Analytical performance analysis for blind quantum source separation with time-varying coupling

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***Abstract*—Classical, i.e. non-quantum, blind source separation (BSS) methods estimate unknown source signals by using only mixed signals obtained by transferring these source signals through a mixing transform, which typically has unknown parameter values. We developed quantum versions of BSS, in which the mixing parameter values almost always remained fixed over time. We here show that rapidly varying mixing is much more complex to handle, because unknown *quantum* states cannot be cloned, i.e. copied. We avoid this issue thanks to a specific separating system based on a master-slave structure. We provide an original analytical analysis of the performance of that structure, depending on the mismatch between its master and slave inverting blocks.**

I. Prior work and problem statement

Within the information processing (IP) domain, various fields developed very rapidly during the last decades. One of these fields is Blind Source Separation (BSS), which led to various classes of methods, e.g. Independent Component Analysis (ICA) [2]. All BSS investigations were initially performed in a "classical", i.e. non-quantum, framework. Another growing field within the overall IP domain is Quantum Information Processing (QIP) [15]. QIP is closely related to Quantum Physics (QP). It uses abstract representations of systems whose behaviors are requested to obey the laws of QP. This made it possible to develop new and powerful IP methods, that manipulate the states of so-called quantum bits, or qubits.

In 2007, we bridged the gap between classical (B)SS and QIP/QP in [3], by introducing a new field, Quantum Source Separation (QSS), and especially its blind version (BQSS). The QSS problem consists of restoring (the information contained in) unknown individual source quantum states, eventually using only the mixtures (in SS terms [4]) of these states which result from their undesired coupling. It is thus closely related to system inversion. The blind (or unsupervised) version of this problem corresponds to the case when the parameter values of the mixing operator are initially unknown and are estimated by using only mixtures of source quantum states, i.e. without knowing these source states (see also [4] for (B)QSS applications).

We initially developed a first class of BQSS methods, based on a separation principle that has some relationships with classical ICA (see especially [3], [4], [6]). Then, in [5], [7], we introduced another class of BQSS methods, using a new separation principle based on the disentanglement of output quantum states of the separating system. This class of methods yields attractive features as compared with the previous one (see especially [12]). We then proposed modified BQSS methods also based on the above disentanglement principle: see [8], [9], [10], [11], [12].

The coupling/mixing parameter values remained fixed over time in all above-mentioned investigations of BQSS and of the associated so-called Blind Quantum Process Tomography (BQPT) [8], [9], [10], [11], i.e. blind quantum system identification. Very recently [13], we started to introduce the first extensions of these BQSS (and BQPT) approaches to configurations where these parameter values vary. Such configurations were somewhat studied for classical BSS. However, in [13], we especially showed that they are much more complex for BQSS when mixtures evolve rapidly over time. This is due to the quantum nature of the data, as explained further in this paper. In [13], we proposed a solution to that problem. However, that investigation was limited to the development of the *principles* of the proposed system, in terms of structure and associated adaptation procedure. We hereafter proceed much further, by providing a detailed analytical derivation of the *performance* of that approach, due to the possible mismatch between the so-called master and slave parts of the proposed separating system.

Within the overall information processing and transmission domain, the third field to be considered here is wireless communication. New challenges in this field and solutions are discussed in the survey [14]. One of these major challenges is still the radio frequency congestion. By looking at the US or European frequency allocation chart, one can realize the importance of this problem. To create new wireless applications, researchers are exploring several axes such as Dynamic Spectrum Allocation (DSA) and Opportunistic Spectrum Allocation (OSA) over licensed and non-licensed bands, or the use of mm waves. The mm waves could be a good solution for the Internet of Things (IoT) in site communication. However, mm waves can only be deployed for Line Of Sight (LOS) communication. Recent studies and projects have shown the efficiency of Free Space Optical (FSO) communication using visible or infrared light, or lasers. However, FSO is also suffering from LOS and other issues [1]. Moreover, although we are at an early stage of quantum signal processing and communication systems, engineers and scientists are hoping a lot of outcomes from such systems. In this framework, quantum extensions of blind system identification and inversion methods are of high interest, due to their close relationship with the blind equalization problem in classical wireless communication. We therefore investigate such extensions in this paper.

