can be considered as a linear filter). These methods were generally based on high order statistics [8, 16, 12, 3].

Recently, some subspace methods have been explored to solve the blind identification or separation of sources problem [4, 5, 14, **?**, 6]. The advantage of these methods is: by using only second order statistics (but more sensors than sources), we can separate the sources (with some assumptions concerning the channel filters) or identify the convolutive mixture up to an instantaneous mixture. The subspace methods are very elegant methods from theoretical point of view, but in general case, the convergence of these algorithms are relatively slow due to the minimization of large size matrices.

In [14], we proposed a subspace method for a convolutive mixture model based on LMS algorithm. Unfortunately, that algorithm was very slow due to the large size of the matrices and the use of LMS method. That algorithm requires mores than 7000 iterations to converge. In this paper we propose another criterion also based on subspace approach but this criterion is minimized using conjugate gradient algorithm [2]. The convergence of the proposed method is relatively fast, and may be achieved in less than 1000 iterations.

This new algorithm can be decomposed in two steps: in the first step, by only using second-order statistics, we reduce the convolutive mixture problem to an instantaneous mixture; then in the second step, we must only separate sources consisting of a simple instantaneous mixture (typically, most of the instantaneous mixture algorithms are based on fourth-order statistics).

2 Channel model

Let us consider p unknown and statistical independent sources $S(n)$ observed by using q sensors $Y(n)$, with $q > p$.

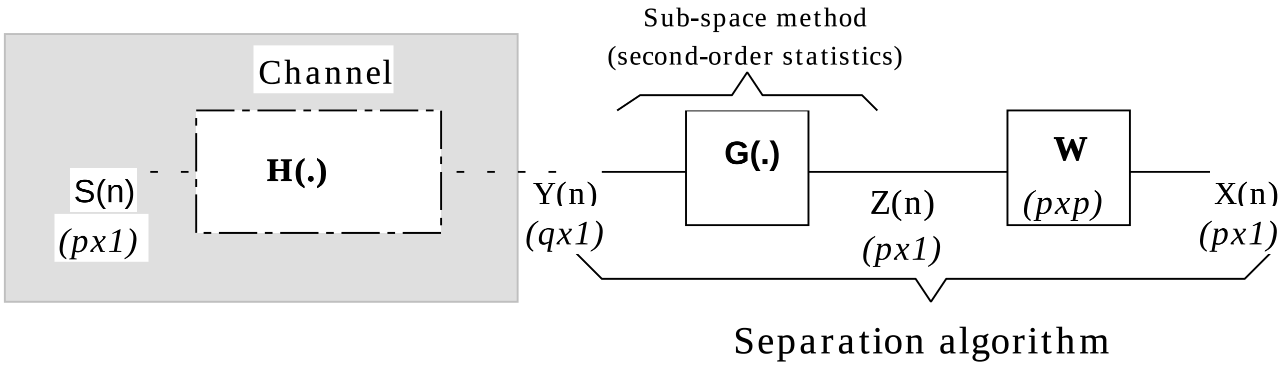


Figure 1: General structure.

Denote the channel effect by a $q \times p$ polynomial matrix $\mathbf{H}(z) = (h_{ij}(z))$, entries of which $h_{ij}(z)$ are finite impulse response (FIR) filters, and by M the highest degree of the filters $h_{ij}(z)$. In the sequel, M will be called the degree of the filter matrix $\mathbf{H}(z)$. Denote by $\mathbf{H}(i)$ the real $q \times p$ matrix corresponding to the filter matrix $\mathbf{H}(z)$ at time i:

$$\mathbf{H}(z) = (h_{ij}(z)) = \sum_{i=0}^{M} \mathbf{H}(i) z^{-i}. \tag{1}$$

The mixture vector $q \times 1$, at time n, is given by:

$$Y(n) = \sum_{i=0}^{M} \mathbf{H}(i) S(n-i), \tag{2}$$

where $S(n-i)$ is the $p \times 1$ source vector at the time $(n-i)$. Let us use the following notations:

$$Y_N(n) = \begin{pmatrix} Y(n) \\ \vdots \\ Y(n-N) \end{pmatrix}, \tag{3}$$

$$S_{M+N}(n) = \begin{pmatrix} S(n) \\ \vdots \\ S(n-M-N) \end{pmatrix}. \tag{4}$$

By using $N > q$ observations of the mixture vector, we can formulate the model (2) in another form:

$$Y_N(n) = \mathbf{T}_N(\mathbf{H}) S_{M+N}(n), \tag{5}$$

where $\mathbf{T}_N(\mathbf{H})$ is the Sylvester matrix corresponding to $\mathbf{H}(z)$. The Sylvester matrix $q(N+1) \times p(M+N+1)$ is given by [9]:

$$\mathbf{T}_N(\mathbf{H}) = \begin{bmatrix} \mathbf{H}(0) & \mathbf{H}(1) & \mathbf{H}(2) & \dots & \mathbf{H}(M) & 0 & 0 & \dots & 0 \\ 0 & \mathbf{H}(0) & \mathbf{H}(1) & \dots & \mathbf{H}(M-1) & \mathbf{H}(M) & 0 & \dots & 0 \\ \vdots & \vdots & & & & & & & \\ 0 & \dots & \dots & & 0 & \mathbf{H}(0) & \mathbf{H}(1) & \dots & \mathbf{H}(M) \end{bmatrix}. \tag{6}$$

3 Criterion and constraint

It is obvious from (4) and (5), that the source separation will be achieved by estimating $\mathbf{S}_{M+N}(n)$. By consequence the separation can be done by estimating a $(M+N+1)p \times q(N+1)$ left inverse matrix $\mathbf{G}$ of the Sylvester matrix, which exists if the matrix $\mathbf{T}_N(\mathbf{H})$ has a full rank.

It was proved in [1] that the rank of $\mathbf{T}_N(\mathbf{H})$ is given by:

$$\text{Rank } \mathbf{T}_N(\mathbf{H}) = p(N+1) + \sum_{i=1}^{p} M_i, \tag{7}$$

where M_i is the degree of the ith column of $\mathbf{H}(z)$. The degree of a column is defined as the highest degree of the filters in this column. It is easy to prove using (7) that the Sylvester matrix has a full rank and it is left invertible if each column of the polynomial matrix $\mathbf{H}(z)$ has the same degree and $N > Mp$.

Suppose that $\mathbf{G}$ is the left inverse of $\mathbf{T}_N(\mathbf{H})$ then we can remark:

$$\begin{aligned} \mathbf{G}\mathbf{Y}_N(n) &= \mathbf{S}_{M+N}(n), \\ \mathbf{G}\mathbf{Y}_N(n+1) &= \mathbf{S}_{M+N}(n+1). \end{aligned} \tag{8}$$

Let us denote by G_i the ith block row[1] of G. By using (8), we can easily demonstrate that:

$$\begin{aligned} \mathcal{G}\mathcal{Y}(n) &= (G_1, G_2, \ldots, G_{(M+N+1)}) \begin{pmatrix} Y_N(n) & 0 & \ldots & 0 \\ -Y_N(n+1) & Y_N(n) & 0 & \ldots \\ 0 & -Y_N(n+1) & \ldots & \vdots \\ \vdots & \ldots & \ldots & \vdots \\ 0 & \ldots & -Y_N(n+1) & Y_N(n) \\ 0 & \ldots & 0 & -Y_N(n+1) \end{pmatrix}, \\ &= 0. \end{aligned}$$

where $\mathcal{G} = (G_1, G_2, \ldots, G_{(M+N+1)})$ is a $p \times q(N+1)(M+N+1)$ matrix. From the previous equation (9), a simpler criterion can be derived:

$$\min_{\mathcal{G}} \mathcal{G} \sum_{n=n_0}^{n_1} \mathcal{Y}(n)\mathcal{Y}^T(n)\mathcal{G}^T. \tag{9}$$

The sum operation is added to improve the performances of the experimental results. In addition the choice of number n_0 and n_1 depends on the data and it has some influence on the convergence speed of the algorithm (in our experimental study, we used $20 < n_1 - n_0 < 50$).