The remainder of this paper is organized as follows. We first define the considered quantum mixing/coupling model in Section II. Then, in Section III, we summarize the principles of the BQSS method intended for constant mixing that we proposed in [7] and that is used as the starting point in this paper. Moreover, we analyze its constraints in detail. The extension of the above BQSS method to the considered varying mixing is defined in Section IV, whereas its performance is analyzed in Section V. Conclusions are eventually drawn from this overall investigation in Section VI.

II. Mixing/coupling model

As stated above, computations of the field of QIP use qubits instead of classical bits [15]. In [5], we first detailed the required concepts for a single qubit and then presented the type of coupling between two qubits that we consider and that defines the "mixing model", in (B)SS terms, of our investigation. We hereafter summarize the major aspects of that discussion, which are required in the current paper.

A qubit with index i considered at a given time t_0 has a quantum state. If this state is pure, it belongs to a two-dimensional space $\mathcal{E}_i$ and may be expressed as

$$|\psi_i(t_0)\rangle = \alpha_i|+\rangle + \beta_i|-\rangle \tag{1}$$

in the basis of $\mathcal{E}_i$ defined by the two orthonormal vectors that we hereafter denote $|+\rangle$ and $|-\rangle$, whereas α_i and β_i are two complex-valued coefficients constrained to meet the condition

$$|\alpha_i|^2 + |\beta_i|^2 = 1 \tag{2}$$

which expresses that the state $|\psi_i(t_0)\rangle$ is normalized.

In the BQSS configuration studied in this paper, we first consider a system composed of two qubits, called "qubit 1" and "qubit 2" hereafter, at a given time t_0. This system has a quantum state. If this state is pure, it belongs to the four-dimensional space $\mathcal{E}$ defined as the tensor product (denoted $\otimes$) of the spaces $\mathcal{E}_1$ and $\mathcal{E}_2$ respectively associated with qubits 1 and 2, i.e. $\mathcal{E} = \mathcal{E}_1 \otimes \mathcal{E}_2$. We hereafter denote $\mathcal{B}_+$ the basis of $\mathcal{E}$ composed of the four orthonormal vectors $|++\rangle$, $|+-\rangle$, $|-+\rangle$, $|--\rangle$, where e.g. $|+-\rangle$ is an abbreviation for $|+\rangle\otimes|-\rangle$, with $|+\rangle$ corresponding to qubit 1 and $|-\rangle$ corresponding to qubit 2. Any pure state of this two-qubit system then reads

$$\begin{aligned} |\psi(t_0)\rangle &= c_1(t_0)|++\rangle + c_2(t_0)|+-\rangle \\ &\quad + c_3(t_0)|-+\rangle + c_4(t_0)|--\rangle \end{aligned} \tag{3}$$

and has unit norm. It may also be represented by the corresponding vector of complex-valued components in basis $\mathcal{B}_+$, which reads

$$C_+(t_0) = [c_1(t_0), c_2(t_0), c_3(t_0), c_4(t_0)]^T \tag{4}$$

where ^{T} stands for transpose. In particular, we study the case when the two qubits are independently prepared, i.e. initialized, with states defined by (1) respectively with $i = 1$ and $i = 2$. We then have

$$|\psi(t_0)\rangle = |\psi_1(t_0)\rangle \otimes |\psi_2(t_0)\rangle \tag{5}$$

$$\begin{aligned} &= \alpha_1\alpha_2|++\rangle + \alpha_1\beta_2|+-\rangle \\ &\quad + \beta_1\alpha_2|-+\rangle + \beta_1\beta_2|--\rangle. \end{aligned} \tag{6}$$

Besides, we consider the case when the two qubits, which correspond to two spins 1/2, have undesired coupling after they have been initialized according to (5). The considered coupling is based on the Heisenberg model with a cylindrical-symmetry axis presently collinear to the applied magnetic field. This common axis is chosen as the "quantization axis", called Oz. In basis $\mathcal{B}_+$, the evolution of this system's quantum state from t_0 to t is described by a matrix M specific to the considered type of coupling, by means of the relationship

$$C_+(t) = MC_+(t_0) \tag{7}$$

where $C_+(t)$ is the counterpart of (4) at time t and defines the coupled (or "mixed", in BSS terms) state $|\psi(t)\rangle$ of the two-qubit system at that time. For the considered type of coupling, our previous calculations show that

$$M = QDQ^{-1} = QDQ \tag{8}$$

with

$$Q = Q^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

and D equal to

$$\begin{bmatrix} e^{-\mathrm{i}\omega_{1,1}(t-t_0)} & 0 & 0 & 0 \\ 0 & e^{-\mathrm{i}\omega_{1,0}(t-t_0)} & 0 & 0 \\ 0 & 0 & e^{-\mathrm{i}\omega_{0,0}(t-t_0)} & 0 \\ 0 & 0 & 0 & e^{-\mathrm{i}\omega_{1,-1}(t-t_0)} \end{bmatrix} \tag{10}$$

where i is the imaginary unit. The four real (angular) frequencies $\omega_{1,1}$ to $\omega_{1,-1}$ in (10) depend on the physical setup and their values are unknown in practice. We hereafter first consider the case when these parameters, the interval $(t - t_0)$ between the preparation and use of the considered states, and hence the overall coupling model are "constant over time", i.e. take the same values for all successive pairs of source state and associated mixed state, respectively denoted as $|\psi(t_0)\rangle$ and $|\psi(t)\rangle$, where the values of t_0 and t are specific to each such pair.

III. A BQSS method for constant coupling

In [5], [7], [12], to uncouple qubit states mixed according to the above model with constant parameters, we introduced a BQSS method which is the basis for the extended method targeted at time-varying mixing considered further in this paper. Therefore, we first summarize the main features of this method for constant mixing hereafter.

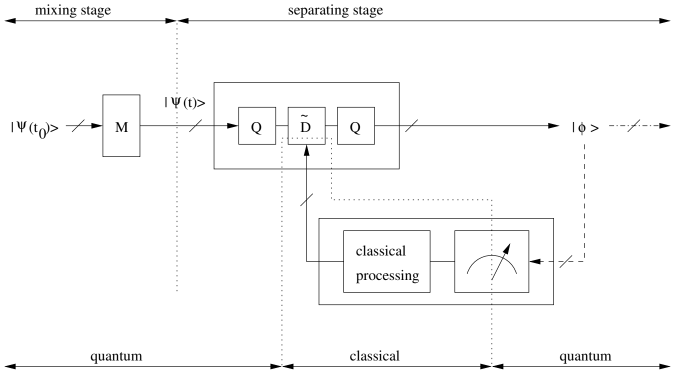

Fig. 1. Global (i.e. mixing + separating) configuration for constant mixing, including a quantum-processing inverting block and a classical-processing adapting block. Each quantum state $|\Phi\rangle$ is used only once (no cloning): see Section III-C.

A. Inverting block of separating system

The inverting block of the separating system is the part of this system which is to be used eventually (i.e. after this block has been adapted, as explained below) to derive the output quantum state $|\Phi\rangle$ of this system from its input quantum state, which is the above-defined coupled state $|\psi(t)\rangle$. That block appears in the upper right part of Fig. 1 and is used in the two BQSS methods considered in this paper. It uses quantum processing means only. The output quantum state of that block and therefore of our complete separating system is denoted as

$$|\Phi\rangle = c_1|++\rangle + c_2|+-\rangle + c_3|-+\rangle + c_4|--\rangle. \quad (11)$$

It may also be represented by the corresponding vector of components of $|\Phi\rangle$ in output basis $\mathcal{B}_+$, denoted as

$$C = [c_1, c_2, c_3, c_4]^T. \quad (12)$$

We then have

$$C = UC_+(t) \quad (13)$$

where U defines the unitary quantum-processing operator applied by our separating system to its input $C_+(t)$. As justified below, we choose this operator U to belong to the class defined by

$$U = Q\tilde{D}Q \quad (14)$$

$$\text{with} \quad \tilde{D} = \begin{bmatrix} e^{i\gamma_1} & 0 & 0 & 0 \\ 0 & e^{i\gamma_2} & 0 & 0 \\ 0 & 0 & e^{i\gamma_3} & 0 \\ 0 & 0 & 0 & e^{i\gamma_4} \end{bmatrix} \quad (15)$$

where γ_1 to γ_4 are free real-valued parameters.