It was proved for similar criterion [10, **?**] that the minimization of this kind of cost function (9) does not give the Moore-Penrose generalized inverse (pseudoinverse) of the Sylvester matrix $\mathbf{T}_N(H)$, but a $(M+N+1)p \times q(N+1)$ matrix $\mathbf{G}$ which satisfies that $\mathbf{G}\mathbf{T}_N(\mathbf{H})$ is a block diagonal matrix:

[1] G_i is $p \times q(N+1)$ matrix and $G = (G_1^T, \ldots, G_{M+N+1}^T)^T$.

$$\mathbf{GT}_N(\mathbf{H}) = \begin{pmatrix} \mathbf{A} & 0 & \dots & \dots & 0 \\ 0 & \mathbf{A} & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \dots & 0 & \mathbf{A} \end{pmatrix}, \tag{10}$$

where $\mathbf{A}$ is an arbitrary $p \times p$ matrix.

It is clear that as the algorithm converges, the estimated sources are instantaneous mixtures (according to a matrix $\mathbf{A}$) of actual sources: in fact using (5) and (10), we find that:

$$\mathbf{G}Y_N(n) = \begin{pmatrix} AS(n) \\ \vdots \\ AS(n-M-N) \end{pmatrix}. \tag{11}$$

To avoid the spurious solution $\mathbf{G} = \mathbf{0}$ and force the matrix $\mathbf{A}$ to be an invertible matrix[2], we propose the minimization subject to the constraint:

$$\mathbf{G}_1 \mathbf{R}_Y(n) \mathbf{G}_1^T = \mathbf{I}_p, \tag{12}$$

where $\mathbf{G}_1$ is the first block row $(p \times q(N+1))$ of $\mathbf{G}$, $\mathbf{R}_Y(n) = EY_N(n)Y_N(n)^T$ is the covariance matrix of $Y_N(n)$ and $\mathbf{I}_p$ is a $(p \times p)$ identity matrix. If the above constraint is verified then:

$$\mathbf{G}_1 \mathbf{R}_Y(n) \mathbf{G}_1^T = \mathbf{A} \mathbf{R}_S(n) \mathbf{A}^T = \mathbf{I}_p, \tag{13}$$

where $\mathbf{R}_S(n) = ES(n)S(n)^T$ is the source covariance matrix. $\mathbf{R}_S(n)$ is a full rank diagonal matrix thanks to the statistical independence of the p sources from each other. As consequence of (13), matrix $\mathbf{A}$ becomes invertible.

Experimentally, the cost function (9) is minimized using a conjugate gradient algorithm [2]. The algorithm proposed by Chen *et al.* in [2] can minimize a cost function $f(V)$ with respect to a vector (V). From theoretical point of view, this algorithm can converge in a number of iterations which is less than the dimension of V. In our case, the cost function (9) must be minimized with respect to a $p \times q(N+1)(M+N+1)$ matrix $\mathcal{G}$. As consequence, the cost function (9) should be decomposed into p cost functions, each one only depends on one line of $\mathcal{G}$. Afterwards, we can easily apply the conjugate gradient algorithm to minimize our criterion[3].

Finally, the constraint (12) can be satisfied easily by a simple Cholesky decomposition, than $\mathbf{G}_1$ can be normalized by $\mathbf{G}_1^* = (\mathbf{G}_1 \mathbf{R}_Y(n) \mathbf{G}_1^T)^{-1/2} \mathbf{G}_1$ at each iteration. In addition, the source separation of the instantaneous residual mixture is achieved according to the method proposed in [15].

4 Experimental results

Even if the convergence of this algorithm is attained in small number of iterations (in general case, less than 1000 iterations are needed), the convergence time is relatively important due to the minimization of large size matrices. For that reason, we present in this section some experimental results in the case of two sources. Actually, we are looking to improve the algorithm convergence, so we can separate more than two sources with reasonable time.

[2] So the separation of the residual instantaneous mixture becomes possible using any algorithm for the separation of instantaneous mixture

[3] Because the limitation of the page number, we can not give more details in this article.

The experimental study shows that for two stationary sources, the convergence of the subspace criterion (9) is attained with about 800 iterations (see figure 2).

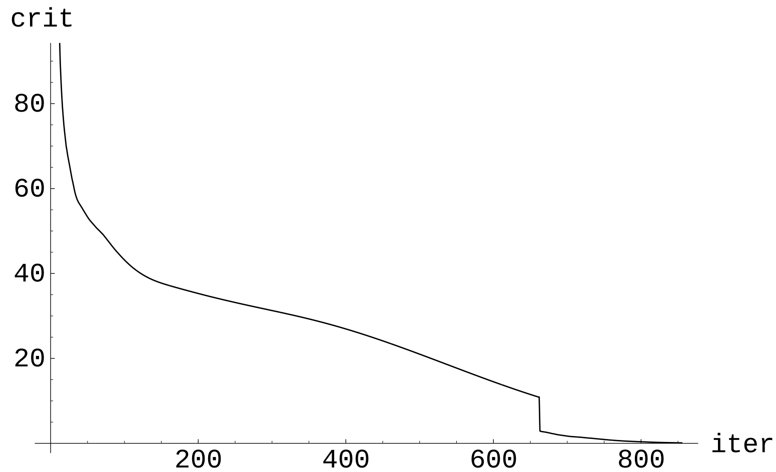


Figure 2: The convergence of the sub-space criterion

In that experiment, four sensors $q = 4$ and two stationary sources $p = 2$ were used:

- The first source is an independent identically distributed (iid) signal with an uniform probability density function (pdf).
- The second signal is output of an AM filter $h(z) = 1 + .5z^{-1} - .4z^{-2} + .2z^{-3}$, who has an iid with uniform pdf signal as input.

The channel effect $\mathbf{H}(z)$ is considered as a FIR filter of fourth degree ($M = 4$):

$$H(z) = \begin{pmatrix} -1 - 2z^{-1} + z^{-2} + 1.5z^{-3} + z^{-4} & z^{-1} + z^{-2} + 2z^{-3} + 1.5z^{-4} \\ 2 - 4z^{-1} + 4z^{-2} & 1 - 2z^{-1} + 1.5z^{-2} + z^{-3} + 0.5z^{-4} \\ -1 - z^{-1} + 0.4z^{-2} + 3z^{-3} - z^{-4} & 3 - 2z^{-2} + 2z^{-3} + z^{-4} \\ -2 + z^{-2} + 4z^{-3} - 1.5z^{-4} & 1 + 2z^{-1} - 2.5z^{-2} - z^{-3} + 0.4z^{-4} \end{pmatrix} \quad (14)$$

We can see in figure 3 that the objective of first step of the algorithm was achieved, with $G.T_N(H)$ being a block diagonal matrix (where A is a 2×2 matrix, see (10)).