B. Adapting block of separating system

The above type of inverting block was selected because it can perfectly restore the quantum source state $|\psi(t_0)\rangle$ for adequate values of its free parameters γ_1 to γ_4: setting them so that $\tilde{D} = D^{-1}$ yields $U = M^{-1}$, which results in $C = C_+(t_0)$ and $|\Phi\rangle = |\psi(t_0)\rangle$. However, the condition $\tilde{D} = D^{-1}$ cannot be used as a *practical* procedure for directly assigning $\tilde{D}$, because D is unknown. Instead, a procedure for adapting the parameters γ_1 to γ_4 of $\tilde{D}$ by using only one or several values of the available mixed state $|\psi(t)\rangle$ is therefore required, which corresponds to a *blind* (quantum) source separation problem.

The BQSS method developed to this end in [5], [7], [12] uses the output disentanglement separation principle introduced in those papers and based on the concept of quantum state entanglement. The resulting cost functions and algorithms are skipped here, due to space limitations. The associated structure of the complete separating system is shown in Fig. 1. It adapting block (see feedback path in lower part of Fig. 1) receives successive values of the output quantum state $|\Phi\rangle$ and first converts them into classical-form data by means of spin component measurements. Repeatedly creating such data yields estimates of probabilities associated with these spin components (see [5], [7], [12]). Classical processing is then applied to these estimates, to control γ_1 to γ_4.

C. Phases of operation

In the above method, the states $|\Phi\rangle$ derived by the inverting block of the separating system for successive values of the source state $|\psi(t_0)\rangle$ may be used in two ways: (i) as the final outputs of the complete separating system, (ii) as the inputs of the adapting block. This deserves special care, because these $|\Phi\rangle$ are *quantum* states, which must therefore fulfill the no-cloning theorem [15], which has no equivalent in the classical framework, and which is related to the "fan out" operation for the output of a circuit: *a single instance* of an unknown quantum state (here $|\Phi\rangle$) cannot be copied to be used as the inputs of several subsequent sub-systems. Hence, a single instance of $|\Phi\rangle$ cannot be sent both to the output of our complete separating system (dash-dotted line in rightmost part of Fig. 1) and to its internal adapting block (dashed line in lower part of Fig. 1).

In [5], [7], [12], we solved this problem for constant coupling by introducing a two-phase procedure defined as follows. This procedure first adapts the separating system parameters γ_1 to γ_4 (e.g. with the method mentioned in Section III-B), at this stage using the outputs $|\Phi\rangle$ of the inverting block only as the inputs of the adapting block. This corresponds to the "adaptation phase" in the operation of the separating system. One then freezes these parameters γ_1 to γ_4 and does not send states $|\Phi\rangle$ anymore to the adapting block. One then starts the "inversion phase" of the operation of the separating system. During this phase, the above fixed values of γ_1 to γ_4 thus remain such that, for any value of the source state $|\psi(t_0)\rangle$, the corresponding output $|\Phi\rangle$ of the inverting block is equal to that source state $|\psi(t_0)\rangle$ (possibly up to some indeterminacies due to the considered adaptation method, and assuming this method provided a relevant solution). During the inversion phase, these outputs $|\Phi\rangle$ of the inverting block are therefore used only as the outputs of the complete separating system, to be sent to a subsequent system.

Note that, with this basic version of our two-phase procedure, the source states $|\psi(t_0)\rangle$ first used to adapt the separating system are lost from the point of view of the final application, i.e. their estimates $|\Phi\rangle$ are not provided by the separating system to the subsequent system (see [13] for a more complex version, in which the states $|\psi(t_0)\rangle$ lost above are prepared again after adaptation, for inversion only, not to be lost).

IV. A BQSS METHOD FOR TIME-VARYING COUPLING

A. Considered configuration

The above approach was developed for the case when the mixing parameters $\omega_{1,1}(t-t_0)$ to $\omega_{1,-1}(t-t_0)$ remain constant over time. These parameters may progressively evolve in more complex situations, e.g. if there is a drift (i) of the applied magnetic field, upon which $\omega_{1,1}$ to $\omega_{1,-1}$ depend as shown in [4], or (ii) of the time interval, denoted as $(t-t_0)$ above, between the preparation of a source state $|\psi(t_0)\rangle$ and the use of the associated mixed state $|\psi(t)\rangle$. We hereafter address these more complex situations. We assume that the mixing parameters evolve slowly enough to be considered as constant over the short time period required for performing the above-defined adaptation phase intended for constant mixing. However, among the two cases that we started to investigate in [13], we here focus on the most complex one, which is when the mixing parameters evolve so quickly that the separating system parameters should be adapted as often as possible to track mixture evolution, that is, permanently (and without preparing "lost states" $|\psi(t_0)\rangle$ again after adaptation, for inversion only).

B. A method based on a master-slave structure

The separating system structure of Fig. 1 cannot be kept here, because it would permanently use the outputs $|\Phi\rangle$ only to adapt the inverting block, thus being unable to provide any restored source states $|\Phi\rangle$ to the final application, as explained in Section III-C. In [13], we solved this problem by introducing a different separating system architecture, which simultaneously creates two quantum states. This "master-slave" architecture, shown in Fig. 2, includes two inverting blocks which have the same structure. The output of one of these blocks, called the master inverting block (top-right part of Fig. 2), is used to control the adaptation of that block, using the same feedback loop as in Fig. 1. The control signals sent to the sub-block $\tilde{D}$ of that master inverting block thus again have a *classical* form. They can therefore be sent*, in addition,* to the sub-block $\tilde{D}$ of the slave inverting block (bottom-right part of Fig. 2). Both inverting blocks thus receive the "same" control signals[1] and, assuming they have the same dependence with respect to their control signals, they provide the same output state $|\Phi\rangle$ when both receiving the same mixed state $|\psi(t)\rangle$. The slave inverting block thus provides a free instance of the state $|\Phi\rangle$, which is used as the final output of our separating system and therefore e.g. sent to the subsequent quantum-processing circuit involved in applications which use our BQSS approach.

This approach yields two constraints. First, it is relevant only if one can implement two instances of the inverting block which have the same, or at least almost the same, behavior when they receive the same control signals. As detailed in [13], it is encouraging to know that a similar approach has been applied in the framework of *classical* processing. Moreover, for our BQSS problem, Section V below provides a detailed analysis of the influence of the mismatch between the master and inverting blocks, which was not studied at all in [13].

The other constraint entailed by the above approach is that two instances of the same mixed state $|\psi(t)\rangle$ should be available, in order to derive the corresponding two states respectively by means of the master and slave inverting blocks. As stated above, due to the no-cloning theorem, creating these *two* instances of $|\psi(t)\rangle$ is not obvious, as they cannot be derived by just copying a single instance of $|\psi(t)\rangle$. Among the two solutions to this problem proposed in [13], we here consider the one shown in the left-hand part of Fig. 2. Briefly, it consists of starting from *classical-form* data, which can therefore be copied and then converted *twice* into quantum-form data, then eventually yielding *two* instances of states $|\psi(t_0)\rangle$ and $|\psi(t)\rangle$.

V. ANALYTICAL PERFORMANCE ANALYSIS

As explained above, the practical performance of the proposed master-slave approach for BQSS described in Section IV-B depends on the "mismatch" between the actual (hereafter arbitrary) parameter values of the master inverting block, denoted as γ_{1m} to γ_{4m} below and obtained by adapting that block, and those of the slave inverting block, denoted as γ_{1s} to γ_{4s} below, when these two blocks receive the same control signals. In this section, we derive original analytical expressions defining the influence of the above mismatch on resulting errors related to the output state $|\Phi\rangle$ of the separating system. To this end, we consider the case when

$$\gamma_{js} = \gamma_{jm} + \mu\mu_j \qquad \forall\ j \in \{1, \ldots, 4\} \tag{16}$$

where μ is a fixed positive factor and each value μ_j defines the specific value of the mismatch for each parameter γ_{js}. One may either consider a fixed value for each of these μ_j or define μ_j as a zero-mean random variable describing the statistics of the considered mismatch phenomenon. In the latter case, one may constrain μ_j to be normalized in some way, e.g. to have unit variance or to be uniformly distributed over the normalized interval $[-1/2, 1/2]$. The parameter μ then defines the overall magnitude of the mismatches for all parameters γ_{js} (which is the reason why μ is not absorbed in the definition of the mismatch variables μ_j in (16)).