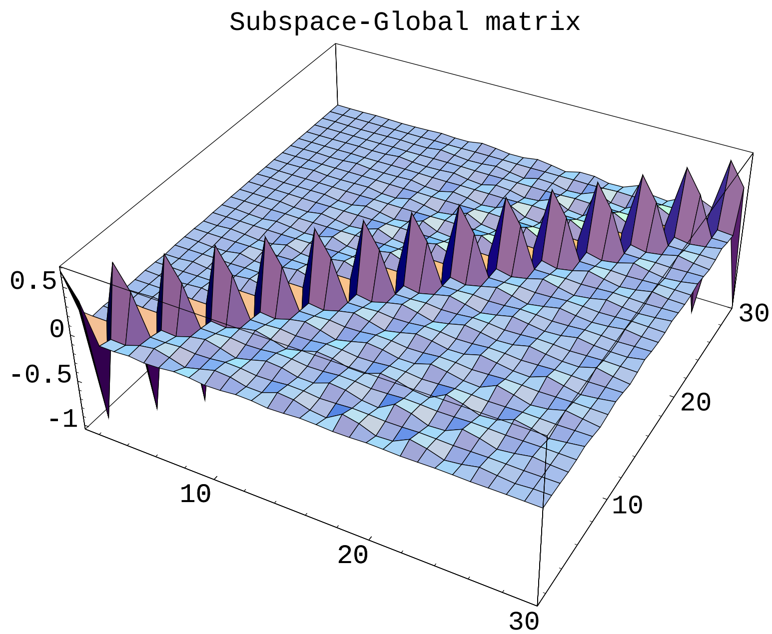


Figure 3: Performance results: $G.T_N(H)$ should be a block diagonal matrix.

When the minimization of the cost function (9) is achieved, the two ($p = 2$) output signals $z_i(n)$ are given by $Z(n) = (z_1(n), z_2(n))^T = \mathbf{A}S(n)$. The performance of this instantaneous residual mixture separation [15] is shown in figure 4.

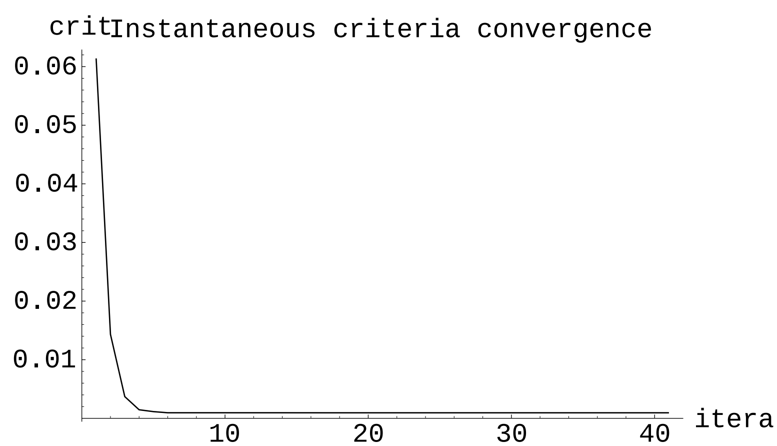
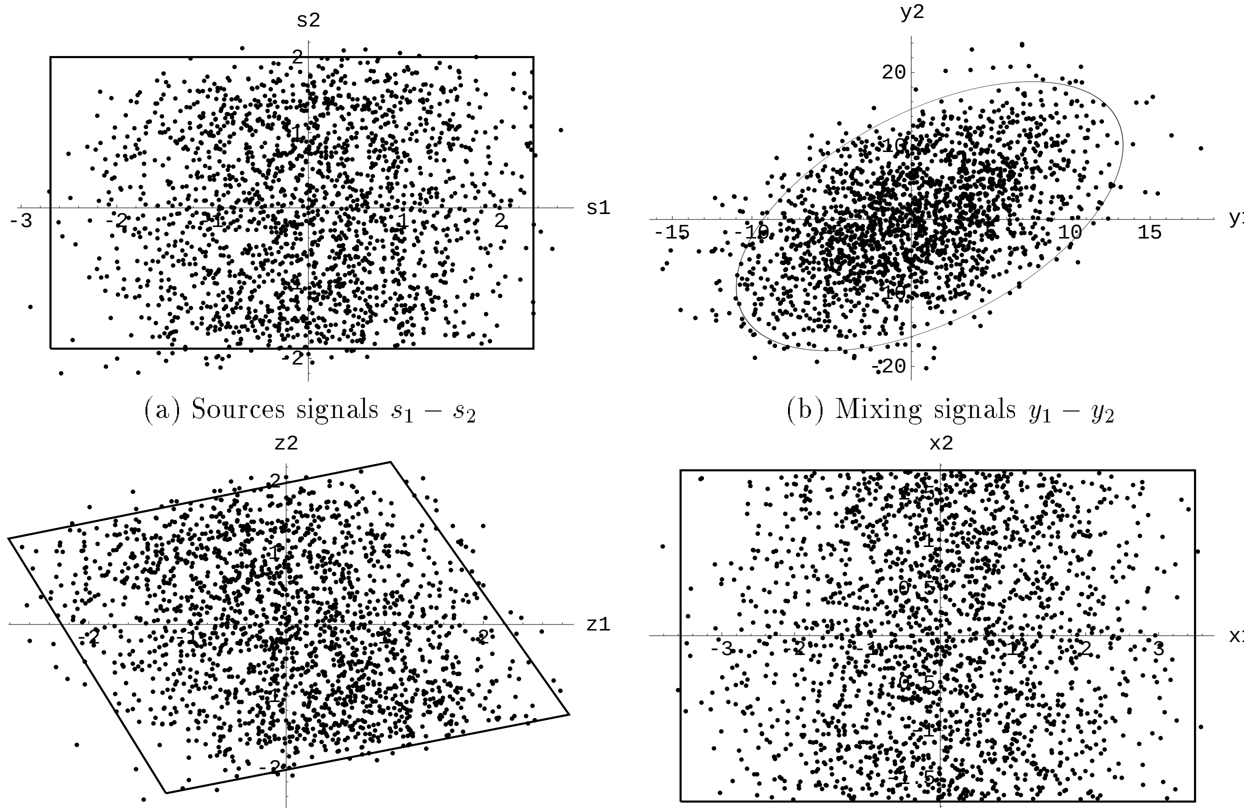