As shown in [12], for a given source state $|\psi(t_0)\rangle$, the coefficients of the output state (11) of the *master* inverting block (therefore here denoted with a subscript "m") read

[1] More precisely, the complete time axis is here split into adjacent short time periods, and adaptation is performed separately during each of these periods. The control signals of the master and slave inverting blocks are the same at the end of each of these adaptation phases, due to the following operation. For each of these phases, the control signals of the master adapting block may evolve during that phase, precisely to perform adaptation (e.g. using the algorithm mentioned in Section III-B). At the end of this adaptation phase, their final values are the result of that phase. This result is sent to the slave adapting block, so that it uses it during the next adaptation phase. This result is thus the set of control signals which is "the same for", i.e. shared by, the master and slave adapting blocks.

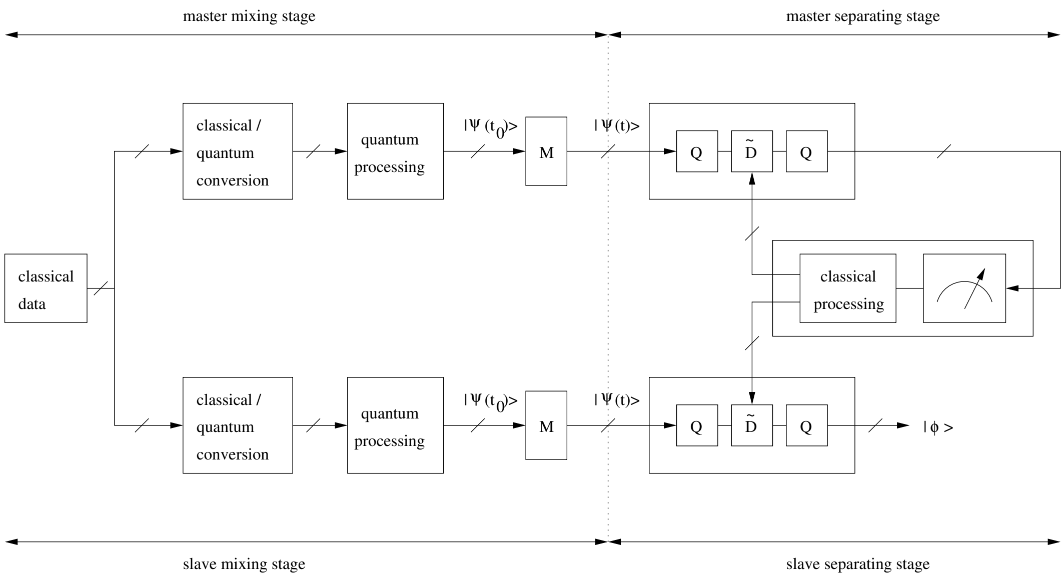

Fig. 2. Global configuration using a master-slave structure.

$$
\begin{aligned}
c_{1m} &= \alpha_1\alpha_2 e^{\mathrm{i}\delta_{1m}} && (17)\\
c_{2m} &= \frac{1}{2}e^{\mathrm{i}\delta_{2m}}\left[(\alpha_1\beta_2+\beta_1\alpha_2)\right.\\
&\qquad \left.+(\alpha_1\beta_2-\beta_1\alpha_2)e^{\mathrm{i}(\delta_{3m}-\delta_{2m})}\right] && (18)\\
c_{3m} &= \frac{1}{2}e^{\mathrm{i}\delta_{2m}}\left[(\alpha_1\beta_2+\beta_1\alpha_2)\right.\\
&\qquad \left.-(\alpha_1\beta_2-\beta_1\alpha_2)e^{\mathrm{i}(\delta_{3m}-\delta_{2m})}\right] && (19)\\
c_{4m} &= \beta_1\beta_2 e^{\mathrm{i}\delta_{4m}} && (20)
\end{aligned}
$$