Figure 4: Performances of the instantaneous residual mixture separation.

Finally, to demonstrate the behavior of our algorithm and its performances, we plot the different signals in their own plane, as in figure 5.



(a) Sources signals $s_1 - s_2$

(b) Mixing signals $y_1 - y_2$

(c) First step of the sub-space algorithm $z_1 - z_2$

(d) Estimated signals $x_1 - x_2$

Figure 5: Experimental results.

In figure 5, we remark that the sources $s_1(n)$ and $s_2(n)$ are statistically independent as are estimated signals $x_1(n)$ and $x_2(n)$ (for more information concerning the relationship between the distribution of signals and their statistical relationships with each other, see [17]). In addition, from figure 5 (c) we can say that these signals may be obtained by mixing independent signals with help of an instantaneous mixtures. Finally, we can see the mixing signals in the figure 5 (b).

5 Conclusion

In this paper, we present a new sub-space algorithm to solve the problem of blind separation of sources for convolutive mixture. This algorithm is based on the minimization, using the conjugate gradient algorithm, of a sub-space criterion based on second-order statistics.

The minimization of that criterion can not achieve the separation, but it can transfer the convolutive mixture into an instantaneous mixture. In addition, the separation of the residual instantaneous mixture can be done using any instantaneous mixture algorithm, typically based on fourth-order statistics. By consequence, we find that most of the channel parameters can be estimated using only second-order statistics. The actual version of the algorithm is relatively fast. In general case the convergence of the sub-space criterion is attained in less than 1000 iterations.

We succeeded in separating two stationary sources, with about -22 dB of residual crosstalk. Currently, we are trying to separate more than two stationary or non-stationary sources (for example: speech signals).