with

$$\delta_{jm} = \gamma_{jm} - \gamma_{jd} \qquad \forall\ j \in \{1,\ldots,4\} \tag{21}$$

where we define the "desired values" of γ_{1m} to γ_{4m} as

$$
\begin{aligned}
\gamma_{1d} &= \omega_{1,1}(t-t_0) && (22)\\
\gamma_{2d} &= \omega_{1,0}(t-t_0) && (23)\\
\gamma_{3d} &= \omega_{0,0}(t-t_0) && (24)\\
\gamma_{4d} &= \omega_{1,-1}(t-t_0). && (25)
\end{aligned}
$$

The BQSS method defined in [7] and [12] (BQPSS-D1 method) for adapting all γ_{jm} parameters was shown to yield some phase indeterminacies for the above coefficients c_{1m} to c_{4m}. Therefore, we hereafter consider only their moduli. Tedious calculations then show that they read

$$
\begin{aligned}
|c_{1m}| &= |\alpha_1\alpha_2| && (26)\\
|c_{23m}| &= \frac{1}{\sqrt{2}}\left[(A_1^2+A_2^2)+\cos(\delta_{3m}-\delta_{2m})\varepsilon_{23}(A_1^2-A_2^2)\right.\\
&\qquad \left.+\sin(\delta_{3m}-\delta_{2m})2\varepsilon_{23}A_1A_2\sin(\xi_2-\xi_1)\right]^{1/2} && (27)\\
|c_{4m}| &= |\beta_1\beta_2| && (28)
\end{aligned}
$$

where we use the polar representations

$$
\begin{aligned}
\alpha_1\beta_2 &= A_1 e^{\mathrm{i}\xi_1} && (29)\\
\beta_1\alpha_2 &= A_2 e^{\mathrm{i}\xi_2} && (30)
\end{aligned}
$$

and where the compact notation c_{23m} means that (27) yields c_{2m} if $\varepsilon_{23}=1$ and c_{3m} if $\varepsilon_{23}=-1$.

The output of the *slave* inverting block also yields (26)-(28) with (21)-(25), except that all subscripts "m" are replaced by "s" (for ε_{23}, the same value, and hence notation, is used both in the master and slave inverting blocks, because we hereafter compare the two values of a coefficient c_j with the same index j in both inverting blocks). Combining the "slave version" of (27) with (16) then allows one to express output parameters of the slave inverting block with respect to those of the master block and to the mismatch between them. This yields

$$
\begin{aligned}
|c_{23s}| &= \frac{1}{\sqrt{2}}\left[(A_1^2+A_2^2)\right.\\
&\quad +\cos(\delta_{3m}-\delta_{2m}+\mu(\mu_3-\mu_2))\varepsilon_{23}(A_1^2-A_2^2)\\
&\quad +\sin(\delta_{3m}-\delta_{2m}+\mu(\mu_3-\mu_2))2\varepsilon_{23}A_1A_2\\
&\quad \left.\times\sin(\xi_2-\xi_1)\right]^{1/2}. && (31)
\end{aligned}
$$

For any index j of a coefficient c_j, with $j \in \{1,\ldots,4\}$, the signed error between the master and slave values of the modulus of this coefficient c_j is then defined as

$$\Delta|c_j| = |c_{js}| - |c_{jm}|. \tag{32}$$

The master and slave versions of (26) and (28) then show that $\Delta|c_j|$ is equal to zero for c_1 and c_4. On the contrary, the two non-zero values of $\Delta|c_j|$ for c_2 and c_3, compactly denoted as $\Delta|c_{23}|$ hereafter, are obtained by inserting (27) and (31) in (32). This shows that these errors are nonlinear functions of the parameter μ which defines the magnitude of the mismatch between the master and slave inverting blocks.

This was expected because μ may be arbitrarily increased in this theoretical analysis, whereas the moduli of the coefficients c_j are bounded to one (since they are coefficients of a quantum pure state) and $\Delta|c_j|$ is therefore bounded.

When the mismatch variables μ_j are considered to be random, one may then calculate the statistical average of each squared above-defined error $\Delta|c_j|$. The corresponding root mean squared error (RMSE) then reads

$$RMSE(|c_j|) = \sqrt{E\{(\Delta|c_j|)^2\}} \quad \forall\, j \in \{1, \ldots, 4\} \quad (33)$$

where $E\{.\}$ stands for expectation. This error is thus defined for a single source state $|\psi(t_0)\rangle$ and a single set of parameters γ_{jm} in the master inverting block. Statistical averages over random distributions of these parameters may also be defined in the same way as above.

A case of high practical interest is when the mismatch, and hence μ, are low. In this case, we derive an approximation of the above errors $\Delta|c_{23}|$ by first developing the cosine and sine functions of $(\delta_{3m} - \delta_{2m} + \mu(\mu_3 - \mu_2))$ in (31), so as to extract $\mu(\mu_3 - \mu_2)$, and by then deriving a power series expansion with respect to the latter quantity. Lengthy calculations thus yield

$$\Delta|c_{23}| = \mu(\mu_3 - \mu_2)F + \mathcal{O}([\mu(\mu_3 - \mu_2)]^2) \quad (34)$$

with a factor F defined as

$$\begin{aligned} F \;=\; & \frac{1}{2\sqrt{2}}\varepsilon_{23}[-\sin(\delta_{3m} - \delta_{2m})(A_1^2 - A_2^2) \\ & + \cos(\delta_{3m} - \delta_{2m})2A_1A_2\sin(\xi_2 - \xi_1)] \\ & /[(A_1^2 + A_2^2) + \varepsilon_{23}\{\cos(\delta_{3m} - \delta_{2m})(A_1^2 - A_2^2) \\ & + \sin(\delta_{3m} - \delta_{2m})2A_1A_2\sin(\xi_2 - \xi_1)\}]^{1/2}. \quad (35) \end{aligned}$$

Using (33), the corresponding RMSE reads

$$RMSE(|c_{23}|) \simeq \mu|F|\sqrt{E\{(\mu_3 - \mu_2)^2\}}. \quad (36)$$

For small values of the mismatch parameter μ (and when $F \neq 0$), this error therefore grows linearly with respect to μ. Moreover, if μ_2 and μ_3 are uncorrelated zero-mean random variables,

$$E\{(\mu_3 - \mu_2)^2\} = E\{\mu_3^2\} + E\{\mu_2^2\}. \quad (37)$$

In particular, if μ_2 and μ_3 are normalized in terms of unit variance, then $E\{(\mu_3 - \mu_2)^2\} = 2$. If, instead, μ_2 and μ_3 are uniformly distributed over the normalized interval $[-1/2, 1/2]$, then $E\{(\mu_3 - \mu_2)^2\} = 1/6$. For the above first-order approximations of errors too, statistical averages over random values of $|\psi(t_0)\rangle$ and/or γ_{jm} may also be computed.

Moreover, the above errors were calculated for arbitrary values of the γ_{jm} parameters, for the sake of generality. Specific results may then be derived from above, by focusing on the case when the considered adaptation method, applied to the master inverting block, exactly converges to one of the desired points. As shown in [7], [12], this corresponds to

$$\delta_{3m} - \delta_{2m} = m\pi \quad (38)$$

where m is an integer. The associated errors are straightforwardly obtained by inserting (38) in (32), (33) (34) and (36). Their expressions are skipped here, due to space limitations.

VI. Conclusion

Blind Quantum Source Separation methods were almost only developed for a constant mixing (i.e. coupling) operator. In this paper, we showed that rapidly varying mixing is much more complex to handle, because unknown *quantum* states cannot be cloned, i.e. copied. For such mixing, we especially provided an original analytical analysis of the performance of the master-slave structure that we proposed for the separating system, depending on the mismatch between its master and slave inverting blocks. Our future works will especially consist of developing a software simulation of this system, to numerically validate the above analytical mismatch analysis.

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Erratum: replace two terms $E\{r_i\}E\{q_i\}$ in (33) of [3] by $E\{r_i q_i\}$, since q_i depends on r_i.

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