Acknowledgments

The authors are grateful to Prof. Philippe Loubaton (Univ. de la Marne la Vallée, France) for discussions and comments.

References

[1] R. Bitmead, S. Kung, B. D. O. Anderson, and T. Kailath. Greatest common division via generalized Sylvester and Bezout matrices. *IEEE Trans. on Automatic Control*, 23(6):1043–1047, December 1978.

[2] H. Chen, T. K. Sarkar, S. A. Dianat, and J. D. Brule. Adaptive spectral estimation by the conjugate gradient method. *IEEE Trans. on Acoustics, Speech and Signal Processing*, ASSP-34(2):272–284, April 1986.

[3] N. Delfosse and P. Loubaton. Adaptive blind separation of convolutive mixtures. In *Proceding of ICASSP*, pages 2940–2943, Atlanta, Georgia, May 1996.

[4] D. Gesbert, P. Duhamel, and S. Mayrargue. Subspace-based adaptive algorithms for the blind equalization of multichannel fir filters. In M.J.J. Holt, C.F.N. Cowan, P.M. Grant, and W.A. Sandham, editors, *Signal Processing VII, Theories and Applications*, pages 712–715, Edinburgh, Scotland, September 1994. Elsevier.

[5] A. Gorokhov and P. Loubaton. Second order blind identification of convolutive mixtures with temporally correlated sources: A subspace based approch. In *Signal Processing VIII, Theories and Applications*, pages 2093–2096, Triest, Italy, September 1996. Elsevier.

[6] A. Gorokhov and P. Loubaton. Subspace based techniques for second order blind separation of convolutive mixtures with temporally correlated sources. *IEEE Trans. on Circuits and Systems*, 44:813–820, September 1997.

[7] C. Jutten and J. Hérault. Blind separation of sources, Part I: An adaptive algorithm based on a neuromimetic architecture. *Signal Processing*, 24(1):1–10, 1991.

[8] C. Jutten, L. Nguyen Thi, E. Dijkstra, E. Vittoz, and Caelen J. Blind separation of sources: An algorithm for separation of convolutive mixtures. In *International Signal Processing Workshop on Higher Order Statistics*, pages 273–276, Chamrousse, France, July 1991.

[9] T. Kailath. *Linear systems.* Prentice Hall, 1980.

[10] A. Mansour. *Contributions à la séparation de sources.* PhD thesis, INPG Grenoble, 12-January 1997.

[11] A. Mansour and C. Jutten. Fourth order criteria for blind separation of sources. *IEEE Trans. on Signal Processing*, 43(8):2022–2025, August 1995.

[12] A. Mansour and C. Jutten. A simple cost function for instantaneous and convolutive sources separation. In *Actes du XVème colloque GRETSI*, pages 301–304, Juan-Les-Pins, France, 18-21 septembre 1995.

[13] A. Mansour and C. Jutten. A direct solution for blind separation of sources. *IEEE Trans. on Signal Processing*, 44(3):746–748, March 1996.

[14] A. Mansour, C. Jutten, and P. Loubaton. Subspace method for blind separation of sources and for a convolutive mixture model. In *Signal Processing VIII, Theories and Applications*, pages 2081–2084, Triest, Italy, September 1996. Elsevier.

[15] A. Mansour, A. Kardec Barros, M. Kawamoto, and N. Ohnishi. A fast algorithm for blind separation of sources based on the cross-cumulant and levenberg-marquardt method. In *Fourth International Conference on Signal Processing (ICSP'98)*, pages 323–326, Beijing, China, 12-16 October 1998.

[16] L. Nguyen Thi and C. Jutten. Blind sources separation for convolutive mixtures. *Signal Processing*, 45(2):209–229, 1995.

[17] G. Puntonet, C., A. Mansour, and C. Jutten. Geometrical algorithm for blind separation of sources. In *Actes du XVème colloque GRETSI*, pages 273–276, Juan-Les-Pins, France, 18-21 september 1995.